# 1993 PAPER ONE

**1.a)** Solve $3^{2x+1}-3^{x+1}-3^{x}+1=0$

**b)** Solve the simultaneous equations

$x + 2y - 3z = 0$
$3x + 3y - z = 5$
$x - 2y + 2z = 1$

*Ans*: (a) $x = -1$, $x = 0$; (b) $x = 1$, $y = 1$, $z = 1$

**2**. When the quadratic expression $ap^2 + bp + c$ is divided by $p - 1$, $p - 2$ and $p + 1$, the remainders are 1, 1 and 25 respectively. Determine the values of $a$, $b$ and $c$. Hence the factors of the expression. (Ans: $a = 4$, $b = -12$, $c = 9$; factors: $(2p - 3)$ and $(2p - 3)$)

**b**) Express $2x^3 + 5x^2 - 4x - 3$ in the form $(x^2 + x - 2)\,Q(x) + Ax + B$; where $Q(x)$ is a polynomial in $x$ and $A$ and $B$ are constants. Determine the values of A and $B$ and the expression $Q(x)$.
(Ans: $A = -3$, $B = 3$), $Q(x) = 2x + 3$

**3**. i) Show that $\ln 2^r$, $r = 1, 2, 3$ is an arithmetic progression.
ii) Find the sum of the first 10 terms of the progression. (Ans: 38.1231)
iii) Determine the least value of m for which the sum of the first $2m$ terms exceeds 883.7. (Ans: $m = 25$)
**b**) Given that the equations $y^2 + py + q = 0$ and $y^2 + my + k = 0$ have a common root. Show that $(q - k)^2 = (m - p)(pk - mq)$.

**4**. Solve the simultaneous equations

$z_1 + z_2 = 8$
$4z_1 - 3iz_2 = 26 + 8i$

Using the values of $z_1$ and $z_2$, find the modulus and argument of $z_1 + z_2 - z_1z_2$ (Ans: $z_1 = 8 + 2i$; $z_2 = -2i$)

**5**. Use the Maclaurin's theorem to show that the expansion $e^{-x}\sin x$ up to the term in $x^3$ is $\frac{x}{3}\left(x^2-3x+3\right)$.

Hence evaluate $e^{-\pi/3}\sin\frac{\pi}{3}$ to 4 decimal places

(Ans: 0.334)

**6**. Differentiate with respect to *x:*

i) $\tan^{-1}\left(\frac{6x}{1-2x^2}\right)$ (Ans: $\frac{6+12x^2}{1+32x^2+4x^4}$

ii) $\left(\cos x\right)^{2x}$

(Ans: $=2\left(\cos x\right)^{2x}\left(\ln\cos x-x\tan x\right)$

**b**) Write down the expression of the volume $v$, and surface area $s$ of a cylinder of radius r and height h
If the surface area is kept constant, show that the volume of the cylinder will be maximum when $h = 2r$

**7**. Find: i) $\int\ln\left(x^2-4\right)dx$

Ans: $x\ln\left(x^2-4\right)-2x+2\left[\ln\left(\frac{x+2}{x-2}\right)\right]+c$

ii) $\int\frac{dx}{3-2\cos x}$

Ans: $\frac{2}{\sqrt{5}}\tan^{-1}\left[\sqrt{5}\tan\frac{x}{2}\right]+c$

iii) Use the substitution of $x=\frac{1}{u}$ to evaluate $\int_1^2\frac{dx}{x\sqrt{x^2-1}}$

Ans: $\frac{\pi}{3}$

**8**. A curve is given by the parametric equations $x = 4\cos 2t$, $y = 2\sin t$.

i) Find the equation of the normal to the curve at $t=\frac{5}{6}\pi$ (Ans: $y = 4x - 7$)
ii) Sketch the curve for $-\frac{\pi}{2}<t<\frac{\pi}{2}$
iii) Find the area enclosed by the curve and the $y$ – axis (Ans: 7.6425 sq. units)

**9**. Show that $\cos 3\theta = 4\cos3\theta - 3\cos\theta$.

Hence solve the equation $4x^3-3x-\frac{\sqrt{3}}{3}=0$

(Ans: 0.9492, -0.2037, -0.746)

b) Find all the solutions of the equation $5\cos x-4\sin x=6$ in the range $-180^0 \le x \le 180^0$

(Ans: -18.2, -59.1)

**10**. Given that $\tan^{-1}(\alpha) = x$ and $\tan^{-1}(\beta) = y$ by expressing α and β as tangent ratios, of $x$ and $y$, and manipulating the ratios, show that $x+y=\tan^{-1}\left(\frac{\alpha+\beta}{1-\alpha\beta}\right)$. Hence or otherwise:

(i) Solve for $x$ in $\tan^{-1}\left(\frac{1}{x-1}\right)+\tan^{-1}\left(x+1\right)=\tan^{-1}\left(-2\right)$,

(Ans: $x = -2, 2$)

(ii) Without using table or calculators, determine the value of $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)=\tan^{-1}\left(\frac{1}{8}\right)$

(Ans: $\frac{\pi}{4}$)

**11**.a) A tangent from the point $T(t^2, 2t)$ touches the curve $y^2 = 4x$. Find:

(i) The equation of the tangent (Ans: $ty - x - t^2 = 0$)
(ii) The equation of line L parallel to normal at $(t^2, 2t)$ and passing through (1, 0), (Ans: $y + tx - t = 0$)
(iii) The point of intersection $X$ of line $L$ and the tangent. (Ans: $(0, t)$)

b) A point $P(x, y)$ is equidistant from $X$ and $T$. Show that the locus of $P$ is $t^4 - 3t - 2(x + y) = 0$

The proof is not valid. Loci is $t^4 - 3t - 2(xt + y) = 0$

**12**. The equation of a circle, centre $O$ is given by $x^2 + y^2 + Ax + By + C = 0$, where $A$, $B$ and C are constants. Given that $4A = 3B$, $3A = 2C$ and $C = 9$,

(a) Determine i) the coordinates of the centre of the circle (Ans: (-3, -4)

ii) The radius of the circle (Ans: 4 units)

**b**) A tangent is drawn from the point $Q(3, 2)$ to the circle. Find

(i) the coordinates of $P$, the point where the tangent meets the circle

(Ans: (-4.16, -0.17) OR (0.83, -5.16))

(iii) the area of the triangle $QPO$

(Ans: 14.96 sq. units)

**13**. $A$ and $B$ are points whose position vectors are $\boldsymbol{a} = 2\boldsymbol{i} + \boldsymbol{k}$ and $\boldsymbol{b} = \boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$ respectively. Determine the position vector of the point $P$ that divides $AB$ in the ratio 4: 1 (Ans: $\frac{1}{5}[6i - 4j + 16k]$)

**b**) Given that $\boldsymbol{a} = \boldsymbol{i} - 3\boldsymbol{j} + 3\boldsymbol{k}$ and $\boldsymbol{b} = -\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}$, determine

**(i)** the equation of the plane containing $\boldsymbol{a}$ and $\boldsymbol{b}$

(Ans: $-3x + 5y + 6z = 0$)

(ii) the angle the line $\frac{x-4}{4} = \frac{y}{3} = \frac{z-1}{2}$ makes with the plane in (i) above (Ans. 19.4°)

**14**. The table below shows the marks scored in General paper by some students in mock examination from a certain school:

| Marks | Number of students |
|---|---|
| 31 – 40 | 12 |
| 41 – 50 | 18 |
| 51 – 60 | 14 |
| 61 – 70 | 8 |
| 71 – 80 | 6 |
| 81 – 90 | 2 |

a) i) Draw a histogram to represent the scores

ii) From your histogram, estimate the mode. (46.5)

**b**) Calculate the mean, median and standard deviation

(Ans: 52.833, 50.5, 13.646)

# 1993 PAPER TWO

**1**. Show that the equation $3x^3 + x - 5 = 0$ has a real root between $x = 1$ and $x = 2$

**(a)** Use linear interpolation to find the first approximation to this root (Ans: 1.045)

**(b)** Using the Newton Raphson formula twice, find the value of the root correct to 2 dps . (Ans: 1.09)

**2 (a**). When $x = 0.8$, $e^x = 2.2255$ and $e^{-x} = 0.4493$ correct to 4decimal places.

(i) Round off the values of $e^x$ and $e^{-x}$to 2decima places.

(Ans: 2.23, 0.44)

(ii) Truncate the values of $e^x$ and $e^{-x}$ to 2 decimal places

(Ans: 2.22, 0.44)

(iii)If the maximum possible error in the values $e^x$ and $e^{-x}$ is ± 0.00005, what are the corresponding maximum and minimum values of the quotient $e^x$/ $e^{-x}$? Give your answers correct to 3 decimal places). (Ans: 4.953)

(b) Show that the iterative formula for solving the equation $x^3 = x + 1$ is $x_{n+1} = \sqrt{\left(1 + \frac{1}{x_n}\right)}$. Starting with $x_0 = 1$, find the solution of the equation to four significant figures. Draw a flowchart that computes and prints the root of the equation. (Ans: 1.324678713)

**3** a) Forces 2**i** – 3**j**, 7**i** + 9**j**, -6**i** – 4**j**, -3**i** –2**j** act on a lamina at points (1, -1), (1, 1), (-1, -1), (-1, 1) respectively. Determine

(i) The resultant of the forces (Ans: $(0i + 0j)$)

(ii) The sum of their moments about (0, 0) what effect do the forces have on the body? (Ans: 4 units)

**b**) Two beads $A$ and $B$ start together from a point $O$ and slide down in a vertical plane along smooth straight wires inclined at angles 30º and 60º respectively. The wires are on the same side of the vertical. Taking **i** and **j** as unit vectors in the horizontal and vertical directions respectively, Show that the acceleration of the bead $B$ relative to bead $A$ is $\frac{g}{2}\mathbf{j}$; where $g$ is the acceleration due to gravity.

**4** ABCD is a square lamina of side a from which a triangle ADE is removed. E being a point in CD of a distance t from C.

(i) Show that the center of mass of the remaining lamina is at a distance $\frac{a^2 + at + t^2}{3(a+t)}$ from BC

(ii) Hence, show that if this lamina is placed in a vertical plane with $CE$ resting on a horizontal table, equilibrium will not be possible if $t$ is less than $\frac{a(\sqrt{3}-1)}{2}$

**5a** A particle moving with simple harmonic motion has speeds of $5ms^{-1}$ and $8ms^{-1}$ at distances 16m and 12m respectively from its equilibrium position. Find the amplitude and the period of the motion.

(Ans: 20m, $t = 4\pi$ seconds

**b**) A particle of mass 3kg is moving on the curve described by $\mathbf{r} = 4\sin 3t\mathbf{i} + 8\cos 3t\mathbf{j}$ where **r** is the position vector of the particle at time $t$.

(i) Determine the position and velocity of the particle at the time $t = 0$. (Ans: $-8j$, $12i$)

(ii) Show that the force acting on the particle is $-27r$.

**6(a) A** body of mass 10 kg rests on a smooth horizontal plane. Horizontal forces of magnitudes $2\sqrt{3}$, 16, 5 and $F$ Newtons act on the body in the directions $030^0$, $120^0$, $0^0$ and $270^0$ respectively. Given that the acceleration of the body is 3 $ms^{-2}$. Find the value of $F$ and give the direction of the acceleration.

(Ans: $F = 46.829$, $\theta = 87.577°$)

**b**) A car of mass 1500kg is pulling a trailer of mass 600kg up a road inclined at an angle $\alpha = \sin^{-}(0.1)$ . The resistance to motion for both the trailer and car is 0.15 N per kg. If they are retarding at $0.5ms^{-2}$, find:

(i) The tractive force exerted by the engine;

(Ans: 1323N)

(ii) The tension in the coupling between the car and the trailer. (Ans: 378 N)

**7.** A particle of mass m is placed on a smooth face of a wedge, which stands on a smooth horizontal plane. The face of the wedge is inclined at an angle α to the horizontal and the mass of the wedge is 4m. If the system is released from rest, show that the speed of the particle relative to the wedge after one second is $\dfrac{5g\sin\alpha}{4+\sin^2\alpha}$

**8.** A particle is projected with speed $v$ at an angle $\theta$ above the horizontal. If the particle passes through the point $P(x, y)$

(i) Write down expressions of $x$ and $y$ in terms of $v$, $\theta$ and time $t$. (Ans: $y = v\sin\theta\cdot\dfrac{x}{v\cos\theta} - \dfrac{1}{2}g\left[\dfrac{x^2}{v^2\cos^2\theta}\right]$)

(ii) Hence or otherwise show that

$y = x\tan\theta - \dfrac{gx^2}{2v^2\cos^2\theta}$; where $g$ is the acceleration due to gravity.

(iii) Show that the range on the horizontal through the point of projection is $R = \dfrac{v^2\sin 2\theta}{g}$

b) A mortar fires shells at different angles of projection from point $O$. If the speed of projection is $\sqrt{50g}$ where $g$ is the acceleration due to gravity and the shell is projected so as to pass through the point $B(10, 20)$

i) Find the possible angles of projection.

