## SENIOR FIVE INTRODUCTION TO NUMERICAL METHODS

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## CHAPTER ONE: LINEAR INTERPOLATION AND EXTRAPOLATION

If the two known points are given by the coordinates ( $x_{0}, y_{0}$ ) and ( $x_{1}, y_{1}$ ), the linear interpolant is the straight line between these points. For a value $x$ in the interval $\left(x_{0}, x_{1}\right)$, the value $y$ along the straight line is given from the equation

$$
\frac{y-y_{0}}{x-x_{0}}=\frac{y_{1}-y_{0}}{x_{1}-x_{0}}
$$

Note: The gradient between $\left(x_{0}, y_{0}\right)$ and ( $\mathrm{x}, \mathrm{y}$ ) is equal to the gradient between ( $x_{0}, y_{0}$ ) and ( $x_{1}, y_{1}$ )

http://en.wikipedia.org/wiki/Linear_interpolation

Solving this equation for $y$, which is the unknown value at $x$, gives

$$
\begin{aligned}
& y-y_{0}=\left(\mathrm{x}-\mathrm{x}_{0}\right) \frac{y_{1}-y_{0}}{x_{1}-x_{0}} \\
& y=y_{0}+\left(y_{1}-y_{0}\right) \frac{x-x_{0}}{x_{1}-x_{0}}
\end{aligned}
$$

Note: If the unknown value is not in between two known values but next to these then we still make use of the gradient method to find the unknown but this is then called Linear extrapolation. The line is extrapolated to cover the next point.

Exercise:

1. Given the values below:

| x | 0 | 5 | 10 | 15 | 20 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| t | 0 | 12 | 25 | 39 | 54 |

Use linear interpolation or extrapolation to find: (i). t when $\mathrm{x}=12$, (ii) x when $\mathrm{t}=45$, (iii) t when $\mathrm{x}=23$, (iv) x when $\mathrm{t}=60$.
2. The postal system charges airmail going overseas according to their weights. The standard weights are $200 \mathrm{mg}, 400 \mathrm{mg}$ and 600 mg . The charges are $700 /=, 1200 /=$ and $3000 /=$ respectively.
(i). How much does a mail weighing 520 mg cost?
(ii). A person has $3100 /=$, what maximum weight of the parcel is acceptable from her for posting.
3.(a) The table below gives the values of angles and their corresponding sine function:

| x | $28.1^{0}$ | $28.2^{0}$ | $28.3^{0}$ | $28.4^{0}$ |
| :--- | :--- | :--- | :--- | :--- |
| $\sin \mathrm{x}$ | 0.4710 | 0.4726 | 0.4741 | 0.4756 |

Using linear interpolation or extrapolation find: (i). $\sin ^{-1}(0.7431)$ (ii) $\sin ^{-1}(0.47097)$, (iii). $\sin \left(28.18^{\circ}\right) \quad$ (iv). $\sin \left(28.49^{\circ}\right)$

## CHAPTER TWO: APPROXIMATION OF ROOTS:

## (i) Graphical Method.

If the graph of $y=f(x)$ is drawn, then the root, $x_{0}$, lies at the point where it cuts the $x$-axis.
This is because the curve crosses from either the positive $y$ values to the negative $y$ values or otherwise past a value of $\mathrm{y}=0$ whose corresponding x value is the root. (See graphs below)


$\mathrm{f}(\mathrm{a})<0$ and $\mathrm{f}(\mathrm{b})>0$
(From negative to positive)
$\mathrm{f}(\mathrm{a})>0$ and $\mathrm{f}(\mathrm{b})<0$
( From positive to negative)

When we construct an accurate graph as shown below the value of the root $\mathrm{x}_{0}$ is clearly seen and can be read off from here.


## Exercise:

1. Draw a suitable table to locate the intervals within which the roots of $x^{2}+x-3=0$ lie. Use graphical method to obtain the roots.
2. Obtain the roots of the equation $x-2 \sin x=0$ in the interval 0 to $\pi$ using graphical method. (Work in radians)
3. Show graphically that $\operatorname{Sin} \mathbf{x}=\mathbf{1} / \mathbf{x}$ has two roots in the interval $0<\mathrm{x}<\pi$. Obtain the roots. Two graphs can be drawn for $\mathbf{y}_{\mathbf{1}}=\operatorname{Sin} \mathbf{x}$ and $\mathbf{y}_{\mathbf{2}}=\mathbf{1} / \mathbf{x}$ and find the point of intersection of the graphs or draw one graph of $\mathbf{y}=\operatorname{Sin} \mathbf{x}-\mathbf{1 / x}$ and find where it cuts the x -axis. In each case we read off the $x$-value as our root.
( Work in radians)
4. Solve graphically the root of the equation $\ln x+x-2=0$ in the interval $1 \leq x \leq 2$.

## (ii) Approximation of roots by linear interpolation.

When the interval within which the root lies is known, the value $\mathrm{x}_{0}$ for which $\mathrm{f}\left(\mathrm{x}_{0}\right)=0$ can be calculated from linear interpolation. It can be improved upon by testing the sign of the calculated value and applying the interpolation again on the side with different signs. This process is repeated until a more accurate value is attained. However unless requested, calculations should be done just once.

## Example:

Show that the root of $x^{3}-3 x-12=0$ lies between 2 and 3 . Hence use linear interpolation to calculate the root to 3d.ps.
Let $f(x)=x^{3}-3 x-12=0$. $f(2)=8-6-12=-10<0, f(3)=27-9-12=6>0$, thus change in signs implies existence of a root between 2 and 3 .

| $x$ | 2 | $x_{0}$ | 3 |
| :--- | :--- | :--- | :--- |
| $f(x)$ | -10 | 0 | 6 |

$\frac{x_{0}-2}{0-{ }^{-10}}=\frac{3-2}{6-10}$
$\Rightarrow x_{0}=2+10 / 16, x_{0}=2.625$. For a better approximation we find $f(2.625)=-1.787$ which implies that the root is between 2.625 and 3 because of the sign change. We perform a second interpolation in the interval $[2.625,3]$.
$\Rightarrow \mathrm{x}_{1}=2.7111$. Likewise, $\mathrm{f}(2.7111)=-0.2065=>$ root lies in the interval $[2.7111,3]$

Continue from here $\qquad$ until when the x values are consistent to 3 d.p.s

