PHYSICS

HEAT TRANSFER

CONDUCTION

Conduction is the flow of heat through matter from hotter regions to colder regions without movement of matter as a whole.

Mechanism of heat conduction

(b) In non-metals (poor conductors)

When a non-metal is heated at one end, the atoms at the heated end gain kinetic energy and vibrate with greater amplitude. This vibrational energy is transmitted through the atomic bond to the neighboring atoms forming a wave that transmits energy from one end of the non-metal to the other end.

(a) In metals (good conductors)

When a metal is heated at one end, the atoms at the heated end gain kinetic energy and vibrate with greater amplitude. This vibration energy is transmitted through the atomic bonds to the neighboring atoms forming a wave that transmits energy from the end of the metal to the other end.

Metals also posses delocalized (free mobile) electrons. When a part of a metal is heated, these free electrons gain thermal energy and their velocities increase. They distribute this energy by collision with positive ions in the lattice and increase the ions' vibrational energy. Because electrons are light, they are able to move quickly to the cooler parts of the solid. So this mode of heat transfer is much faster.

Question: Why are metals better conductors of heat than non-metals?

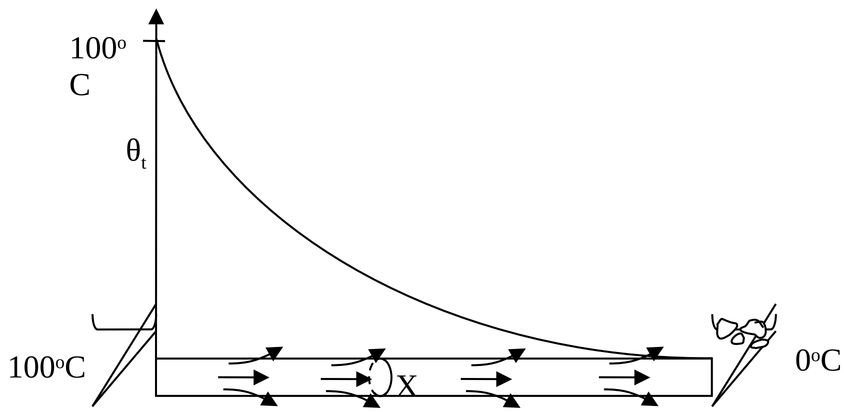
In metals, heat is transmitted by movement of free mobile electrons and by inter-atomic vibrations. In non-metals, heat is transmitted only through inter-atomic vibrations since non-metals don't have free electrons.

Since electrons are very small and highly mobile, heat transfer due to motion of electrons is faster than heat transfer due to interatomic vibrations. Therefore, metals are better conductors of heat than non-metals.

Temperature Distribution along a Conductor.

(a). Along an unlagged good conductor

The temperature distribution is not uniform along the conductor because of the heat lost to the Surroundings



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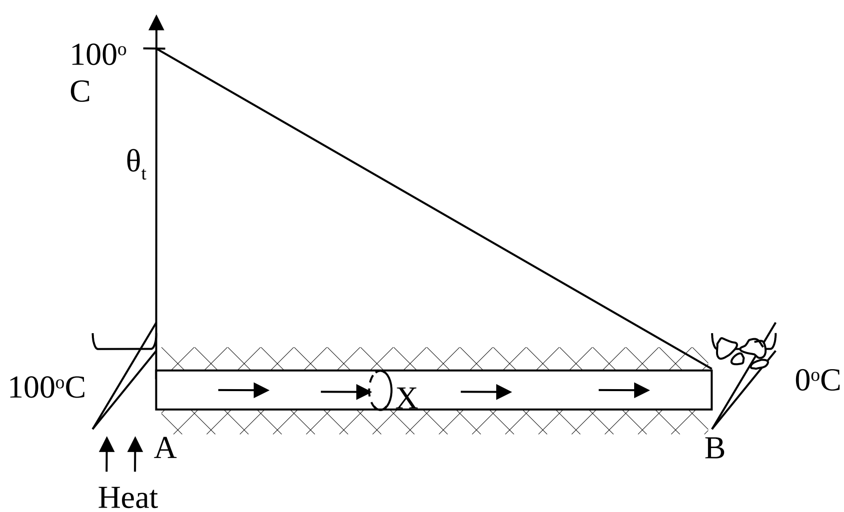
(b) Lagged Bar

The temperature distribution is uniform along the conductor because no is heat lost to the surrounding.