(Ans: 81.87°, 71.57)

ii) Deduce that the difference between the corresponding times taken to travel from $O$ to $B$ is $\dfrac{10-2\sqrt{5}}{\sqrt{5}}$

**9**. **a**) The rate of change of atmospheric pressure, $P$ with respect to altitude, h in kilometers is proportional to the pressure. If the pressure at 6000 meters is half of the pressure at $P_0$ at sea level. Find the formula for the pressure at any height. (Ans: $P = P_0 \cdot e^{-0.0001155h}$)

**b**) Solve the differential equation

$$(x^2+1)\frac{dy}{dx} + y^2 + 1 = 0,\ x = 0,\ y = 1.$$

(Ans: $\tan^{-1}(y) + \tan^{-1}(x) = \dfrac{\pi}{4}$)

**10**. A random variable $X$ has the probability density function

$$f(x) = \begin{cases} k(1-x^2); & 0 < x \le 1, \\ 0 & ;Other\ wise. \end{cases}$$

Where $k$ is a constant. Find:

i) The value of the constant $k$ (Ans: $k = 1.5$)

ii) The mean of $X$ (Ans: $\dfrac{3}{8}$)

iii) The variance of $X$ (Ans: $\dfrac{19}{320}$)

**b**) The number of times a machine breaks down every month is a discrete random variable $X$ with a probability density function

$$P(X = x) = \begin{cases} k\left(\frac{1}{4}\right)x; x = 0, 1, 2, 3, ... \\ 0 \quad ;\ Other\ wise. \end{cases}$$

Where $k$ is a constant

Determine the probability that the machine will not break more than two times a month. (Ans: $\dfrac{63}{64}$)

**12**. The life time of a bulb is normally distributed with a mean of 800 hours and standard deviation of 80 hours. The manufacturer guarantees to replace bulbs which blow after less than 660 hours.

i) What percentage of bulbs will he have to replace under the guarantee? (Ans: 4.01%)

ii) The manufacturer is only willing to replace a maximum of 1% of the bulbs. What should be the guaranteed lifetime of the bulbs? (Ans: 613.92 hrs)

iv) Instead of reducing the guaranteed lifetime as in (ii), the mean lifetime was increased by superior technology. What should be the new mean so that only 1% are replaced if the guaranteed lifetime remains 660 hours but the standard deviation is reduced to 70 hours?(Ans: 822.82)

**13**. Nkeza makes 5 practice runs in the 100 m sprint. A run is successful if he runs it in less than 11 seconds. There are 8 chances out of 10 that he is successful.

Find the probability that

**a**) i) He records no success at all, (Ans. 0.003)

ii) He records at least 2 successes (Ans: 0.9933)

**b**) If he is successful in the 5 practice runs, he makes two additional runs. The probability of success in either additional runs is 0.7. Determine the probability that Nkeza will make 7 successful runs in total. (Ans: 0.16073)

**14**. Ten shops in Kampala which attract a similar number and type of customers are ranked in times of quality of service, size of verandah and price of items. Rank 1 indicates best service, largest verandah and lowest price of commodities. The results, including monthly average sales and are given below:

| Shop | Quality of service | Size of verandah | Price of commodities | Sales (kg) |
|---|---|---|---|---|
| A | 3 | 3 | 6 | 20 |
| B | 7 | 5 | 10 | 10 |
| C | 4 | 10 | 7 | 31 |
| D | 6 | 7 | 2 | 47 |
| E | 8 | 2 | 4 | 37 |
| F | 2 | 1 | 5 | 38 |
| G | 5 | 8 | 3 | 38 |
| H | 9 | 6 | 8 | 15 |
| I | 10 | 4 | 10 | 21 |
| J | 1 | 9 | 1 | 42 |

a) By calculation, determine whether the price of commodities or the size of the verandah is the more important factor affecting sales.

(Ans ρ = 0.8758; Since there is a highly positive correlation between price of commodities and sales, therefore, it is the price of commodities other than the size of the verandah that affects sale.)

b) Is there any evidence that the size of the verandah influences the quality of service?

(Ans ρ = 0.-0.1879; Null hypothesis; H0;   = 0 (there is no correlation between size of the verandah and quality of service)

Alternative; H1; | | > 0 (there is correlation between the two)

But from the tables, | | = 0.65

Since  | |= 0.188 < 0.65

Therefore there is no sufficient evidence at 5% level of significance to show that the size of the verandah influences the quality of service.

c) Is there evidence that a shop with lower priced commodities offer poor quality service (e.g. by employing fewer sales people)?

Ans: Since $|\rho|$= 0.558 < 0.65, there is no sufficient evidence at 5% level of significance to conclude that a shop with lower priced commodities offer poor quality services

# 1994 PAPER ONE

1\. a) Solve $2\sqrt{(x-1)}-\sqrt{(x+4)}=1$

(Ans: 5, $\frac{13}{9}$)

b) solve the simultaneous equations:

$x + y + z = 2$

$\frac{x+2y}{-3}=\frac{y+2z}{4}=\frac{2x+z}{5}$ (Ans: 1, -2, 3)

**2**) a) If $\log_2 x+\log_4 x+\log_{16} x=\frac{21}{16}$, find $x$

(Ans: $x$ = $x=2^{3/4}$)

**b**) In an arithmetic progression $u_1 + u_2 + u_3 + ....$, $u_4$ = 15 and $u_{16}$ = -3. Find the greatest integer $N$ such that $u_N \geq 0$. Determine the sum of the first $N$ terms of the progression. (Ans: 136.5)

**3**. a) Given that $\alpha$ and $\beta$ are the roots of the equation $x^2 + px + q = 0$, express $(\alpha - \beta^2)$ $(\beta - \alpha^2)$ in terms of $p$ and $q$. Deduce that for one root to be the square of another, $p^3 - 3pq + q^2 + q = 0$ must hold

**b**) Determine the expansion of $\frac{(x+4)}{(x^2-1)}$ in ascending powers of $x$ up to the term containing $x^r$ for $|x| <1$.

(Ans: Hence

$$\frac{x+4}{(x+1)(x-1)}=-4-x-4x^2-x^3-.....\left[\frac{5+3(-1)^r}{2}\right]x^r$$

Provided $|x| < 1$ i.e. $-1 < x < 1$)

**4**. **a**) Given that $z = 3 + 4i$, find the value of the expression $z+\frac{25}{z}$. (Ans: 6)

**b**) Given that $\left|\frac{z-1}{z+1}\right|=2$, show that the locus of the complex number is $x^2+y^2+\frac{10x}{3}+1=0$. Sketch the locus.

**5**. a) Express $\sqrt{\left(\frac{\sin 2\theta-\cos 2\theta-1}{2-2\sin 2\theta}\right)}$ in terms of tan $\theta$.

(Ans: $\frac{1}{\sqrt{(\tan\theta-1)}}$)

**b**) Find the general solution of the equation √3 sin $\theta$ – cos $\theta$ + 1 = 0 (Ans: $\theta=2n\pi-\frac{2\pi}{3}$)

**c**) Factorise cos $\theta$ – cos 3$\theta$ – cos 7$\theta$ + cos 9$\theta$ in the form $A$cos$k$ $\theta$ sin$l$ $\theta$ sin$m$ $\theta$, where $A$, $k$, $l$ and $m$ are constants. (Ans: $=-4\cos 5\theta\sin 3\theta\sin 2\theta$)

**6**. Given that sin $x$ + sin $y = \lambda_1$ and cos $x$ + cos $y = \lambda_2$, show that

i. $\tan\frac{(x+y)}{2}=\frac{\lambda_1}{\lambda_2}$

ii. $\cos(x+y) = \dfrac{\lambda_2^{\,2} - \lambda_1^{\,2}}{\lambda_2^{\,2} + \lambda_1^{\,2}}$

**b**) Solve the simultaneous equations:

$\cos x + 4\sin y = 1$

$4\sec x - 3\,\mathrm{cosec}\, y = 5$ for values of $x$ and $y$ between $0^0$ and $360^0$

(Ans: $x = 78.5°,\ 281.5°$, $y = 11.5°,\ 168.5°$ )

**7**. Prove that the tangents to the parabola $y^2 = 4ax$ at the points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ meet at the point $T(apq, a(p+q))$.

i) If $M$ is the midpoint of $PQ$, prove that $TM$ is bisected by the parabola

ii) If $P$ and $Q$ vary on the parabola in such a manner that $PQ$ is always parallel to the fixed line $y = mx$, show that $T$ always lies on the fixed line parallel to the $x$-axis

**8**. The coordinates of a point $P(x, y)$ on the curve are given parametrically by the equations $x = a\cos\theta$, $y = b\sin\theta$ where $a$ and $b$ are constants and $\theta$ is the parameter.

Find:

i. The Cartesian equation for the curve and identify the curve (Ans: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$)

ii. The equation of the tangent to the curve at the point where the parameter $\theta = \varphi$.

(Ans: $ay\sin\varphi + bx\cos\varphi - ab = 0$)

iii. The relation between $\varphi_1$ and $\varphi_2$ if the tangents at the points $(a\cos\varphi_1, b\sin\varphi_1)$, $(a\cos\varphi_2, b\sin\varphi_2)$ are at right angles to one another.

(Ans: $\tan\varphi_1 = \dfrac{-b^2}{a^2\tan\varphi_2}$)

**9**. a) Differentiate:

i. $e^{ax}\sin bx$. (Ans: $= e^{ax}\sin bx\left(a + b\cot bx\right)$ )

ii. $\dfrac{(x+1)^2(x+2)}{(x+3)^3}$, giving your answer in the simplest form (Ans: $\dfrac{dy}{dx} = \dfrac{(x+1)(5x+9)}{(x+3)^4}$ )

**b**) Given that $y = e^{\tan^{-1}x}$, show that

$(1+x^2)\dfrac{d^2y}{dx^2} + (2x-1)\dfrac{dy}{dx} = 0$.

Hence or otherwise, determine the first four non-zero terms of the Maclaurin's expansion of $y$.

(Ans: $= 1 + x + \dfrac{x^2}{2} + \dfrac{x^3}{6} + \ldots.$ )

**10**. **a**) Evaluate $\int_1^{\sqrt{2}} (x + \tan x)\,dx$.. (Ans: 1.0003 (4 dps))

*The accuracy can be improved by increasing the number of sub-intervals.*

**b**) Use the trapezium rule with six sub-intervals to estimate $\int_0^{\pi} \sin \tfrac{x}{3}\,dx$. Correct to 3 decimal places.

Find the error in your estimation and suggest how the accuracy of your result can be improved. (Ans: (0.0033)

**11**. a) Determine the equation of the normal to the curve $y = \dfrac{1}{x}$ at the point $x = 2$. Find the coordinates of the other point where the normal meets the curve again.

**(Ans:** $(-\tfrac{1}{8}, -8)$ **)**

**b**) Find the area of the region bounded by the curve $y = \dfrac{1}{x(2x+1)}$, the $x$-axis and the lines $x = 1, x = 2$

(Ans: 0.1823 sq units)

**12**. Show that the curve $y = \dfrac{x+1}{x^2 + 2x}$ has no turning points. Sketch the curve. Give the equations of the asymptotes.

**13**. A vector $XY$ of magnitude $a$ units makes an angle of $\alpha$ with the horizontal. Another vector $YZ$ beginning from the end point Y, inclined at an angle $\beta$ to the same horizontal axis is of magnitude $b$ units. If θ is the angle between the positive directions of the two vectors, where $\theta = \beta - \alpha$ is acute, show that he resultant vector $XZ$ has a magnitude $xz$ equal to $\sqrt{(a^2 + b^2 + 2ab\cos\theta)}$ units and is inclined at an angle $\alpha + \sin^{-1}\left(\frac{b\sin\theta}{xz}\right)$ to the horizontal. Hence or otherwise calculate the magnitude and direction of the resultant vector of the vectors $XY$ and $YZ$, inclined at $30^0$ and $75^0$ to the horizontal and of magnitude 9 and 6 units respectively

(Ans: 47.7642)

**14**. The table below shows the weights of Freshers in 1991/1992 academic year who underwent medical examination at the university hospital

| Weight (in kg) | Number of students |
|---|---|
| 44 – 44 | 3 |
| 45 – 49 | 10 |
| 50 – 54 | 15 |
| 55 – 59 | 10 |
| 60 – 64 | 4 |
| 65 – 69 | 5 |
| 70 – 74 | 4 |
| 75 – 79 | 6 |
| 80 – 84 | 1 |

a) Calculate,

i) The mode (Ans: 52)

ii) The median and mean weight of the students.

(Ans: 55, 57.948)

b) Draw a cumulative frequency graph. Hence deduce the interquartile range of the weights (Ans: 16)

**15**. a) Four and five digit numbers greater than 6000 are obtained by arranging the digits 3, 4, 5, 6, 7. How many of these are

i) Odd numbers (Ans: 30)
ii) Even numbers (Ans 48)

b) Nine out of twelve members of the school's Geography club are to be taken out on a study tour. Given that there are seven boys and five girls and at least three girls have to go for the tour.

(i) Find the number of ways in which the selection of students can be done (Ans: 16 ways)

(ii) If there are two sisters in the club, who are definitely selected to go, in how many ways can the remaining students be selected? (Ans: 161 ways)

# 1994 PAPER TWO

**1.(a)** Given the following table of values:

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 0 | 5 | 10 | 15 | 20 |
| $t$ | 0 | 12 | 25 | 39 | 54 |

Use linear interpolation to find

i. $t$ when $x = 12$ (Ans: 30.6)
ii. $x$ when $t = 45$ (Ans: 17)

b) Show graphically that there is only one positive real root of the equation $xe^{-x} - 2x + 5 = 0$.
Using the Newton-Raphson method, find this root correct to one decimal place (Ans: 2.5967)

**2** a) Given $x = 3$, $y = 12$, $z = 6$, all to the nearest integer, find the maximum value of

i) $\frac{x+y}{z}$, ii) $\frac{xy}{z}$, iii) $\frac{x}{y} - \frac{y}{z}$.

