SENIOR SIX HOLIDAY REV ISION EXERCISES MAY 2019

APPLIED MATHEMATICS(Mechanics, Numerical and Statistics)

MECHANICS;*Use* $g=9.8ms^{-2}$ *where applicable.*

1(a) Two particles are travelling along a straight line AB of distance 20metres apart. At the same instant one particle starts from rest at A and moves towards B with a constant acceleration of $2ms^{-2}$ and the other particle starts from rest at B and moves towards A with a constant acceleration of $5ms^{-2}$.Find how far from A do the particles meet. Ans. 40/7 m

(b)A car moving in a straight line with a constant acceleration covers a distance of r metres during the fourth second of its motion and q metres during the fifth second of its motion. Find in terms of r and q its (i)acceleration, (ii)initial velocity.(iii)distance covered in the sixth second of its motion.

2(a)A ball is projected vertically upwards with a speed of $25ms^{-1}$. Four seconds later another ball is projected vertically upwards exactly from the same point and at the same speed. If the balls collide and $g=10ms^{-2}$, calculate the height at which the collision takes place.

(b)A stone is thrown vertically upwards with a speed of u ms^{-1}. T seconds later, another stone is projected vertically upwards from the same point with the same speed. If the stones meet in the air space and g is acceleration due to gravity, show that the height at which the stones meet is $\frac{4u^2-g^2T^2}{8g}$ metres.

3(a)An object with position vector 6 **i**-8 **k** moves with a constant speed of $5\sqrt{17}$ m/s in the direction 2 **i**-2 **j**+3 **k**. Find its distance from the origin after 2 seconds.

(b)A particle of mass 3kg initially at rest and located at a point A(1,0.-2) is acted upon by a force of 3t **i**-6 **j**+12t **k** Newtons for a time of t seconds. Find the

(i) speed of the particle after one second (ii) distance covered by the particle after one second.(iii) distance of the particle from the origin after one

second(iv)work done by the force in one second.(v)power developed in one second.

4(a)A rectangle ABCD is acted on by forces 4N, 6N and 8N along the sides BA, CD and CB respectively. The order of the letters show the directions of the given forces. If side AB= 5cm and AD= 3cm, find the magnitude and direction of the resultant force.

(b)ABCDEF is a regular hexagon in which forces of magnitudes 2N,3N,4N and 5N act along AC,AE, AF and ED respectively. The order of the letters indicate the directions of the forces. If AD is horizontal, find the magnitude and direction of the resultant force.

5(a)Point A is vertically below point B. A particle of mass 0.2kg is projected from A vertically upwards with a speed of 20m/s and passes through point B with a speed of 8m/s. Use the principle of conservation of energy to find distance AB. Ans. 17.14m

(b)In each minute a pump draws a volume of $2.4m^3$ of water from a well 5m below the ground, and issues it at ground level through a pipe of cross – sectional area $0.005m^2$. If the density of water is $1000kgm^{-3}$, find the (i) speed with which the water leaves the pipe. Ans.8m/s

(ii) power output of the pump. Ans. $3.24x10^3$ w (iii) power in put of the pump if the pump is 75% efficient.ans. $4.32x10^3$ w

6 .A mass of 2kg lies on a rough plane which is inclined at an angle of 30^0 to the horizontal. One end of a light inextensible string is attached to this mass and the string passes up the line of greatest slope and over a smooth pulley fixed at the top of the slope; a freely suspended mass of 5kg is attached to its other end .The system is released from rest as the 2kg mass accelerates up the slope , it experiences a constant resistance of 14N down the slope . Find the
(a)acceleration of the masses and the tension in the string.ans;$3.6ms^{-2}$,31N

(b)reaction of the pulley on the string. Ans; $31\sqrt{3}$ N

7. A fixed pulley carries a string which has a load of mass 8kg attached to one end and a light pulley to the other end. This light pulley carries another string which has a load of mass 5kg at one end and another load of mass 3kg at the other end. If $g=10ms^{-2}$ and the system is released from rest, calculate the (a)acceleration of the 3kg mass. (b)tension in the strings.

8(a)At time t=0 a particle is projected from a point O on a horizontal plane with a speed 14m/s in a direction inclined at $\tan^{-1}\frac{3}{4}$ above the horizontal. The particle just clears the top of a vertical wall the base of which is 8m from O. Calculate the (i)time at which the particle passes over the wall. Ans;5/7 seconds

(ii)height of the wall .ans; 3.5m

(b)A girl projects a ball from a height of 1.5m above the level ground with a speed of 10m/s and hits a bottle standing on a wall 4m high and 5m from her. If $g=10ms^{-2}$, find the two possible angles of projection. 45^0 ,71.6^0.

If the bottle has a horizontal velocity of 3m/s after being struck by the ball, find where it hits the ground. Ans; $\frac{6\sqrt{5}}{5}$ metres from the wall.

9. A particle of weight 40N is suspended by a light inextensible string from a light ring. The ring can slide along the rough horizontal rod. The coefficient of friction between the rod and the ring is ½. A force acting on the particle upwards at an angle of 30^0 to the horizontal, keeps the particle in equilibrium with the ring about to slide. Find the acting force and the tension in the string.

10. A particle of mass 3kg is attached to one end of a light elastic string .The other end of the string is attached to point O on a rough inclined plane at 30^0 to the horizontal. The particle is released from O and slides down the plane until it comes to sudden rest at point A. The natural length of the string is 1m, coefficient of friction 0.1 and modulus of elasticity is 30N.Using the principle of conservation of energy, find the extension of the string when the particle comes to sudden rest.

NUMERICAL;

9(a) By trapezium rule, use six strips to estimate $\int_0^2 (x+\cos x)dx$,correct to four significant figures. (b)Find the actual value of $\int_0^2 (x+\cos x)dx$,correct to four significant figures. (c)Find the relative error in part (a) above and state how you can reduce the error.

10. Use Newton –Raphson method to derive a formula for finding the root of the equation 10cosx-x=0. If the initial estimation of the root is 1.57, estimate the root of the equation ,correct to three decimal places.

11.Show that the root of the equation x-2sinx=0 lies between 1 and 2.Use linear interpolation once to find the initial approximation of the root,correct to 2d.places.Hence by Newton- Raphson , find the root of the equation ,correct to three d.places.

STATISTICS;

12. The table below shows the ages(yrs) of some of the people in a town council.

Ages	<5	<10	<15	<20	<25	<30
Frequency	4	8	7	10	6	5

(a) Calculate the (i)mean and standard deviation. (ii)number of people whose age is below the mean age.(b)Draw an ogive and use it to estimate the 80% middle range of the ages of the distribution.

13.The continuous random variable X has a probability distribution function

$$f(x)=\begin{cases} cx^2;0\le x\le 2 \\ 2c(4-x);2\le x\le 4 \end{cases}$$

Find (a) the value of c and sketch f(x) (b)the median value of x(c)the cumulative probability function, F(x) and P(1.5<x<3.5) (c)the mean and variance of x.