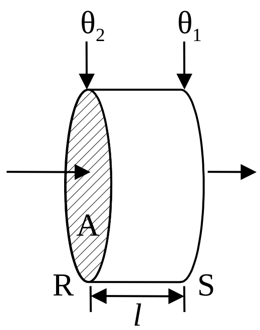


The difference between the temperature distributions is due to the fact that, when the bar is not lagged, heat escapes through its sides by convection. Thus, the heat flowing per second past a cross-section like X, is less than that entering at A. Therefore the heat flow rate through a section in the bar decreases from the hot end to the cold along the bar.

When the bar is lagged, the escape of heat through the sides is negligible so that the heat flow rate along the bar is constant.

The temperature gradient along the bar is greatest where the heat flow rate is greatest.

Factors that determine the rate of heat flow.



Consider a parallel-sided slab of material of thickness *l*, and area A, with face R at temperature θ_2 and face S at temperature θ_1, where $\theta_2 > \theta_1$.

The temperature gradient across the slab is

$$\frac{\theta_2-\theta_1}{l}$$

Assuming there is no heat loss through the sides of the slab, the heat flow rate through the slab is proportional to the cross-sectional area, A and to the temperature gradient.

$$i.e \frac{Q}{t} \propto A\left(\frac{\theta_2-\theta_1}{l}\right)$$

$$\therefore \frac{Q}{t} = kA\left(\frac{\theta_2-\theta_1}{l}\right)$$

where k is a constant known as ***thermal conductivity*** for the material of the slab.

We may write

$$k=\frac{Q/t}{A\left(\frac{\theta_2-\theta_1}{l}\right)}$$

Hence, thermal conductivity can be defined as ***the heat flow rate per unit area per unit temperature gradient***.

Definition:

Coefficient of thermal conductivity is the rate of heat flow normal to opposite faces of a one metre cube of material when the temperature difference across its faces is one kelvin.

The S.I unit of thermal conductivity is $Wm^{-1}K^{-1}$

Therefore the factors that affect rate of heat flow are;

(i) Temperature gradient. $\left(\frac{\theta_2-\theta_1}{l}\right)$

(ii) Cross-section area (A).

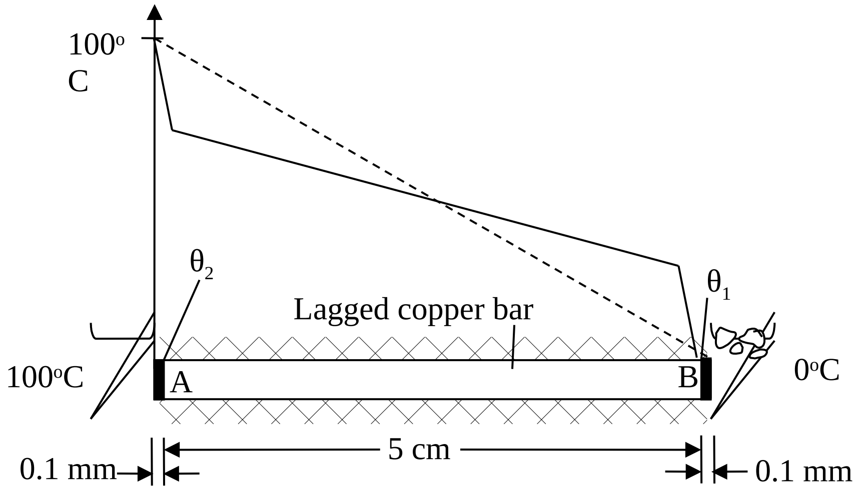
(iii) Nature of the material used (K).

In the table below are some examples of thermal conductivities:

Material	Copper	Silver	Lead	Glass	Water	Air
Conductivity ($Wm^{-1}K^{-1}$)	385	410	35	0.78	0.556	0.024

Effect of a Thin Layer of Bad Conductor at the ends of a good conductor.

We shall take an example of a lagged bar of thermal conductivity k, cross-sectional area A and length 5 cm, coated with a 0.1 mm layer of dirt at each end.