(Ans: 0.8696, 7.9545, -1.4649)

**b**) A trader in tea and coffee makes an annual profit in tea of Sh. 1080 million with a margin of error of ±10% and an annual loss in coffee of Sh. 560 million with a margin of error of ±5%

i. Find the range of values representing his real income. (Ans: $384 \le P \le 656$)
ii. Given that his annual income tax is Sh. 75 million, express this as a percentage of his gross income giving your answer as a range of value.
(Ans: 11.4% to 19.5%)

**3**. To a motorist travelling due north at 40 kmh$^{-1}$ the wind appears to come from the direction N 60$^0$ E at 50 kmh$^{-1}$.

i. Find the true velocity of the wind.
(Ans: 45.82 kmh$^{-1}$, N 70.9°W)
ii. If the wind velocity remains constant, but the speed of the motorist is increasing, find his speed when the wind appears to be blowing from the direction N 45$^0$ E. (Ans: 58.3 kmh$^{-1}$)

**4**. Strings *AC* and *BC* are both of natural length 5*l*. *AC* is inelastic and *BC* has a modulus of elasticity of λ. *A* and *B* are attached to points in a horizontal line, distance 5*l* apart. A mass *M* is attached to *C* and the system is in equilibrium in a vertical plane with *BC* of length 6*l*. Find λ and the tensions in *CA* and *CB*.
(Ans: λ = 17.165, 3.433*M* N; 7.347*M* N)

**5**. a) A bullet travelling at 150 ms$^{-1}$ will penetrate 8 cm into a fixed block of wood before coming to rest. Find the velocity of the bullet when it has penetrated 4 cm of the block. (Ans: 106ms$^{-1}$)

b) A particle of mass 2 kg, initially at rest at (0, 0, 0) is acted upon by the force $\begin{pmatrix} 2t \\ t \\ 3t \end{pmatrix} N$

Find i) its acceleration at time *t*. (Ans: $a = \begin{pmatrix} t \\ \frac{1}{2}t \\ \frac{3}{2}t \end{pmatrix}$

ii) its velocity after 3 seconds.

(Ans: $v = \frac{9}{2}\mathbf{i} + \frac{9}{4}\mathbf{j} + \frac{27}{4}\mathbf{k}$ )

iii) the distance the particle has traveled after 3 seconds. (Ans: 8.4 m)

**6**. a) *A*, *B* ,*C* and *D* are the points (0, 0), (10,0), (7, 4) and (3, 4) respectively. If *AB*, *BC*, *CD* and *DA* are made of a thin wire of uniform mass, find the coordinates of the centre of gravity. (Ans: 5, 1.5)

**b**) i) If instead *AB* is a uniform lamina, find its centre of gravity, *G*. (Ans: 5, 1.7)

ii) If the lamina is hung from *B*, find the angle *AB* makes with the vertical. (Ans: 18.8°)

**7**. a) A particle *P* of mass *M* lies on a smooth horizontal table and is attached to two light elastic strings fixed to the table at points *A* and *B*. The natural lengths of the strings are *AP* = 12*l*, *PB* = 5*l* and their moduli of elasticity are *mg* and $\frac{5mg}{2}$ respectively. *AB* = 12*l*. Show that when *P* is in equilibrium, *AP* = 6*l*.

*P* is now held at *C* in the line *AB* with *AC* = 5*l* and then released. Show that the resulting motion is simple Harmonic with period $4\pi\sqrt{\frac{l}{3g}}$. Find the maximum speed. (Ans: $\frac{1}{2}\sqrt{3gl}$ )

**8**. Anon – uniform beam A*B*, 4.5m long is balanced horizontally on two supports *P* and Q such that A*P* = 0.4m and Q*B* = 0.6m. When a mass of 20kg is placed at either end, the beam is on the point of toppling.

Find (i) the distance from A at which the weight of the beam acts. (Ans: 1.8m)

(ii) Weight of the beam. (Ans: 56 N)

(iii) The distance from A at which the 20kg mass must be placed for the reactions of the supports to be equal. (Ans: 2.25m from A)

**9. a)** Solve the differential equation $\frac{dt}{d\theta} + t\cot\theta = 2\cos\theta$ given that t = 3 when $\theta = \frac{\pi}{2}$.

(Ans: $t = \frac{1}{2}\operatorname{cosec}\theta\left(5 - \cos 2\theta\right)$ )

**b**) The mass of a man together with his parachute is 70 kg. When the parachute is fully open, the system experiences an upward force proportional to the velocity of the system. If the constant of proportionality is 1/10 and the system is descending at the speed of 10 ms$^{-1}$ when the parachute opens out, determine the speed of the parachute three minutes later. (Ans: 7.738 ms$^{-1}$)

**10**. The packets of omo sold in a shop are of four categories namely, small, medium, large and giant. On a particular day, the stock is such that the ratio of small: medium: large: giant is equal to 4:2:1:1. The costs of the packets are in the ratio small: medium: large: giant equal to 350:500:800:1400 respectively.

a) 30 packets are sold randomly on that particular day, the total cost of the sales being *s* shillings.

Calculate i) the expected value of *s*. (Ans: 17250)

ii) the standard deviation of *s*.

(Ans: 10310.7953)

b) Ten packets are picked at random. Determine the probability that six are medium size packets.

(Ans: 0.0162)

**11**. A continuous random variable *X* has a probability density function:

$f(x) = kx(3 - x)$ for $0 \le x \le 2$,
$f(x) = k(4 - x)$ for $2 \le x \le 4$,
$f(x) = 0$ Else where.

Find i) The value of *k*, (Ans: *k* = 3/16)

ii) The mean, (Ans: 1.75)

iii) *F(x)*, the cumulative distribution function,

(Ans:

$$F(x) = \begin{cases} 0; & x \le 0 \\ \frac{3}{16}\left(\frac{3}{2}x^2 - \frac{1}{3}x^3\right); & 0 \le \times \le 2 \\ \frac{3}{4}x - \frac{3}{32}x^2 - \frac{1}{2}; & 2 \le \times \le 4 \\ 1 & x > 4 \end{cases}$$

iv) $P(1 \le x \le 3)$ (Ans. 0.6875)

**12**. **a**) Two biased tetrahedrons have each their faces numbered 1 to 4. The chances of getting any one face showing uppermost is inversely proportional to the number on it. If the two tetrahedrons are thrown and the number on the uppermost face noted, determine the probability that the faces show the same number.

**(**Ans: 0.328)

**(b)** If it is a fine day, the probability that Alex goes to play football is $9/10$ and the probability that Bob goes is $3/4$. If it is not fine, Alex's probability is $1/2$ and Bob $1/4$. Their decisions are independent. In general it is known that it is twice as likely to be fine as not fine.

i) determine the probability that both go to play

(Ans: 59/120)

ii) if they both go to play, what is the probability that it is a fine day? (Ans: 54/59)

**13**. **a**) Among the spectators watching a football match, 80% were the home teams' supporters while 20% were the visiting teams' supporters. If 2500 of the spectators are selected randomly, what is the probability that there are more than 540 visitors in this sample? (Ans: 0.0215)

**b**) A factory manager states that the average time taken to make one unit of a product is 48 minutes. A sample of 49 trials were taken and the average time was 49 minutes with a standard deviation of 2 minutes.

i. Test the manager's claim at 99% level of confidence.

ii. Determine the 80% confidence limits of the mean production time per unit.

(Ans: 47.634 and 48.366.)

**14**. **a**) In many government institutions, officers complain about typing errors. A test was designed to investigate the relationship between typing speed and errors made.

Twelve typists A, *B*, C, L, .... L were picked at random to type the same text. The table below shows the ranking of the typists according to speed and errors made. [N.*B*. lowest ranking in errors indicate least errors made]

| TYPIST: | A | *B* | C | D | E | F | G | H | I | J | K | L |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SPEED: | 3 | 4 | 2 | 1 | 8 | 11 | 10 | 6 | 7 | 12 | 5 | 9 |
| ERROR: | 2 | 6 | 5 | 1 | 10 | 9 | 8 | 3 | 4 | 12 | 7 | 11 |

Calculate the rank correlation coefficient: Test the assertion made by officers and comment on your result. [$\sigma = 0.71$ and $\tau = 0.58$ are Spearman's and Kendall's' correlation coefficients respectively, at 1% level of significance based on 12 observations]

(Ans: 0.8182)

**b**) The cost of travelling a certain distance away from the city centre is found to depend on the route and the distance a given place is away from the centre. The table below gives the average rates of travel charged for distances to be traveled away from the city centre:

| Distance, s(km) | 9 | 12 | 14 | 21 | 24 | 30 | 33 | 45 | 46 | 50 |
|---|---|---|---|---|---|---|---|---|---|---|
| Rates charged, r (shs) | 750 | 1000 | 1150 | 1200 | 1350 | 1250 | 1400 | 1750 | 1600 | 2000 |

i. Plot the above data on a scatter diagram and draw line of best fit through the points of the scatter diagram.

ii. Determine the equation of the line in (i) above in the form $r = \beta s + \acute{\alpha}$, where $\acute{\alpha}$ and $\beta$ are constants. Use your result to estimate to the nearest shilling the cost of travelling a distance of 40 km.

(Ans: $r = \frac{1250}{41}s + 475.6$ ; $r$ = 1695/-)

# 1995 PAPER ONE

## SECTION A (40 *marks*)

**1.** Solve the simultaneous equations:

$2^x + 4^y = 12$

$3(2)^x - 2(2)^{2y} = 16$. (Ans: $x = 2, y = 1$)

Hence show that $(4)^x + 4(3)^{2y} = 100$.

b) Given that α and β are roots of the quadratic equation $ax^2 + bx + c$, determine an equation whose roots are $\alpha + \beta$ and $\alpha^3 + \beta^3$. Hence or otherwise solve the equations

$\alpha + \beta = 2$

$\alpha^3 + \beta^3 = 26$

(Ans: $\alpha = 3$ when $\beta = -1$ or $\alpha = -1$ when $\beta = 3$)

**2**. The first term of an arithmetic progression (A.P) is 73 and the 9th is 25. Determine

i) The common difference of the A.P. (Ans: -6)

ii) The number of terms that must be added to give a sum of 96. (Ans: 96)

**b**) A geometric progression (G.P.) and an arithmetic progression (A.P) have the same first term. The sums of their first, second and third terms are 6, 10.5 and 18 respectively. Calculate the sum of their fifth terms. (Ans: 57)

**3**. a) Determine the possible values of $x$ in the equation $\log_2 x + \log_x 64 = 5$. (Ans: 8)

b) Jack operates an account with a certain bank which pays a compound interest rate of 13.5 % per annum. He opened the account at the beginning of the year with sh. 500,000 and deposits the same amount of money at the beginning of every year. Calculate how much he will receive at the end of 9 years. After how long will the money have accumulated to sh. 3.32 million? (Ans: 4.6 years)

**4** a) Express each of the following complex numbers $z_1 = (1 - i)(1 + 2i)$, $z_2 = \frac{2+6i}{3-i}$ and $z_3 = \frac{-4i}{1-i}$ in the form $a + bi$. (Ans: $3 + i$, $2i$, $2 - 2i$ resp.)

ii) Find the modulus and argument of $Z_1Z_2Z_3$ given in (a) (i) above. (Ans: $8 + 16i$, 63.4349°)

b) Find the square root of $12i - 5$.

(Ans: $2 + 3i$ or $-2 - 3i$)

**5**. a) Express $\sin\theta + \sin 3\theta$ in the form $m\cos\theta\sin n\theta$ where $m$ and $n$ are constants. (Ans: $2\sin 2\theta\cos\theta$)

b) Find the general solution of

$\cos 7\theta + \cos 5\theta = 2\cos\theta$.

(Ans: $\theta = 2n\pi, 2n\pi \pm \frac{\pi}{2}$, $\mathbf{2}n\pi \pm \frac{\pi}{3}$)

c) Prove that $\frac{\sin A + \sin 4A + \sin 7A}{\cos A + \cos 4A + \cos 7A} = \tan 4A$.

**6**. Show that $f(x) = \frac{x(x-5)}{(x-3)(x+2)}$ has no turning points.

Sketch the curve $y = f(x)$. If $g(x) = \frac{1}{f(x)}$, sketch the curve of $y = g(x)$ on the same axes. Show the asymptotes and where $f(x)$ and $g(x)$ intersect.

**7**. The tangent at any point $P\left(ct, \frac{c}{t}\right)$ on the hyperbola $xy = c^2$ meets $x$ and $y$ axes at $A$ and $B$ respectively. $O$ is the origin.

a) Prove that
   i) $AP = PB$
   ii) The area of triangle $AOB$ is constant

**b**) If the hyperbola is rotated through an angle of $-45^0$ about $O$, find the new equation of the curve.
(Ans: $x^2 - y^2 = 2c^2$)

**8**. a) Prove that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where $ABC$ has all the angles acute and $R$ is the radius of the circumcircle.

**b)** From the top of vertical cliff 10 m high, the angle of depression of ship $A$ is $10^0$ and of ship $B$ $15^0$. The bearings of $A$ and $B$ from the cliff are $162^0$ and $202½^0$ respectively. Find the bearing of B from $A$.
(Ans: 301.5°)

**9**) a) i) Show that $\frac{d}{dx}\left(a^x\right) = a^x \ln a$

(ii) Find $\int 3^{\sqrt{2x-1}}\, dx$. (Ans: $\frac{3^{\sqrt{2x-1}}}{\ln 3}\left(\sqrt{2x-1} - \frac{1}{\ln 3}\right) + c$)

**b**) A shell is formed by rotating the portion of the parabola $y^2 = 4x$ for which $0 \le x \le 1$ through two right angles about its axis.

Find:
   i) the volume of the solid formed.
(Ans: 6.2832 cubic units)
   ii) The area of the base of the solid formed.
(Ans: 12.5664 sq units)

**10**. Express $\frac{x^3 - 3}{(x-2)(x^2+1)}$ as partial fractions.

Hence or otherwise find $\int \frac{x^3 - 3}{(x-2)(x^2+1)} dx$.

(Ans: $x + \ln(x-2) + \frac{1}{2}\ln(x^2+1) + \tan^{-1} x + c$)

**b**) Use the trapezium rule to evaluate the integral $\int_2^3 \left(\frac{x}{x^2+1}\right) dx$ using five sub-intervals. Give your answer correct to 4 decimal places. Find the error in your estimation. (Ans: 0.0001)

**11**. a) i) If $x^2\sec x - xy + 2y^2 = 15$, find $\frac{dy}{dx}$.

(Ans: $\frac{y - x^2 \sec x \tan x - 2x \sec x}{4y - x}$)

ii) Given that $y = \theta - \cos\theta$; $x = \sin\theta$; show that $\frac{d^2y}{dx^2} = \frac{1+\sin\theta}{\cos^3\theta}$.