End A of the bar is not at 100°C, and neither is end B at 0°C. To see this, suppose that the conductivity of the dirt is a thousandth that of copper. Then the heat flow rate per second is

$$\frac{Q}{t}=\frac{kA}{1000}\frac{(100-\theta_2)}{0.1x\,10^{-3}}=kA\frac{(\theta ¿¿ 2-\theta_1)}{0.05}=\frac{kA}{1000}\frac{(\theta_1-0)}{0.1x\,10^{-3}} ¿$$

from which $100-\theta_2 = 2\theta_2-\theta_1 = \theta_1$

Thus, $\theta_1 = 40^oC$ and $\theta_2 = 60^oC$

This is why scale, deposited in boilers, water heaters, etc greatly reduces heat transfer.

In the same way, in winter, when a layer of ice forms on the surface of a pond it greatly reduces heat transfer from the water underneath so that aquatic life continues to thrive throughout the season.

Rate of heat flow through a composite wall

Imagine a composite wall consisting of materials A, B and C. If there is no heat loss through the sides, the heat flow rate per unit area is the in each material.

Let their respective thermal conductivities be k_A, k_B and k_C and their respective thicknesses l_A, l_B and l_C and the external temperatures be θ_1 and θ_4, where $\theta_1 < \theta_4$. Then

$$q=k_A\frac{(\theta ¿¿ 4-\theta_3)}{l_A}\Rightarrow\theta_4-\theta_3=\frac{q\,l_A}{k_A}\ldots\ldots\ldots\ldots(1)¿$$

$$q=k_B\frac{(\theta ¿¿ 4-\theta_3)}{l_B}\Rightarrow\theta_3-\theta_2=\frac{q\,l_B}{k_B}\ldots\ldots\ldots\ldots(2)¿$$

$$q=k_C\frac{(\theta ¿¿2-\theta_1)}{l_C} \Rightarrow \theta_2-\theta_1=\frac{q\,l_C}{k_C}\ldots\ldots\ldots\ldots(3)¿$$

Eq(1) + Eq(2) + Eq(3) gives

$$\theta_4-\theta_1=q\left(\frac{l_A}{k_A}+\frac{l_B}{k_B}+\frac{l_C}{k_C}\right)$$

$$q=\frac{(\theta ¿¿4-\theta_1)}{\left(\frac{l_A}{k_A}+\frac{l_B}{k_B}+\frac{l_C}{k_C}\right)}¿$$

Thus, only the outside temperatures, the thicknesses and the respective conductivities need to be known for the rate of heat flow through a composite wall to be known.

Exercise:

1. A composite wall is formed of a 2.5cm copper plate (k_C = 385 $Wm^{-1}K^{-1}$), a 3.2mm layer of asbestos (k_A = 0.75 $Wm^{-1}K^{-1}$), and a 5cm layer of fibre glass (k_G = 0.038 $Wm^{-1}K^{-1}$). This is subjected to an overall temperature difference of 600^{o}C. Calculate the heat flow rate per unit area through the composite structure.

2. Calculate the theoretical percentage change in heat loss by conduction achieved by replacing a single-sheet glass window by a double-sheet window consisting of two galss sheets separated by 10 mm of air. In each case glass is 2 mm thick.
[Ratio of the thermal conductivities of glass and air is 3:1]
3. A closed metal vessel contains water (i) at 30^{o}c and then (ii) at 75^{o}c . The vessel has a surface area of 0.5m^2 and a uniform thickness of 4mm. If the outside temperature is 15^{o}c, calculate the heat loss per minute by conduction in each case. {Thermal conductivity of metal = 400$Wm^{-1}K^{-1}$}

4. A bar 0.20m in length and cross-sectional area 2.5 x $10^{-4}m^2$ is ideally lagged. One end is maintained at100^{0}C while the other end is maintained at 0^0c by immersion in melting ice. Calculate the rate at which the ice melts owing to the flow heat along the bar.

(*Thermal conductivity of the metal bar = 4.0 x $10^2Wm^{-1}K^{-1}$,*

specific latent heat fusion of ice = 3.4 x 10^5Jkg^{-1})

4. Estimate the rate of heat loss from a room through a glass window of area 2m^2 and thickness 3mm when the temperature of the roomis 20^{0}c and that of the the air outside is 5^{0}c .

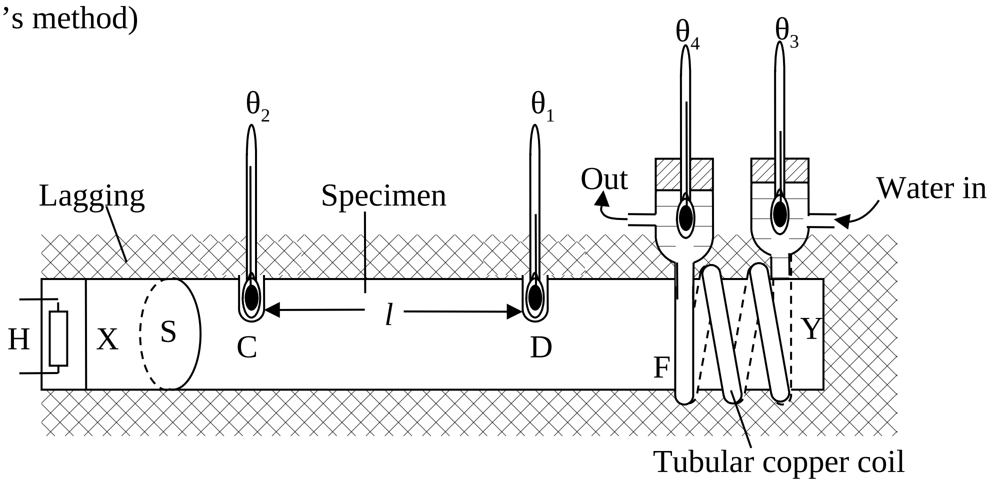
5. One face of a sheet of cork, 3mm thick, is placed in contact with one face of a sheet of glass 5mm thick, both sheets being 20cmsquare.Theouter faces of this square composite sheetare maintained at 100^{0}c and 20^{0}c, the glass being at the higher mean heat temperature. Find
(a) the temperature of the glass –cork interface, and
(b) the rate at which heat is conducted across the sheet, neglecting edge effects.
{Thermal conductivity of cork = 6.3 x $10^{-2}Wm^{-1}k^{-1}$, thermal conductivity of glass 7.2x $10^{-1}Wm^{-1}k^{-1}$}

Conditions for measurement of coefficient of thermal conductivity of a substance.
Before thermal conductivity of a substance can be measured, two conditions must be satisfied;

-*There must me a measurable rate of heat flow,* $\frac{dQ}{dt}$. For good heat conductors such as metals, this is always measurably high. The challenge is obtaining a measurable temperature gradient. This is solved by using a specimen that is considerably longer than its cross-sectional area.

-*There must be a measurable temperature gradient,* $\frac{\theta_1-\theta_2}{t}$.This is always measurably high with poor heat conductors. The challenge is to obtain a measurable rate of heat flow.T his solved by choosing a specimen that is by far thinner than its cross-sectional area such as a thin disc. Hence it is clear that for poor and good thermal conductors, different methods must be used to measure their coefficient of thermal conductivity.

Measurement of good thermal Conductor, e.g a metal
(Searle's method)



During conductivity measurement the following conditions should fulfilled:

- Heat must flow through the specimen at a measurable rate
- The temperature gradient along the specimen must be steep

The temperature gradient along the specimen must be steep
The first condition is naturally fulfilled in good conductors, but to achieve the second one, the specimen is made in form of a lagged bar of reasonable length compared to its diameter. In the above setup XY is the specimen, heated by at H and cooled by water circulating through a tubular copper coil at Y.
The diameter of the bar is measured to determine its cross-sectional area, A, and the apparatus is set up as shown. To measure the temperature gradient, thermometers are placed in holes C and D bored in the bar at a measured distance, *l*, apart. These holes are filled with mercury for good thermal contact.

The apparatus is kept running until all the temperatures have become steady. Then the cooling water circulating is collected over a measured time interval and the mass of it, m, flowing per second is found.