**b**) Determine the maximum and minimum values of $x^2e^{-x}$
(Ans: 0, 0.5413)

**12.a**) Obtain the first two non- zero terms of the Maclaurin's series for sec $x$. (Ans: $1 + \frac{x^2}{2}$)

**b**) Show by Taylor's expansion that the first four terms of $\cos(x+h) = \cos x - h\sin x - \frac{h^2}{2}\cos x + \frac{h^3}{6}\sin x$, where $h$ and $x$ are in radians and $h$ rather small. Use the expansion to evaluate cos $3.9^0$ correct to two decimal places, using $x = 0$

**13**. The position vector of the points A and B with respect to the origin O are $\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix}$ respectively. Determine the equation of the line $AB$.

(Ans: $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$)

**b**) Find the equation of the plane $OPQ$ where $O$ is the origin and $P$ and $Q$ are the points whose position vectors are $\begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ respectively.

(Ans: $-2x + z = 0$)

**c**) i) Given that $R$ is the point at which line $AB$ meets the plane $OPQ$, find the coordinates of $R$.
(Ans: R(7,-7,14))

ii) Show that the point $S$ (1, -1, 2) lies on $\overline{OR}$

**14** The data below shows the amount of cotton (in 1000's of bales) produced by Grower's unions over a certain period of time.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 70 | 41 | 34 | 55 | 45 | 66 | 73 | 77 | 80 | 30 |
| 50 | 45 | 72 | 50 | 27 | 70 | 55 | 70 | 85 | 70 |
| 30 | 50 | 60 | 53 | 40 | 45 | 35 | 55 | 20 | 81 |
| 25 | 51 | 35 | 62 | 60 | 30 | 45 | 35 | 50 | 89 |
| 53 | 23 | 28 | 65 | 68 | 50 | 65 | 34 | 35 | 76 |

i) Beginning with the 20 − 29 class and using class intervals of equal widths, construct a frequency table for the data

Using the frequency table,

ii) draw a cumulative frequency curve for data and hence estimate the median production. (Ans: 54)

iii) calculate the mean and standard deviation of the production. (Ans: 53.7, 17.9822)

**15.** b) i) Evaluate ${}^{80}P_5 \div {}^{80}C_6$ (Ans: 9.6)

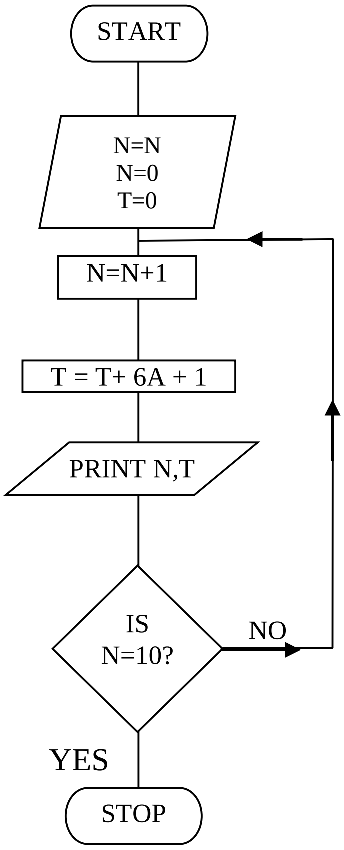
ii) Solve for $n$ in ${}^{n}C_4 = {}^{n}C_2$ (Ans: 6)

iii) A committee of five students to comprise the school council is to be selected from eight male students and five female students. Find how many possible committees can be obtained. (Ans: 1280 ways)

# 1995 PAPER TWO

## SECTION A

**1.** a)




Perform a dry run of the flow chart shown above. What is the outcome in words? After the dry run, state the relationship between $N$ and $T$. (Ans: $N^3 = T$)

**(b)** By sketching the graphs of $2x$ and $\tan x$, show that the equation $2x = \tan x$ has one real root between $x = 1.1$ and $x = 1.2$. Use linear interpolation to find the value of the root correct to 2 decimal places. (Ans: 1.17)

**2**.(a) Show that the Newton Raphson formula for finding the root of the equation $2x^3 + 5x - 8 = 0$ is $\frac{4x_n^{\ 3}+8}{6x_n^{\ 2}+5}$.

b**)** Taking the first approximation to the root of the above equation as 1.2, draw a flow diagram which reads and prints the number of iterations and root. Carry out a dry run of the flow chart and obtain the root with an error of 0.001. (Ans: 1.087)

**3.** In the gulf waters, a battleship steaming northwards at 16 km$^{-1}$is 5 km southwest of a submarine.
Find two possible courses which the submarine could take in order to intercept the battleship if its speed is 12km $^{-1}$. (Ans: N 64.5$^0$W and N25.5$^0$W)

**4**. A particle P starts from a point with a position vector $2\mathbf{j} + 2\mathbf{k}$ with a velocity $\mathbf{j} + \mathbf{k}$. A second particle $Q$ starts at the same time from the point whose position vector is $^{-}11\mathbf{i} - 2\mathbf{j} - 7\mathbf{k}$ with a velocity of $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ . Find

i) the time when the particles are closest together (Ans: 6.2)

ii) the shortest distance between the particles. (Ans: 5.079 units)

iii) How far each particle has travelled by this time. (Ans: $8.2\mathbf{j} + 8.2\mathbf{k}$; $1.4\mathbf{i} + 4.2\mathbf{j} + 5.4\mathbf{k}$)

**5(a)**. A particle of mass $2m$ rests on a rough plane inclined to the horizontal at an angle of $\tan^{-1}(3\mu)$, where $\mu$ is the coefficient of friction between the particle and the plane. The particle is acted upon by a force of $P$ Newtons.

i. Given that the force acts along the line of greatest slope and that the particle is on the point of slipping up, show that the maximum force possible to maintain the particle in equilibrium is $P_{\max} = \frac{8\mu mg}{\sqrt{1+9\mu^2}}$

ii. Given that the force acts horizontally in a vertical plane through a line of greatest slope and that the particle is on the point of sliding down the plane, show that the minimum force required to maintain the particle in equilibrium is $P_{\min} = \frac{4\mu mg}{1+3\mu^2}$

**6.** Two uniform rods A$B$, AC, each of weight W and length 10 cm are smoothly hinged at A. the ends $B$ and C rest on a smooth horizontal plane. An inextensible string joins $B$ and $C$ and the system is kept in equilibrium in a vertical plane with the string taut. An object of weight 2w climbs the rod AC to a point E such that $AE$ = 8cm. given that angle $BAC = 2\theta$. Determine in terms of $w$ and $\theta$

i) the reaction at the ends $B$ and C. (Ans: $\frac{14w}{5}$ )

ii) the tension in the string. (Ans: $\frac{14w}{5}$ )

Hence show that the reaction at the hinge A is given by $\frac{w}{10}\sqrt{49\tan^2\theta + 4}$

**7**. A particle of mass $\frac{4}{3}m$ is attached to one end of a string of length, $l$, the other end being attached to a fixed point A. the particle falls from rest at B at the same horizontal level as A. if AB = $\frac{\sqrt{3}}{2}l$ ,

i) Show that the impulse on the string when it tightens is $\frac{2}{3}m\sqrt{gl}$ .

ii) Find the inclination of the string to the vertical when the kinetic energy of the particle is $3/2$ of that at the point when the string first tightens.

(Ans: 48: 59°)

**8 a**) A bullet travelling at 150 $ms^{-1}$ will penetrate 8 cm into a fixed block of wood before coming to rest. Find the velocity of the bullet when it has penetrated 4 cm of the block. (Ans: 106$ms^{-1}$)

**(b**) A particle of mass 2 kg, initially at rest at (0, 0, 0) is acted upon by the force $\begin{pmatrix} 2t \\ t \\ 3t \end{pmatrix} N$

Find i) its acceleration at time *t*.

(Ans: $v = \frac{t^2}{2}\mathbf{i} + \frac{t^2}{4}\mathbf{j} + \frac{3t^2}{4}\mathbf{k} + c$ )

ii) its velocity after 3 seconds.

(Ans: $v = \frac{9}{2}\mathbf{i} + \frac{9}{4}\mathbf{j} + \frac{27}{4}\mathbf{k}$ )

iii) the distance the particle has traveled after 3 seconds. (Ans: 8.4m)

**10**. A note contains one 200sh note, three 100sh notes and *n* 50sh notes. A note is selected at random from the bag, its value noted and then replaced. The process is repeated many times. If the average of the values of the notes after many trials is 110sh, determine

i) the value of *n*, (Ans: $n = 1$)

ii) the expected value of the sum of two notes selected at random without replacement.

(Ans: 220)

**11**. **a**) In an examination, only two papers, namely mathematics and physics were done. The failure rates were 45% and 40% respectively.

The number of candidates who sat for the examination was 2000. Find the probability that a candidate selected at random

i) failed both mathematics and physics.

(Ans: 0.18)

ii) passed both mathematics and physics

(Ans: 0.33)

iii) passed mathematics and failed physics.

(Ans: 0.22)

**b)** Determine the number of candidates who passed both papers in other grades given that 21.8% and 22.9% passed with distinction in mathematics and physics respectively. (Ans: 246)

**c**) When visiting a friend, john may go by road, air or rail. The probabilities of using road, air or rail are 0.3, 0.8 and 0.6 respectively. The corresponding probabilities of arriving on an agreed time are 0.2, 0.8 and 0.1 respectively. Find the probability of having used the road given that he arrived on time.

(Ans: 0.0789)

**12** A random variable *X* has probability density function

$$f(x) = \begin{cases} \frac{2}{3a}(x+a) \ ; -a < x \le 0 \\ \frac{1}{3a}(2a - x) \ ; 0 < x \le 2a \end{cases}$$

Where *a* is constant. Determine:

i) the value of *a*. (Ans: $a = 1$)

ii) the median of *x*. (Ans: 0.2679)

iii) $P[(x \le 1.5)/(x > 0)]$. (Ans: 0.9375)

iv) the cumulative distribution function $F(X)$. Sketch the graph of $F(X)$

$$(\text{Ans: } \Rightarrow F(x) = \begin{cases} 0 & x < -1 \\ \frac{1}{3}(x^2 + 2x + 1) & -1 < x < 0 \\ \frac{1}{6}(2 + 4x - x^2) & 0 < x < 2 \\ 1 & x \ge 2 \end{cases}$$

**13**. A total population of 700 students sat an examination for which the pass mark was 50. The marks were normally distributed. 28 students scored below 40 marks while 30 scored above 60.

i) Find the mean mark and standard deviation of the students. (Ans: 50.3 and 5.891)

ii) What is the probability that a student chosen at random passed the examination? (Ans: 0.5199)

iii) Suppose the pass mark is lowered by 2 marks, how many more students will pass? (Ans: 92)

**14**. In a certain commercial institution, a speed and error typing examination was administered to 12 randomly selected candidates A, B, C ....L of the institution. The table below shows their speeds (*y*) in seconds and the number of errors in their typing scripts (*x*)

| No. of errors(*x*) | A | *B* | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| | 12 | 24 | 20 | 10 | 32 | 30 | 28 | 15 |
| Speed (*y*)in S | 130 | 136 | 124 | 120 | 153 | 160 | 155 | 142 |

| I | J | K | L |
|---|---|---|---|
| 18 | 40 | 27 | 35 |
| 145 | 172 | 140 | 157 |

i) Plot the data on a scatter diagram.

ii) Draw the line of best fit on your diagram and comment on the likely association between speed and the errors made.

iii) Determine the equation of your line in the form $y = xk + b$ where k and b are constants.

(Ans: $y = \frac{26}{15}x + 102.7$ )

iv) By giving rank 1 to the fastest student, and the student with the fewest errors, rank the above data and use them to calculate the rank correlation coefficient. Comment on your result. (Ans: 0.839)

# 1996 PAPER ONE

**SECTION A (40 *marks*)**

**1**. Solve $3(3^{2x}) + 2(3^x) - 1 = 0$. (Ans: -1)

**2.** Express as equivalent fraction with a rational denominator $\dfrac{\sqrt{2}}{\sqrt{2}+\sqrt{3}-\sqrt{5}}$ (Ans: $\dfrac{3+\sqrt{6}+\sqrt{15}}{6}$)

**3**. Solve the inequality $\dfrac{x-1}{x-2} > \dfrac{x-2}{x+3}$

(Ans: $-3 < x < 7/6$ or $x > 2$)

**4**. Find how many terms of the series $1 + \dfrac{1}{5} + \dfrac{1}{5^2} + \dfrac{1}{5^3} + .....$ must be taken so that the sum will differ from the sum to infinity by less than $10^{-6}$

(Ans: 9)

**5**. Solve the simultaneous equations

$2x - 5y + 2z = 14$

$9x + 3y - 4z = 13$

$7x + 3y - 2z = 3$ (Ans: $x = 1$, $y = -4$ and $z = -4$)

**6**. Find the orthocenter (the point of intersection of the altitudes) of the triangle with vertices at $A$ (-2, 1), $B$ (3, -4) and $C$ (-6, -1). (Ans: $O$ (-2, -4))

**7.** Differentiate with respect to $x$, expressing your results as simply as possible : $\sin^{-1}\left(\dfrac{3+5\cos x}{5-3\cos x}\right)$

(Ans: $\dfrac{-4}{5+3\cos x}$)

**8**. Evaluate $\displaystyle\int_0^{\frac{1}{2}\pi} \sin 2x \cos x\, dx$. (Ans: $\dfrac{2}{3}$)

**SECTION B**

**9. a)** Find $x$ if $\log_x 8 - \log_{x^2} 16 = 1$ (Ans: $x = 2$)

**b**) The sum of $p$ terms of an arithmetic progression is $q$ and the sum of $q$ terms is $p$; find the sum of $p + q$ terms. (Ans: $x = -(p + q)$)

**10**. Given that $z = \sqrt{3} + i$, find the modulus and argument of

i. $z^2$ (Ans: 4, π/3)

ii. $\dfrac{1}{z}$ (Ans: ½, -π/6)

iii. show in an Argand diagram the points representing complex numbers $z$, $z^2$ and $\dfrac{1}{z}$

**b**) In an Argand diagram, $P$ represents a complex number $z$ such that $2|z - 2| = |z - 6\boldsymbol{i}|$
Show that P lies on a circle; find

i. the radius of this circle: (Ans: 4.2164 units)

ii. the complex number represented by its centre.

(Ans: $\dfrac{8}{3} - 2i$)

**11. a)** Find the equation of the circle circumscribing the triangle whose vertices are $A$ (1, 3), $B$ (4, -5) and $C$ (9, -1). Find also its centre and radius.

(Ans: $x^2 + y^2 - \dfrac{-113}{13}x + \dfrac{8}{13}y - \dfrac{41}{13} = 0$)

**b)** If the tangent to the circle, at $A$ (1, 3) meets the $x$-axis at $P$ ($h$, 0) and the $y$-axis at $Q$ (0, $k$), find the values of $h$ and $k$.

(Ans: Centre-$\left(\dfrac{113}{26}, \dfrac{-8}{26}\right)$, radius = 4.71 units)

**12**. a) Given that $7\tan\theta + \cot\theta = 5\sec\theta$, derive a quadratic equation for $\sin\theta$.
Hence or otherwise, find all values of $\theta$ in the interval $0^0 \le \theta \le 180^0$ which satisfy the given equation, giving your answers to the nearest 0.1°, where necessary.

(Ans: $\theta = (19.5^0, 30^0, 150^0, 160.5^0)$)

**b)** The acute angles A and B are such that $\cos\mathbf{A} = ½$, $\sin\mathbf{B} = 1/3$. Show without the use of tables or calculator, that $\tan(A+B) = \dfrac{9\sqrt{3}+8\sqrt{2}}{5}$

**13**. a) Prove that $(\sin 2\theta - \sin\theta)(1 + 2\cos\theta) = \sin 3\theta$.

b) A vertical pole $BAO$ stands with its base O on a horizontal plane, where $BA = c$ and $AO = b$. a point $P$ is situated on a horizontal plane at distance $x$ from $O$ and the angle $APB = \theta$.

Prove that $\tan\theta = \dfrac{cx}{x^2 + b^2 + bc}$

As $P$ takes different positions on the horizontal plane, find the value of $x$ for which $\theta$ is greatest.

(Ans: 18°26')

**14**. A curve is given by $y = \dfrac{(x-1)(x-9)}{(x+1)(x+9)}$

i) Determine the turning point of the curve.

(Ans: minima -$\left(3, -\dfrac{1}{4}\right)$, maxima -(-3, -4))

ii) Determine the equation of the asymptotes of the curve. (Ans: $x = 1$, $x = -9$, $y = 1$)

iii) Sketch the curve

**15. a**) Find the general solution of the equation

$x\dfrac{dy}{dx} - 2y = (x-2)e^x$. (Ans: $y = e^x + cx^2$)

b) The rate of cooling of a body is given by the equation $\dfrac{dT}{dt} = -k(T-10)$ Where $T$ is the temperature in degree Centigrade, $k$ is a constant, and $t$ is the time in minutes.
When $t = 0$, $T = 90$ and when $t = 5$, $T = 60$. Find $T$ when $t = 10$. (Ans: 41.25°)

**16**. a) In the triangle *ABC*, *P* is the point on *BC* such that $BP : PC = \lambda : \mu$

Show that $(\lambda + \mu)AP = \lambda AC + \mu AB$.

b) three non collinear points *A*, *B* and *C* have position vectors **a**, **b** and **c** respectively with respect to an origin *O*. The point *M* on *AC* is such that *AM* : *MC* = 2:1 and the point *N* on AB is such that *AN* : *NB* = 2: 1.

**i)** Show that *BM* = 1/3 **a – b** + 2/3**c**, and find a similar expression for **CN.**

(Ans: $\frac{1}{3}\mathbf{a}+\frac{2}{3}\mathbf{b}-\mathbf{c}$ )

ii) The lines BM and *CN* intersect at *L*. Given that *BL* = *rBM* and *CL* = *sCN*, where *r* and *s* are scalars, express *BL* and *CL* in terms of *r*, *s*, **a**, **b** and **c**. (Ans: $=\frac{1}{3}s\mathbf{a}+\frac{2}{3}s\mathbf{b}-s\mathbf{c}$ )

iii) Hence by using triangle *BLC*, or otherwise, find *r* and *s*. (Ans: $\mathbf{r} = 3/5$ and $\mathbf{s} = 3/5$.)

# 1996 PAPER TWO

## SECTION A

**1.** One end of a light inextensible string of length of 75cm is fixed to a point on a vertical pole. A particle of weight 12N is attached to the other end of the string. The particle is held 21cm away from the pole by a horizontal force. Find the magnitude of this force and the tension in the string.

(Ans: 12.5N, 3.5N)

**2**. A particle with position vector 4**i** + 10**j** + 20**k** moves with a constant speed of $5ms^{-1}$ in the direction of the vector 4**i** + 7**j** + 4**k**. Find its distance from the origin after 9seconds. (Ans: 85 m)

**3**. A cyclist travels 1.25km as he accelerates uniformly at a rate of $Qms^{-2}$ from a speed of $15kmh^{-1}$. Find the value of *Q*. (Ans: $16.7ms^{-2}$)

**4**. In an experiment the following observations were recorded

| | | | | |
|---|---|---|---|---|
| *T*: | 0 | 12 | 20 | 30 |
| *θ*: | 6.6 | 2.9 | -0.1 | -2.9 |

Use linear interpolation to find

i) θ when *T* = 16 (Ans: 1.4)

ii) *T* when θ = ⁻1 (Ans: 23.21)

**5**. A balanced coin is tossed three times and the number of times *X* a "Head" appears is recorded. Complete the following table

| N | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Event | {TTT} | | {HHT,HTH,THH} | |
| P(*X*=*n*) | | | | |

Determine the average or the expected number of heads to appear. (Ans: 1.5)

**6**. In a certain year in the mid -1980s, the production of tea in the common wealth as per the following countries was as shown below.

| Country | Production of tea in millions of kg |
|---|---|
| Bangladesh | 41 |
| India | 635 |
| Indonesia | 108 |
| Kenya | 140 |
| Malawi | 40 |
| Sri Lanka | 212 |
| Tanzania | 17 |
| Uganda | 7 |

Give a pie chart representation of the data

**7**. A bicycle dealer imports 40% and 60% of spare parts from countries A and B respectively. The percentages of parts produced defective in the countries are 0.3% and 0.5% respectively. A spare part is drawn at random from sample of part imported from A and B. Find the probabilities that

i) it is defective and is from country B. (Ans: 0.003)

ii) it is defective. (Ans: 0.0042)

**8**. A population consists of 15 numbers 2, 4, 7, 3, 5, 6, 3, 6, 10, 7, 8, 9, 3, 4, 3. Find:

i) the mode (Ans: 3)
ii) the median. (Ans: 5)
iii) the mean and standard deviation of the population. (Ans: 5.333, 2.3851)

## SECTION B

**9**. **(a)** Find the position of the center of gravity of three particles of masses 1kg, 5kg and 2kg which lie on the y-axis at points (0,2), (0,4) and (0,5) respectively. (Ans: (0, 4))

(b) The area enclosed by the curve $y = x^2$ and the lines $y = 0$, $x = 2$ and $x = 4$, lying in the first quadrant is rotated about the $x$–axis through one revolution . Find the co-ordinates of the center of gravity of the uniform solid so formed.

(Ans: 3.39, 0)

**10.** Initially two ships *A* and *B* are 65 km apart with *B* due east of *A* . A is moving due east at 10 km $h^{-1}$ and B due south at 24kmh$^{-1}$ .The two ships continue moving with these velocities. Find the least distance between the ships in the subsequent motion and the time taken to the nearest minute for such a situation to occur

(Ans: 60 km, 57.6 minutes)

**11.(a)** A conical pendulum consists of a light inextensible string *AB* of length 50cm fixed at A & carrying a bob of mass 2kg at *B*. The bob describes a horizontal circle about the vertical through *A* with a constant angular speed of *A* rad $m^{-1}$.
Find the tension in the string. (Ans: $T = \omega^2$)

**(b)** A smooth surface is inclined at $30^0$ to the horizontal. A body A of mass 2kg is held at rest on the surface by a light elastic string which has one end attached to A and the other to a point on the surface 1.5m away from A up a line of greatest slope. If the modulus of the string is 2g N, Find its natural length . (Ans: 1m)

**12**. A block of mass 6.5kg is projected with a velocity of 4ms$^{-1}$ up a line of greatest slope of a rough plane. Calculate the initial kinetic energy of the block.

(Ans: 52J)

The coefficient of friction between the block and the plane is 2/3 and the plane makes an angle $\theta$ with the horizontal where $\sin\theta = 5/13$. The block travels a distance *d* m up the plane before coming instantaneously to rest. Express in terms of *d*

i) the potential energy gained by the block in coming to rest. (Ans: 25*d* J)

ii) the work done against friction by the block in coming to rest. Hence calculate the value of *d*. (take g = 10ms$^{-1}$). (Ans: $d$ = 0.8)

**13**. i)Show that the iterative formula for solving the equation $2x^2 - 6x - 3 = 0$ is $x_n + 1 = \frac{2x_n^{\ 2} + 3}{4x_n + 6}$

ii) Show that the positive root for $2x^2 - 6x - 3 = 0$ lies between 3 and 4. Find the root correct to 2 decimal places. (Ans: 3.44)

**14**. In a survey of newspaper reading of members of staff of a university, it is found that 80% read NEW VISION (N), 50 percent read MONITOR (*M*) and 30% read the EAST AFRICAN (*E*). Further, 20% read both *M* and *N*, 15% read both *N* and E and 10% read both *M* and E.

a) If a member of staff is chosen at random from the university, find the probabilities

i) that the member reads none of the three papers.

ii) the member is one of those who read at least one of the three papers.

b) Estimate the number of members of staff who read at least two papers if the total number is 500

**c**) What is the probability that given that a member of staff reads two papers, he reads all the three?

(Ans: *impossible to obtain answers for this question)*

**15**. A certain factory produces ball bearings. A sample of the bearings from the factory produced the following results

| Diameter of bearings in mm | Frequency |
|---|---|
| 91 – 93 | 4 |
| 94 – 96 | 6 |
| 97 – 99 | 34 |
| 100 – 93 | 40 |
| 103 – 102 | 13 |
| 106 – 108 | 3 |

i) Determine the mean and diameter of the sample bearings. (Ans: 99.83 mm, 9.3411)

ii) Estimate the mean surface area of the bearings produced by the factory. (Ans: 31293.33)

**16**. The following table gives the marks obtained in Calculus, Physics and Statistics by seven (7) students

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Calculus** | 72 | 50 | 60 | 55 | 35 | 48 | 82 |
| **Physics** | 61 | 55 | 70 | 50 | 30 | 50 | 78 |
| **Statistics** | 50 | 40 | 62 | 70 | 40 | 40 | 60 |

Draw scatter diagrams and determine rank correlation coefficients between the performances of the students in

i. Calculus and Physics. (Ans: 0.902)
ii. Calculus and Statistics. (Ans: 0.643)

Give interpretations to your results.

# 1997 PAPER ONE
## SECTION A

**1**. Solve the equation 4cos$x$ – 2cos2$x$ = 3 for $0^0 \le x \le \pi$.
(Ans: $x$ = 60°)

**2**. Find the values of $k$ for which the equation $\frac{x^2 - x + 1}{x - 1}$ has repeated roots. What are the repeated roots?
**(Ans:** 0 and 0 or 2 and 2)

**3**. By reducing to echelon form, solve the simultaneous equations
$x + y + z = 0$
$x + 2y + 2z = 2$
$2x + y\ 3z = 4$. (Ans: $x$ = -2, $y$ = -1 and $z$ = 3)

**4**. Given that $x = \theta - \sin\theta$,
$y = 1 - \cos\theta$,
Show that $\frac{dy}{dx} = \cot\frac{\theta}{2}$.

**5.** *ABCD* is a square inscribed in a circle $x^2 + y^2 - 4x - 3y = 36$. Find the length of diagonals and the area of the square. (Ans: 84.5 Sq. Units)

**6**. Find. i) $\int \sin^2 x dx$ (Ans: $\frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right] + C$ )

ii) $\int \tan^3 x dx$ (Ans: $\frac{1}{2}\tan^2 x - \ln\cos x + C$ )

**7.** Find the distance of the point (-2, 0, 6) from the plane $2x - y + 3z = 21$. (And: 1.8708 units)

**8.** Determine the volume of the solid generated when the region bounded by the curve $y = \cos 2x$ and the $x$-axis for values of $x$ between 0 and ¾ is rotated about the $x$-axis. (Ans: 1.232 cubic units)

## SECTION B

**9**. If $\log_b a = x$, show that $b = a^{1/x}$ and deduce that

$$\log_a b = \frac{1}{\log_b a}$$

**b**) Solve i) $\log_n 4 + \log_4 n^2 = 3$ (Ans: $n$ = 2 or 4)

ii) $2^{2x-1} + \frac{3}{2} = 2^{x+1}$ (Ans: $x$ = (0, 1.585))

**10(a)**. Given the complex numbers $z_1 = 1 - \boldsymbol{i}$; $z_2 = 7 + \boldsymbol{i}$ represent $z_1 z_2$ and $z_1 - z_2$ on the Argand diagram.