Calculations:

Let k = thermal conductivity of the specimen

$$\text{Then the heat flow rate per second}, \frac{Q}{t}=\frac{kA(\theta_2-\theta_1)}{l}$$

This heat is carried away by the cooling water. If c_w is the specific heat capacity of water, then

$$\frac{kA(\theta_2-\theta_1)}{l}=m\,c_w(\theta_4-\theta_3)$$

$$\therefore k=\frac{m\,c_w(\theta_4-\theta_3)l}{A(\theta_2-\theta_1)}$$

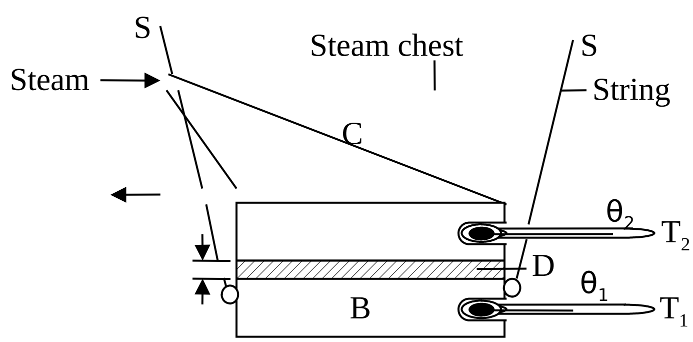
Why this method is suitable for good conductors

- A good conductor always provides a measurably rate of heat flow.
- A long specimen of good conductor provides a steep t(high) temperature gradient.

Precautions taken when measuring thermal conductivity of a metal

- The material must have a uniform cross-sectional area.
- The metal rod should be heavily lagged,
- The length of the metal rod must be made long compared with its diameter so that a measurable temperature gradient is obtained,
- The two holes for the thermometers must be filled with mercury or oil to ensure thermal contact between the thermometers and the metal rod.
- The surface of the metal rod must be polished to minimize radiation losses,
- The rate of heating should be constant.

Measurement of coefficient of thermal conductivity of a poor conductor of heat (Lee’s Method**)**



To get an adequate heat flow rate the specimen, D, is made in form of a thin circular disc.

- The diameter, and the thickness, *l*, of the specimen are first measured and the apparatus is set up as shown in the diagram. B is a thick brass block containing a thermometer.

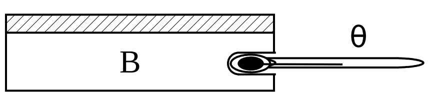
- The whole apparatus is hung in air by three strings, S, attached to B. For good thermal contact, the adjoining faces of C, D and B must be flat and clean. Those of C and B should be polished.
- The specimen is heated by a steam chest, C, whose bottom is a thick brass block, thick enough to accommodate a thermometer.
- When steady conditions have been attained, the temperatures θ_1 and θ_2 are recorded

As brass is an extremely good conductor, θ_1 and θ_2, can be taken as the temperatures of the faces of the specimen.

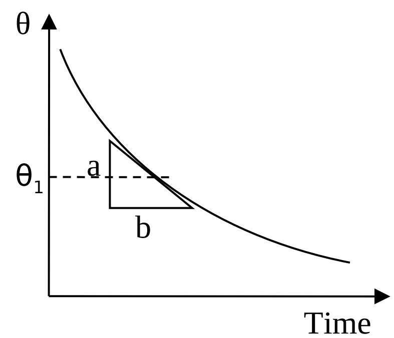
$$\text{Therefore the temperature gradient} = \frac{\theta_2 - \theta_1}{l}$$

Next is to find the heat flow rate through the specimen as follows:

- The specimen, D, is removed and B is heated directly from C until its temperature has risen by about 10°C. Then the specimen alone is placed back on B (See illustration below) and the temperature of B is recorded at intervals and plotted against time as shown on the right below.



(i)



B is now losing heat under the same conditions as in the first part of the experiment. Thus, by drawing a tangent at θ_1 as shown, the rate of heat loss when B was at θ_1 is calculated.

Let k = conductivity of the specimen

A = cross-sectional area of the specimen

m = mass of B

c = specific heat capacity of B

$$\text{Then } kA\frac{(\theta_2 - \theta_1)}{l} = mc\frac{a}{b}$$

$$\therefore k = \frac{mc\,l}{A(\theta_2 - \theta_1)}\, x\, \frac{a}{b}$$

Why this method is suitable for poor conductors

-The large value of surface area and small thickness leads to a measurable rate of heat flow through the specimen (disc),

- The lines of heat flow along the specimen are parallel, that is almost no heat is lost to the surrounding.