Determine the modulus and argument of $\frac{z_1 - z_2}{z_1 z_2}$

(Ans: 0.6325, -124.6952$^0$)

**b**) If $z$ is a complex number in the form ($a + b\boldsymbol{i}$),

solve $\left(\frac{z-1}{z+1}\right)^2 = i$. (Ans: $Z = 1 \pm \mathbf{i}\sqrt{2}$ )

**11**.a) Prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

**b**) Find all the solutions to $2\sin^3\theta = 1$ for $\theta$ between $0^0$ and $360^0$. Hence find the solutions of $8x^3 - 6x + 1 = 0$. (Ans: $x$ = (1.734, 0.766, -0.9367))

**12**. The points $A$, $B$ and $C$ have position vectors (-2**i** + 3**j**), (**i** – 2**j**) and (8**i** – 5**j**) respectively.

i. Find the vector equation of line $AC$
(Ans: $\mathbf{r} = -2\mathbf{i} + 3\mathbf{j} + \lambda(10\mathbf{i} - 8\mathbf{j})$)

ii. Determine the coordinates of $D$ if $ABCD$ is a parallelogram. (Ans: (5, 0))

iii. Write down the vector equation of the line through which point $B$ perpendicular to $AC$ and find where it meets $AC$.
(Ans: $\mathbf{r} = \mathbf{i} - 2\mathbf{j} + t(4\mathbf{i} + 5\mathbf{j})$ )

**13**. Express $f(x) = \frac{2x^2 - x + 14}{(4x^2 - 1)(x + 3)}$ in partial fractions.

Hence evaluate $\int_1^3 f(x)dx$. (Ans: 0.7440)

**14**. Find the equation of the chord joining the points $\left(ct_1, \frac{c}{t_1}\right)$ and $\left(ct_2, \frac{c}{t_2}\right)$ on a hyperbola. Hence

deduce the equation of the tangent at $\left(ct, \frac{c}{t}\right)$.

(Ans: $x + t_1 t_2 y - c(t_1 + t_2) = 0$ )

Find the equation of the tangents to the hyperbola $x = 4t$, $y = 4/t$ which passes through point (4, 3)
(Ans: $x + 4y - 16 = 0$ and $9x + 4y - 48 = 0$)

**15**. Show that the tangents at (-1, 3) and (1, 5) on the curve $y = 2x^2 + x + 2$ passes through the origin. Find the area enclosed between the curve and these two tangents. (Ans: $4/3$ sq.units )

**16**. a) Use Maclaurin's theorem to expand ln(1 + sin $x$) as far as the term in $x^3$. (Ans: $x - \frac{x^2}{2} + \frac{x^3}{6} + ....$)

**b**) Expand $(1 - x)^{\frac{1}{3}}$ as far as the $x^3$. Use your expansion to deduce $\sqrt[3]{24}$ correct to three significant figures.
(Ans: 2.88)

# 1997 PAPER TWO
## SECTION A

**1**. A bag contains 5 white, 3 red and 2 green counters. 3 counters are drawn without replacement. What is the probability that there
  i) is no green counter, (Ans: 0.4667)
  ii) are 2 white counters and a green counter?
(Ans: 1/6)

**2**. Given below is a table of corresponding values of *x* and *y*

| *x* | 0 | 8 | 12 | 20 |
|---|---|---|---|---|
| *y* | 9.2 | 6.0 | 4.4 | 1.5 |

Use linear interpolation to find
(i) y when $x = 15$ (Ans: 3.3125)
(ii) *x* when y = 5.0 (Ans: 10.5)

**3**. A particle with vector 10**i** +3**j**+5**k** moves with constant speed of 6ms$^{-1}$ in the direction **I** +2**j** +2**k**. Find its distance from the origin after 5 seconds.
(Ans: 39.42 m)

**4.** Particles of mass 4, 5 and 6kg are placed at (0, 0), (4, 3) and (5, -2) respectively in the *x*-y plane , Find the co-ordinates of their centre of mass . (Ans: ($\frac{10}{3}$, $\frac{1}{5}$))

**5**. The yields of 13 plots in 1000's of kg were 16, 7, 10, 3, 11, 5, 8, 14, 18, 4, 11, 14 and 90
Find the: (i) Mean (Ans: 10,000)
(ii) Standard deviation (Ans: 44549 )

**6.** A box of mass 2kg is at rest on a plane inclined at 25° to the horizontal. The coefficient of friction between the box and the plane is 0.4. What minimum force applied parallel to the plane would move the box up the plane? (Ans: 15.388N)

**7**. The probability of winning a game is $\frac{4}{5}$ . Ten games are played. What is the:
(i) mean number of successes. (Ans: 8)
(ii) variance (Ans: 1.6)
(iii) probability of at least 8 successes in the ten games. (Ans: 0.6778)

**8**. A mass of 3kg is at rest on a smooth horizontal table .It is attached by a light inextensible string passing over a smooth fixed pulley at the edge of the table to another mass of 2kg, which is hanging freely. The system is released from rest. Determine the resulting acceleration and the tension in the string.

## SECTION B

**9**. (a) The table below shows the likelihood of where A and B spend Saturday evening:

| | *A* | *B* |
|---|---|---|
| Goes to dance | $\frac{1}{2}$ | $\frac{2}{3}$ |
| Visits neighbour | $\frac{1}{3}$ | $\frac{1}{6}$ |
| Stays at home | $\frac{1}{6}$ | $\frac{1}{6}$ |

i) Find the probability that both go out. (Ans: 25/36)
ii) If we know that they both go out, what is the probability that both go to dance? (Ans: 12/25)

b) Four competitors throw a die in turn. What is the probability that
(i) they all score more than a 4) (Ans: 1/81)
(ii) two get less than a 3? (Ans: 2/3)
(iii) the total score is 23? (Ans: 1/324)

**10**. The probability density function of a random variable *X* is given by

$$f(x) = \begin{cases} k(x+2)\ ; -1 < x \le 0 \\ 2k(1-x);\ \ 0 < x \le 1 \\ 0 \quad ;else\ where \end{cases}$$

i. Sketch the function
ii. Find *k* and the mean of *x*. (Ans: $k$ = 3/5, -2/15)
iii. Find the probability $P(0 < x < ½x/ x > 0)$
(Ans: ¾)

**11**. The heights of students in S 1 were according to the following frequency table

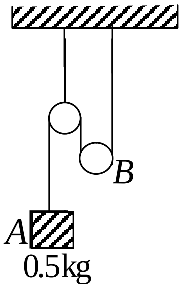
| Class | *f* |
|---|---|
| 151-153 | 2 |
| 154-156 | 14 |
| 157-159 | 13 |
| 160-162 | 13 |
| 163-165 | 2 |
| 166-168 | 1 |

(i) Estimate the mean and standard deviation of the height of students. (Ans: 158.133, 3.222)
(ii) Determine and plot the cumulative frequency distribution for the students' heights. Hence estimate the median, lower and upper quartiles for the heights of the students.
(Ans: = 158 cm, 155.3 cm, 160.5 cm)

**12**. Using the iterative formula show that the 4$^{th}$ root of the number *N* is $x_{n+1} = \frac{3}{4}x_n + \frac{N}{4x_n^{\ 3}}$. Hence show that (45.7)¼ ≈2.600 (correct to 3dps)

**13.a**) Two particles are moving towards each other along a straight line. The first particle has a mass of 0.2kg and moving with a velocity of 4ms$^{-1}$, and the second has a mass of 0.4kg moving with a velocity of 3ms$^{-1}$. On collision, the first particle reverses its direction and moves with a velocity of 2.5ms$^{-1}$ . Find:
I) velocity of the second particle after collision
(Ans: 0.25 ms$^{-1}$)
II) percentage loss in kinetic energy. (Ans: 81.25%)

**b** The diagram shows particle *A* of mass 0.5kg attached to one end of a light inextensible string passing over a fixed light pulley and under a movable light pulley *B*. The other end of the string is fixed as shown below.




(i) What mass should be attached at $B$ for the system to be in equilibrium. (Ans: 1 kg)

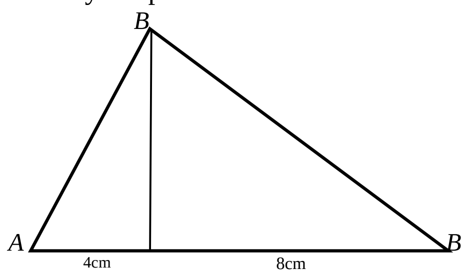
(ii) If $B$ is 0.8kg what are the accelerations of particle $A$ and pulley $B$? (Ans: $g/14$ ms$^{-2}$)

**14**. A particle of masses½ kg is released from rest and slides down a rough plane inclined at 30° to the horizontal. It takes 6 seconds to go 3 meters.

(i) Find the coefficient of friction between the particle and the plane (correct to the 2d.p) (Ans: 0.557)

(ii) What minimum horizontal force is needed to prevent the particle from moving? (Ans: 0.0855)

**15.** Two uniform rods A$B$, $B$C of masses 4kg and 6kg respectively are hinged at $B$ and rest in a vertical position on a smooth floor as shown. A and C are connected by a rope.




**a**)Find the reactions between the rods and the floor at A and C when the rope is taut. (Ans: $14g/3$, $16g/3$)

**b**)If now a body is attached a quarter of the way up C$B$, and the reactions are equal, find the mass of the body. (Ans: 1kg)

**16** At noon, a boat A is 30 km from boat B and its direction from B is $286^0$. Boat A is moving from North East direction at 16kmh$^{-1}$ and boat B is moving in the Northern direction at 10kmh$^{-1}$. By scale drawing or otherwise determine when they are closest to each other. What is the distance between them then? (Ans: 2:26 pm, 11.54 km)

# 1998 MARCH PAPER ONE

## SECTION A

**1.** Solve simultaneously

$$\frac{1}{x}-\frac{1}{y}=\frac{1}{6}$$

$x(5-x)=2y$ (Ans: $(x, y)$ = (9, -18) and (2, 3))

**2**. Prove that $\log_6 x=\frac{\log_3 x}{1+\log_3 2}$. Hence given that $\log_3 2 = 0.631$, find without using tables or calculator $\log_6 4$ correct to 3 significant figures.

**3**. Show that $\cos 4\theta=\frac{\tan^4\theta-6\tan^2\theta+1}{\tan^4\theta+2\tan^2\theta+1}$

**4**. The distance S m of a particle from a fixed point is given by $S=t^2\left(t^2+6\right)-4t\left(t-1\right)\left(t+1\right)$, where $t$ is the time. Find the velocity and acceleration of the particle when $t$ = 1s. (Ans: 0 ms$^{-2}$, 8 ms$^{-1}$)

**5.** Using the substitution $2x + 1 = p$, find $\int_0^1 \frac{xdx}{\left(2x+1\right)^3}$

**6**. ABCD is a quadrilateral with A(2, -2), B(5, -1), C(6, 2) and D(3, 1). Show that the quadrilateral is a rhombus. (Ans: 1/8)

**7.** The points P(4, -6, 1), Q(2, 8, 4), and R(3, 7, 14) lie in the same plane. Find the angle formed between PQ and QR. (Ans: 84.5°)

**8**. Use Maclaurin's expansion to express $\ln(1 + x)^2$ in ascending powers of $x$ up to the term in $x^4$ (Ans: $2x-x^2+\frac{2x^3}{3}-\frac{x^4}{2}$)

## SECTION B

**9**. When $f(x) = x^3 - ax + b$ is divided by $x + 1$, the remainder is 2 and $x + 2$ is a factor of $f(x)$. Find $a$ and $b$ (Ans: $a$ = 5 and $b$ = -2)

**b)** If the roots of the equation $x^2 + 2x + 3 = 0$ are $\alpha$ and $\beta$, form the equation whose roots are $\alpha^2-\beta$ and $\beta^2-\alpha$. (Ans: $x^2 + 2 = 0$)

**10. a)** Show the region represented by $|z - 2 + i| < 1$ on an Argand diagram

**b**) Express the complex number $z = 1-\sqrt{3}i$ in modulus argument form and hence find $z^2$ and $\frac{1}{z}$ in the form $a + b$i. (Ans: $\frac{1}{z}=\frac{1}{4}+\frac{i\sqrt{3}}{4}$)

**11 (a)** Differentiate with respect to $x$

**i)** $2x^x$ (Ans: $2x^x\ln(x+1)$)

**ii)** $\sin^3 2x$ (Ans: $6\sin^2 2x\cos 2x$)

**b)** Find the equation of the tangent and normal to the curve $y=4x^3-6x^2+3x$ at the point (1, 1) (Ans: $y = 3x - 2$)

**12.(a)** Find the solution of 3 cot $\theta$ + cosec $\theta$ = 2 for

$0 \le \theta \le 360^0$. (Ans: $\theta$ = 72.40°, 220.2°)

b) Solve $2 \sin x = \sin(x - 60^0)$ for $-180^0 \le x \le 180^0$. (Ans: $x$ = -30$^0$, 150$^0$)

**13.** $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ are two points on the parabola $y^2 = 4ax$, $PQ$ is a focal chord. Prove that $pq$ = -1 and hence that if the tangents at $P$ and $Q$ intersect at $T$, the locus of $T$ is given by $x + a = 0$. $PM$ and $QN$ are perpendiculars onto $x + a = 0$, $s$ = ($a$, 0). Prove that $M\hat{S}N = 90^0 = P\hat{T}Q$.

**14. a**) Integrate $\dfrac{4x^2}{\sqrt{(1-x^6)}}$ with respect to $x$

(Ans: $\frac{4}{3}\sin^{-1}(x^3) + c$)

**b**) Evaluate $\int_1^3 \frac{x^2+1}{x^3+4x^2+3x}dx$ (Ans: 0.3489)

**15.** Solve a) $y\frac{dy}{dx} = 2x - y$ by using the substitution $y = uv$. (Ans: $\left(\frac{y}{x}+2\right)^2\left(\frac{y}{x}-1\right) = k$)

b) $\frac{dy}{dx} - y\tan x = \cos^2 x$

**(Ans:** $y = \sin x \sec x - \frac{1}{3}\sin^3 x \sec x + k\sec x$**)**

**16**. Given that $\mathbf{OP} = \begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix}$ and $\mathbf{OQ} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$, find the coordinates of the point $R$ such that $\overline{PR} : \overline{PQ} = 1:2$ and the points $P$, $Q$ and $R$ are collinear.

(Ans: $R$(2.5, -1.5, 3.5)

**b**) Show that the vector $5\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ is perpendicular to the line $\mathbf{r} = \mathbf{i} - 4\mathbf{j} + t(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$.

c) Find the equation of the plane through the point with position vector $5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ perpendicular to the vector $3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$. (Ans: $3x + 4y - z = 4$)

# 1998 MARCH PAPER TWO

## SECTION A

**1**. A discrete random variable, $X$, has the following probability distribution:

| $X$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| P($X$) | $\frac{3}{16}$ | ½ | ¼ | $\frac{1}{16}$ |

Find the mean and variance of $X$ (Ans: 2.1875), 0.6523)

**2**. A carton of mass 0.4 kg is thrown across a table with a velocity of 25 ms$^{-1}$. The resistance of the table to its motion is 50N. How far will it travel before coming to rest? What must be the resistance if it travels only 2 meters. (Ans: 2.5m, 62.5N)

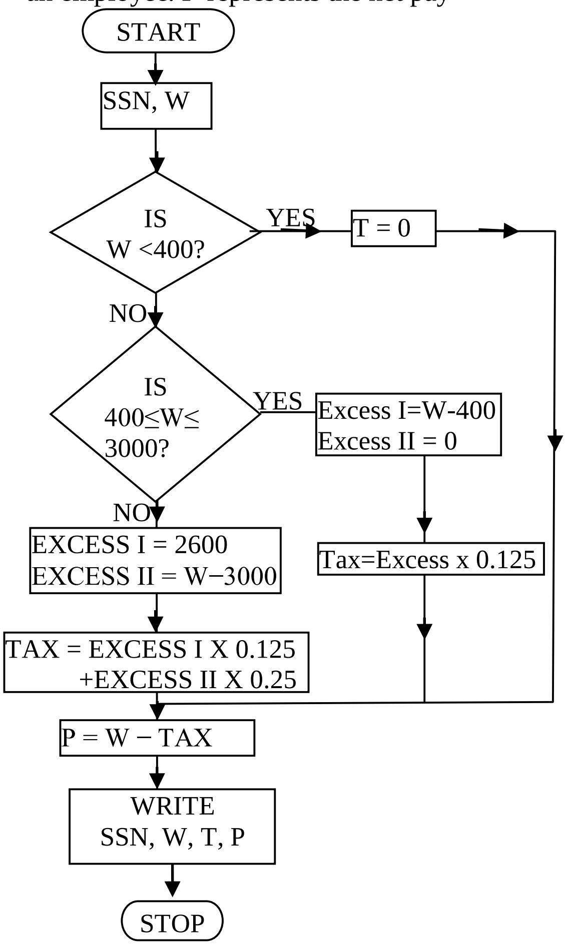
**3.** A motorcycle decelerated uniformly from 20 km$^{-1}$ to 8kmh$^{-1}$ in travelling 896m. Find the rate of deceleration in ms$^{-2}$. (Ans: 0.0145ms$^{-2}$)

**4** A particle of weight 8N is attached to point B of a light inextensible string AB. It hangs in equilibrium with point A fixed and AB at angle of 30$^0$ to the downward vertical. A force F at B acting at right angle to AB, keeps the particle in equilibrium. Find the magnitude of F and the tension in the string. (Ans: 4N, $4\sqrt{3}$N)

**5.** There are 3 black and 2 white balls in each of the two bags. A ball is taken from the first bag and put in the second, then a ball is taken from the second into the first, what is the probability that there are now the same number of black and white balls in each bag as there were to begin with? (Ans: 3/5)

**6.** Using the same graph show that the curves $x^3$ and $2x + 5$ have a common real root. Using the Newton Raphson's formula twice, find the positive root of the equation $x^3 - 2x - 5 = 0$ giving your answer correct to 2dps. (Ans: 2.09)

**7.** The flow chart below shows the social security numbers (SSN) and the monthly wage ($W$ shillings) of an employee. $P$ represents the net pay




Copy and complete the following

| $SSN$ | $W$ | $T$ | $P$ |
|---|---|---|---|
| 280 – 04 | 380 | ..... | ..... |
| 180 – 34 | 840 | ..... | ..... |
| 179 – 93 | 4500 | ..... | ..... |
| 80 – 66 | 5,550 | ..... | ..... |

| | | | |
|---|---|---|---|
| 385 – 03 | 8000 | ..... | ..... |

**8.** 64% of the students at A' level take science subjects and 36% do Arts subjects. The probability of them being successful is ¾ for Science students and 5/6 for Arts students. Find the probability that a student chosen at random will fail. (Ans: 0.22)

## SECTION B

**9(a).** Given that the values $x = 4$, $y = 6$ and $z = 8$ each has been approximated to the nearest integer. Find the maximum and minimum values of

(i) $\frac{y}{x}$, (Ans: 1.857142857, 1.2222)

(ii) $\frac{z-x}{y}$ (Ans: 0.90909090, 0.46153846)

(iii) $(x + y)z$. (Ans: 93.5, 67.5)

**(b).** A Company had a capital of sh.500 million. The profit in a certain year was sh.25.8 million in section A of the company and sh.14.56 million in section *B*. There was a possible error of 5% in section *A* and an 8% error in *B*. Find the maximum and minimum values of the total profits of the sections as a percentage of the capital.

(Ans: 8.56%, 7.58%)

**10.** (a) Show that the Newton Raphson's formula for finding the smallest positive root of the equation $3\tan x + x = 0$ is $\frac{6x_n - 3\sin 2x_n}{6 + 2\cos 2x_n}$

(b) By sketching the graphs of $y = \tan x$, $y = \frac{-x}{3}$ or otherwise, find the first approximation to the required root and use it to find the actual root correct to 3dps (Hint work in radians).

(Ans: 2.456)

**11.** The diameter of a sample of oranges to the nearest cm were:

| Diameter(cm) | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|
| Frequency | 9 | 15 | 21 | 32 | 19 | 13 | 11 |

i) Calculate the mean and standard deviation.

(Ans: 11)

ii) Assuming the distribution is normal, find the minimum diameter if the smallest 10% of the oranges are rejected for being too small.

(Ans: 1.6633)

**12.** A pupil has ten multiple choice questions to answer. There are four alternative answers to choose from. If a pupil answers the questions randomly, find the

i) probability that at least four answers are correct.

(Ans: 0.2241)

ii) most likely number of correct answers (Ans: 2)

b) Otim' chances of passing Physics are 0.60, of Chemistry 0.75 and of Mathematics 0.80.

i) Determine the chance that he passes one subject only. (Ans: 0.17)

ii) If it is known that he passed at least two subjects, what is the probability that he failed Chemistry? (Ans: 0.4815)

**13.** A random variable X has a distribution probability function given by

$$f(x) = \begin{cases} kx, & 0 \le x \le 1 \\ k(4 - x^2), & 1 \le x \le 2 \\ 0, & Elsewhere \end{cases}$$

i) Find the constant $k$ (Ans: 6/13)

ii) Determine $E(X)$ and var $(X)$. (Ans: 1.1923, 0.1399)

iii) Find the cumulative distribution function, $F(x)$ and sketch it.

Ans: $$F(x) = \begin{cases} 0 & x < 0 \\ \frac{3}{13}x^2 & 0 \le x \le 1 \\ \frac{24x - 2x^3 - 19}{13} & 1 \le x \le 2 \\ 1 & x \ge 2 \end{cases}$$

**14**. At the points A (0, - 4), B (2, 1), C (1, 3) and D (-4,-2), there are forces $\begin{pmatrix} -1 \\ -5 \end{pmatrix}$, $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$N respectively

(i) Prove that the resultant is a couple and find its moment

(ii) If the force at D is halved, find the magnitude of the resultant force. Find also the equation of the line of action of the resultant. (Ans: $y = 2x - 13$)

**15.(a)** A particle is projected vertically upwards from a point *O* with speed $\frac{4}{3}v$, After it has travelled a distance $\frac{2}{5}x$ above *O*, on its upward motion, a second particle is projected vertically upwards from the same point and with the same initial speed.

Given that the particles collide at a height $\frac{2}{5}x$ above *O*, $x$ and $v$ being constant, show that

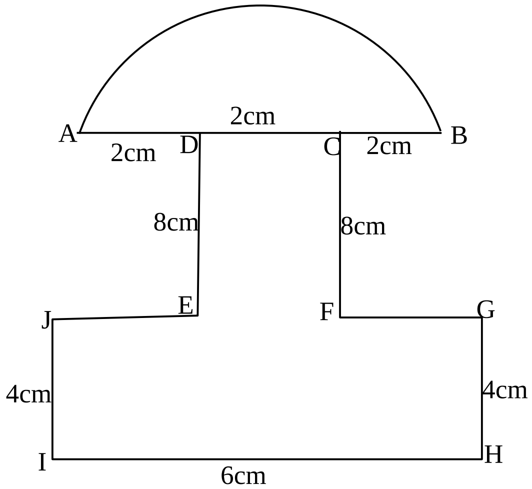
i) at maximum height $H$, $8v^2 = 9gH$,

ii) when the particles collide $9x = 20H$.

**b**) A stone thrown upwards at an angle $\alpha$ to the horizontal with speed,$u$ms$^{-1}$, just clears a vertical wall 4m high and 10m from the point of projection when travelling horizontally. Find the angle of projection.

(Ans: 38.66°)

**16**




The figure ABCDEFGHIJ shows a symmetrical composite lamina made up of a semi-circle, radius 3cm, a rectangle CDEF 2cm × 8cm and another rectangle GHIJ 6cm × 4cm. Find the distance of the centre of gravity of this lamina from *IH*. If the lamina is suspended from *H*, by means of a peg through a hole, calculate the angle of inclination of *HG* to the vertical.

**(Ans:** 6.716 cm from IH, 24.07° to the vertical)

# 1998 NOV/DEC. PAPER ONE

## SECTION A

**1**. Solve $\cos\theta + \sqrt{3}\sin\theta = 2$ for $0^0 \le \theta \le 360^0$
(Ans: $\theta = 60^0$)

**2**. The gradient of a certain curve is given by *kx*. If the curve passes through the point (2, 3) and the tangent that this angle makes is an angle of $\tan^{-1} 6$ with the positive direction of the *x*-axis, find the equation of the curve. (Ans: $y = \frac{3}{2}x^2 - 3$)

**3**. Given that the roots of the equation $x^2 - 2x + 10 = 0$ are α and β, determine the equation whose roots are $\frac{1}{(2+\alpha)^2}$ and $\frac{1}{(2+\beta)^2}$ (Ans: $324x^2 + 1 = 0$)

**4.** By row reducing the appropriate matrix to echelon form, solve the system of equations

$x + 2y - 2z = 0$
$2x + y - 4z = -1$
$4x - 3y + z = 11$

(Ans: $x = 3$, $y = 1$ and $z = 2$)

**5**. Find in the simplest form the derivative of

$\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ (Ans: $\frac{2}{1+x^2}$)

**6.** Show that $\int \tan^n x dx = \frac{\tan^{n-1}}{n-1} - \int \tan^{n-2} x dx$ . Hence or otherwise evaluate $\int \tan^4 x dx$ .

**7.** Calculate the area of triangle with vertices $(^-1, 3)$, (5, 2) and $(4, ^-1)$ (Ans: 7.6811 sq. units)

**8.** *PQRS* is a quadrilateral with vertices P(2, -1), Q(4, -1) and S(2, 1). Show that the quadrilateral is a rhombus.

## SECTION B

**9**. a) Given that $Z_1 = -i + 1$, $Z_2 = 2 + i$ and $Z_3 = 1 + 5i$, represent $Z_2Z_3$, $Z_2 - Z_1$ and $\frac{1}{Z_1}$ on the Argand diagram. Also show the representation of $\frac{Z_2Z_3}{Z_2 - Z_1} + \frac{1}{Z_1}$

**b**) Prove that for positive integer *n*, $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ . Deduce that this formula is also true for negative values of *n*.

**10. a)** Solve $4^x - 2^{x+1} - 15 = 0$ (Ans: $x = 2.3219$)

**b**) Five million shillings are invested each year at a rate of 15% interest. In how many years will it accumulate to more than sh. 50 million?

(Ans: 6 years)

**11(a)** If $y = \tan\left(\frac{x+1}{2}\right)$, show that $\frac{d^2y}{dx^2} = y\frac{dy}{dx}$

**b)** Find the equation of the tangent to the curve $x^2 + y^2 - 2xy = 4x$ at (1, -1) (Ans: $y = -1$)

**12.** The vector equations of lines $P$ and $Q$ are given as $r_p = t(4\mathbf{i} + 3\mathbf{j})$ and $r_q = 2\mathbf{i} + 12\mathbf{j} + 5(\mathbf{i} - \mathbf{j})$.

a) Use the dot product to find the angle between $P$ and $Q$ (Ans: 8.13)

b) If the lines $P$ and $Q$ meet at $M$, find the coordinates of $M$. Find also the equation of the line through $M$ perpendicular to the line $Q$ (Ans: $M(7, 7)$)

**13**. Sketch the curve $y = x - \frac{8}{x^2}$ for $x > 0$, showing any asymptotes. Find the area enclosed by the $x$-axis, the line $x = 4$ and the curve $y = x - \frac{8}{x^2}$. (Ans: 10 sq units)

If this area is now rotated about the $x$-axis through $360^0$, determine the volume of the solid generated, correct to 3 significant figures.