Precautions in the experiment

- The specimen (disc) should be thin in order to obtain a measurable temperature gradient,

- The specimen should have a large cross-sectional area and a small thickness to give adequate rate of heat flow.
- The specimen should be of a disc shape in order to reduce the rate of heat lost to the surrounding from the sides.

More Questions

1. A wall is constructed using two types of bricks. The temperatures of the inner and outer surfaces of the wall are 29 and 21 respectively. The value of the thermal conductivity for the inner brick is $0.4Wm^{-1}K^{-1}$ and that of the outer brick is $0.8Wm^{-1}K^{-1}$
   (i) Explain why in steady state the rate of thermal energy transfer is the same in both layers.
   (ii) If each layer is 12.0cm thick, find the temperature at the interface between the layers.
2. A double glazed window has two glass sheets each of thickness4.0mm separated by a layer of air of thickness 1.5mm. If the two inner air- glass surfaces have steady temperatures of 200c and 4ocrespectively, find the
   (i) Temperatures of the outer air glass surfaces.
   (ii) Amount of heat that flows across an area of the window of $2m^2$
   (conductivity of glass = $0.72wm^{-1}k^{-1}$and that of air = 0.025Wm-1K^{-1}}
3. Two brick walls of thickness 10cm are separated by an air strip of thickness 10cm. The outer surfaces of the brick walls are maintained at 20^0c and 5^0c respectively.
   (i) Calculate the temperature of the inner surfaces of the walls.
   (ii) Compare the rate of heat loss through the layer of air with through a single brick wall.{Thermal conductivity of air is $0.02Wm^{-1}K^{-1}$ and that of brick is $0.6Wm^{-1}K^{-1}$}
4. Metal rods of copper, brass and steel are welded together to form a Y shaped figure. The cross sectional area of each rod is 2cm2. The free end of the copper rod is maintained at 100^0c, while the free ends of brass and steel rods are maintained at 0^0c. If the rods are 046m, 0.13m and 0.12m respectively,
   (i) Calculate the temperature at the junctions
   (ii) Find the heat current in the copper rod.
   { Thermal conductivities of copper, brass and steel are $385Wm^{-1}K^{-1}$, $109Wm^{-1}K^{-1}$, $50.2Wm^{-1}K^{-1}$}
5. A cylindrical iron rod with a base of diameter15cm and thickness 0.30cm has its base coated with a thin film of soot of thickness 0.10cm. It is then filled with water at 100^0c and placed on a large block of ice at 0^0c. Calculate the initial rate at which ice will melt.
   (Thermal conductivity of soot = $0.12Wm^{-1}K^{-1}$}

6. A concrete floor of a hall has dimensions of 10m by 8m. It's covered with a carpet of thickness 2cm. The temperature inside the hall is 22°C while that of the surroundings just below the concrete is 12°C. The thermal conductivities of the concrete and the material of the carpet are 1Wm-1K-1 and 0.05Wm-1K-1 respectively and the thickness of the concrete is 10cm. Calculate,

(i) The temperature of the interface of the concrete and the carpet. (ii) The rate at which heat flows through the floor. [Ans: 14.040C , 1600W]

7. A cooking utensil of thickness 3mm is to be made from two layers; one of aluminum and the other of brass. If one layer is to be 2mm and the other 1mm, determine which combination allows higher rate of flow of heat.[Coefficient of thermal conductivity for brass is 112Wm-1K-1 while that for aluminum is 240Wm-1K-1]

[Ans: The thickness should be 2mm for aluminium and 1mm for brass]

8. A closed metal vessel contains water; (i)At 30°C (ii) At 75°C A vessel has surface area of 0.5m2 and uniform thickness of 4mm. If the outside temperature is 15°C, calculate the heat loss per minute of the vessel trough conduction. [Thermal conductivity is 400 Wm-1K-1] [Ans:4.5 × 107*Jmin*−1, 1.8 × 108*Jmin*−1] 4. A well lagged copper bar between boiling water and ice-water mixture is 12cm long.

(i) Calculate the rate of flow of heat through the copper bar. (ii) Calculate the mass of ice which will melt during 15s. [K for copper = 385 Wm-1K-1, A for the bar = 1.5cm2, Latent heat of fusion of ice = 3.34 x 105kg-1]

[Ans: 48.125W, 2.16g]

9. A copper kettle has a base of thickness 2mm and area 3 x 10-2m2. Estimate the steady difference in temperature between the inner and outer surface of the base which must be maintained to enable enough heat to pass through so that the temperature of 100kg of water rises at a rate of 0.25Ks-1 .Take c for water = 4200Jkg-1K-1 ,for copper = 380 Wm-1K-1]

[Ans: 18.42K]

10. A wall 6m by 3m consists of two layers of bricks of thermal conductivities 0.6 Wm-1K-1 and 0.5 Wm-1K- 1 respectively. The thickness of each layer is 15cm. The inner surface layer A is at 20°C while the outer layer B is at a temperature of 10°C. Calculate the temperature of interface of A and B; and the rate of flow of heat through a wall.

[Ans: 15.450C , 327.6W]

11. A window of height 1m and width 1.5m contains a double glazed unit consisting of two single glass panes each of distance 2mm separated by air. Calculate the rate at which heat is conducted through the window if the temperatures of the external surfaces of glass are 20°C and 30°C.[K for glass = 0.72 Wm-1K-1, K for air =0.025 Wm-1K-1, Thickness of each glass = 4mm] [Ans: 164.63W]

12. A cooking saucepan made of iron has a base area of $0.05m^2$ and thickness of 2.5mm. It has a thin layer of soot of average thickness 0.5mm on its bottom surface. Water in the saucepan is heated until it boils at 100°C. The water boils away at a rate of 0.6kg per minute and the side of the soot nearest to the heat source is at 150°C. Find the thermal conductivity of soot. [K for iron = 66 Wm-1K-1, Specific latent heat of vaporization = 2200kJkg-1] [**Ans**: 6.6 Wm-1K-1]

13 . In double glazing, two thickness of glass G, each 20mm thick are separated by 100mm thickness of air as shown below

Air

G

The thermal conductivities of glass and air are respectively 1,0 $Wm^{-1}K^{-1}$ and 2.0 $Wm^{-1}K^{-1}$ and the outer surfaces of the glass are 20°C and 0°C respectively .Calculate: (i) The thickness of the glass which is thermally equivalent to 100m thickness of air. (ii) The rate of heat flow per unit area through the glass and air

0°C

. [**Ans:** 5m, 3.97Wm-2]

14. In order to minimize heat losses from a glass container, the walls of the container are made of two sheets of glass each 2mm thick, placed 3mm apart; the intervening space being filled with a poorly conducting solid. Calculate the ratio of the rate of conduction of heat per unit area through this composite wall to that which would have occurred had a single sheet of the same glass been used under the same internal and external conditions of temperature. [Assume the conductivity of glass and poorly conducting solid is 0.63 $Wm^{-1}K^{-1}$ and 0.049 $Wm^{-1}K^{-1}$ respectively] [Ans: 7 : 149]

15. A cavity wall is made of bricks 0.1m thick with an air space 0.1m thick between them. Assuming the thermal conductivity of brick is 20 times that of air, calculate (i) The thickness of brick which conducts the same quantity of heat per second per unit area as 0.1m of air. (ii) If the thermal conductivity of brick is 0.5 $Wm^{-1}K^{-1}$, calculate the rate of heat conducted per unit area through the cavity wall when the outside surfaces of the brick walls are respectively 19°C and 4°C.

16. A metal sphere of radius 1.5cm is suspended within an evacuated enclosure whose walls are at 320K. The emissivity of the metal is 0.40. Find the power input required to maintain the sphere at a temperature of 320K, if heat conduction along the supports is negligible.

17. A metal boiler is 1.5cm thick. Find the difference in temperature between the inner and outer surfaces if 40kg of water evaporate from the boiler per metre squared per hour. {Latent heat of vaporization of water = 2268$kJkg^{-1}$, Thermal conductivity of the metal of the boiler = 63$Wm^{-1}K^{-1}$}