(Ans: 42.1497 cubic units)

**14.** From the top of a tower 12.6m high, the angles of depression of ships $A$ and $B$ are $12^0$ and $18^0$ respectively. The bearing of ship A and ship B from the tower are $148^0$ and $209½^0$ respectively. Calculate:

i ) how far apart the ships are from each other (Ans: 53.1412 m)

ii) the bearing of ship $A$ from ship $B$. (Ans: 108.11°)

**15**. a)Prove that $\sin 2x = \frac{2\tan x}{1+\tan^2 x}$

b) Solve for $x$ in:

(i) $\tan x + 3\cot x = 4$

(Ans: Hence $x = (45^o, 71.565^o, 225^o, 251.565^o)$

(ii) $4\cos x - 3\sin x = 2;\ 0 \le x \le 360^0$

(Ans: $x = 29.5^0, 256.7^0$)

**16**. A rumour spreads through a town at a rate which is proportional to the product of the number of people who have heard it and that of those who have not heard it. Given that $x$ is a fraction of the population of the town who have heard the rumour after time, $t$.

i) Form a differential equation connecting $x$, t and a constant, $k$. (Ans: $\frac{dx}{dt} = k(1-x)x.$)

ii) If initially a fraction C of the population had heard the rumour. Deduce that $x = \frac{C}{C+(1-C)^{e^{-kt}}}$

**iii)** Given that 15% had heard the rumour at 9:00 a.m and another 15% by noon, find what fraction of the population would have heard the rumour by 3:00 p.m (Ans: 21%)

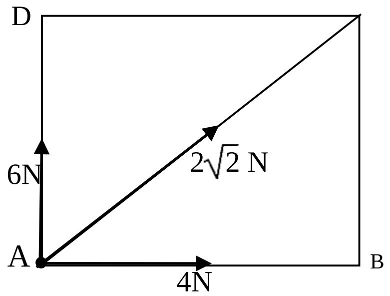
# 1998 NOV/DEC PAPER TWO

## SECTION A

**1.** The probability that two independent events occur together is $\frac{2}{15}$. The probability that either or both events occur is $\frac{3}{5}$. Find the individual probabilities of the two events. (Ans: 1/3, 2/5)

**2.** Find an expression for the power exerted by a force $\mathbf{r} = 4t^2\mathbf{i} + 4t\mathbf{j} + 7t^2\mathbf{k}$ acting on a particle to give it a velocity of $\mathbf{v} = t\mathbf{i} - 3t^2\mathbf{j} + 2t\mathbf{k}$. Find also the acceleration of the particle. (Ans: $6t^3$, $\mathbf{i} - 6t\mathbf{j} + 2\mathbf{k}$ )

**3.**




Three forces of magnitude 6N, 4N, and $2\sqrt{2}$ N act along $AD$, $AB$ and AC respectively as shown above. $ABCD$ is the square. Determine the resultant of the three forces and the angle, which it makes with AB.

(Ans: 10N, 53.13°)

**4.** Given $X = 2.2255$, $Y = 0.449$ correct to the given number of decimal places. State the maximum possible errors in the value of $X$ and $Y$. hence determine the

i) absolute error (Ans: $5.63 \times 10^{-3}$)

ii) limits with in which the value of quotient $\frac{X}{Y}$ lies giving your answers to 2 decimal places

(Ans: 4.95 and 4.96)

**5.** The probability that Bob wins a tennis game is 2/3. He plays 8 games. What is the probability that he wins

i) at least 7 games, (Ans: 0.1951)

ii) exactly 5 games? (Ans: 0.2731)

**6.** An elastic string of natural length 1.2 m and modulus of elasticity 8 N is stretched until the extending force is 16N. Find the extension and the work done.

(Ans: 0.6m, 27 joules)

**7.** By using the Newton Raphson formula and $x_0 = \frac{\pi}{2}$ as the initial approximation to the root of the equation $10\cos x - x = 0$, show that the next approximation is $\frac{5\pi}{11}$

**8**. The table below shows the distribution of marks gained by a group of students in a mathematics test marked out of 50.

| Marks | Frequency |
|---|---|
| 1 − 10 | 15 |

| | |
|---|---|
| 11 − 20 | 20 |
| 21 − 30 | 32 |
| 31 − 40 | 26 |
| 41 − 50 | 7 |

Plot an ogive for the data and use it to estimate the median mark and semi – interquartile range.

(Ans: 25.5, 8.5)

## SECTION B

**9** (a) Show that the iterative formula for finding the 4th root of a number $N$ is given by: $x_{n+1} = \frac{3}{4}\left(xn + \frac{N}{3x_n{}^3}\right)$,

$n$ = 0,1,2,3.

(b) Draw a flowchart that

(i) Reads the number $N$ and the initial approximation $x_n$

(ii) Computes and prints $N$ and its fourth root after 4 iterations and give the root correct to 3dps

(c) Perform a dry run for $N$ = 39.0 and $x_n$ = 2.0

(Ans: 39.0)

**10**. A particle is projected with a speed of $10\sqrt{2g}$ $ms^{-1}$ from the top of a cliff, 50m high. The particle hits the sea at a distance 100m from the vertical through the point of projection. Show that there are two possible directions of projection, which are perpendicular. Determine the time taken from the point of projection in each case.

(Ans: 76.72°, -13.28°, $t$ = 2.321 s)

**11.** A probability density function is given as

$$f(x) = \begin{cases} kx(4 - x^2); & 0 \le x \le 2 \\ 0 & ; Elsewhere \end{cases}$$

Find the

i) value of $k$ (Ans: ¼)

ii) median (Ans: 1.0824)

iii) mean (Ans: 1.0667)

iv) standard deviation (Ans: 0.4422)

**12.** Show that the root of the equation $f(x) = e^x + x^3 - 4x = 0$ lies between 1 and 2. By using the Newton Raphson method. Find the root to 2decial places. (Ans: 1.12)

**13**. To one end of a light inelastic string is attached a mass of 1kg which rests on a smooth wedge of inclination $30^0$. The string passes over a smooth fixed pulley at the edge of the wedge, under a second smooth moveable pulley of mass 2kg and over a third smooth fixed pulley ,and has a mass of 2kg attached to the other end . Find the accelerations of the masses and the moveable pulley and the tension in the string. (Assume the portions of the string lie in the vertical plane). (Ans: $a_1 = \frac{1g}{2}$, $\frac{1}{2}g$ , 0, $T$ = 9.8N).

**14**. Boxes made in a factory have weights which are normally distributed with a mean of 4.5 kg and a standard deviation of 2.0kg. Find the probability of there being a box with a weight of more than 5.4 kg when a box is chosen at random. If a sample of 16 boxes is drawn, find the probability that the mean is between

i) 4.6 and 4.7 kg, (Ans: 0.0761)

ii) 4.3 and 4.7 kg. (Ans: 0.3108)

**15**. The ages of people in a town were as follows

| Age(years) | 0-<5 | 5-<15 | 15-<30 | 30-<50 | 50-<70 | 70-<90 |
|---|---|---|---|---|---|---|
| Number (thousands) | 4.4 | 8.1 | 10.5 | 14.6 | 9.8 | 4.7 |

a. Draw a histogram for this data

b. State the modal age interval (Ans: 0 - < 5)

c. Estimate the:

i. average age of the town (Ans: 36.0125)

ii. number of people under 18 years (Ans: 14600)

iii. median age (Ans: 34.1781)

**16**. (a). Ship $A$ is sailing with speed of Ukmh$^{-1}$ in a direction N $30^0$E. a second ship $B$ is sailing with a speed $v$ kmh$^{-1}$ in a direction N $\theta°$ E. The velocity of ship $A$ relative to $B$ is due North East. Show that

$$u = v(\sqrt{3} + 1)(\cos\theta - \sin\theta)$$

**b**) Ship $A$ changes its course to N60°E, while it continue with the same speed. Ship $B$ continues with the same velocity. The velocity of ship $A$ relative to ship $B$ is now due East. Find tanθ. (Leave your answer in surd form). (Ans: $\frac{\sqrt{3}-1}{\sqrt{3}+1}$)

# 1999 PAPER ONE

## SECTION A

**1**. Given that the equation $2x^2 + 5x - 8 = 0$, has roots α and β, find the equation whose roots are $\frac{1}{(\alpha+2)^2}$ and $\frac{1}{(\beta+2)^2}$ (Ans: $100x^2 - 49x + 4 = 0$)

**2**. The vector equations of two lines are

$$\mathbf{r}_1 = \begin{pmatrix}5\\-6\end{pmatrix} + \lambda\begin{pmatrix}3\\1\end{pmatrix} \text{ and } \mathbf{r}_2 = \begin{pmatrix}4\\1\end{pmatrix} + \gamma\begin{pmatrix}-2\\3\end{pmatrix}.$$

Determine the point where $\mathbf{r}_1$ meets $\mathbf{r}_2$. (Ans: (8, 5))

**3**. Solve $\cos(\theta + 35^0) = \sin(\theta + 25^0)$ for $0 \le \theta \le 360^0$ .
(Ans: Hence $\theta = 15^0, 195^0$ for $0^0 \le \theta \le 360^0$)

**4**. The population of a country increases by 2.75% per annum. How long will it take for the population to triple? (Ans: 40.5 years)

**5**. Differentiate with respect to $x$:
i) $3x \ln x^2$ ii) $\cot 2x$ (Ans: $-2\text{cosec}^2(2x)$ )

**6**. Evaluate $\int_0^1 \frac{\tan^{-1}(x)}{1-x^2}\,dx.$ (Ans: *impossible*)

**7**. A curve is defined by the parametric equations
$x = t^2 - t,$
$y = 3t + 4.$
Find the equation of the tangent to the curve at (2, 10).
(Ans: $y = x + 8$)

**8**. A cliff which is 100m high runs in the S.E - N.W. direction along the coast. From the top of the cliff, the angle of depression of a ship moving at steady speed of 24 kmh$^{-1}$ towards the cost is $08^0$. Calculate the distance of the ship from the coast at the instant. What is the angle of elevation of the cliff from the ship one minute later? (Ans: 17.796°)

## SECTION B (60 *marks*)

**9**. The locus of P is such that the distance OP is half the distance PR, where O is the origin and R is the point (-3, 6).

i) Show that the locus of P describes a circle in the $x$ – $y$ plane. (Ans: $x^2 + y^2 - 2x + 4y - 15 = 0$)

ii) Determine the radius and centre of the circle.
(Ans: $= 2.5495$ Units (4 d.p.))

iii) Where does P cut the line $x = 3$?
(Ans (3, -6) and (3, 2))

**10.a)** Solve the equation $2(3^{2x}) - 5(3^x) + 2 = 0$
(Ans: $x = {}^-0.6309$ or $0.6309$)

**b)** The equations of three planes $P_1$, $P_2$ and $P_3$ are $2x - y + 3z = 3$, $3x + y + 2z = 7$ and $x + 7y - 5z = 13$ respectively. Determine where the three planes intersect. (Ans: (-2, 5, 4))

**11**. If $z$ is a complex number, describe and illustrate on the Argand diagram the locus given by each of the following:

i) $\left|\frac{z+i}{z-2}\right| = 3,$

(Ans: $x^2 + y^2 - \frac{1}{4}y - \frac{9}{2}x + \frac{35}{8} = 0$ ; $\left(\frac{9}{4}, \quad \frac{1}{8}\right)$ ; r = 0.8385)

ii) Arg(z + 3) = $\frac{\pi}{6}$. (Ands: $y = \frac{x\sqrt{3}}{3} + \sqrt{3}$ )

**12a)**. Solve $\sin 3x + \frac{1}{2} = 2\cos^2 x$ for $0 \le x \le 360^0$.
(Ans: Hence $x = 30^0, 60^0, 120^0, 150^0, 240^0$ and $300^0$)

**b).** Given that in any triangle $ABC$,
$\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\cot\left(\frac{A}{2}\right)$, Solve the triangle with two sides 5 and 7 and the included angle 45°.
(Ans: $a = 4.95$, $A = 45^0$, $B = 89.4^0$ and $C = 45.6^0$)

**13**. A research to investigate the effect of a certain chemical on a virus infection crops revealed that the rate at which the virus population is destroyed is directly proportional to the population at that time. Initially, the population was $P_0$ at $t$ months later, it was found to be $\boldsymbol{P}$.

**a)** Form a differential equation connecting $P$ and $t$
(Ans: $\frac{dP}{dt} = -KP$ )

**b)** Given that the virus population reduced to one third of the initial population in 4 months, solve the equation in (a) above. (Ans: $P = P_o \cdot 3^{-t/4}$ )

**c)** Find:

**i)** how long it will take for only 5% of the original population to remain. (Ans: 10.907 months)

**ii)** what percentage of the original virus population will be left after $2\frac{1}{2}$ months? (Ans: 503268%)

**14.** i) Find $\int \frac{x^2}{(x^4-1)}dx$ .

(Ans: $= \frac{1}{4}\ln\left(\frac{x-1}{x+1}\right) + \frac{1}{2}\tan^{-1}(x) + C$ )

ii) Evaluate $\int_0^1 \frac{x}{\sqrt{1+x}}dx$. (Ans: 0.3905)

**15**. A hemispherical bowl of radius $a$ cm is initially full of water. The water runs out through a small hole at the bottom of the bowl at a constant rate such that it empties the bowl in 24s. Given that when the depth of the bowl is $x$ cm and the volume of the water is $\frac{1}{3}$ $\pi x^2(3a - x)$ cm$^3$, show that the Depth of the water at that instant is decreasing at a rate $a^3[36x(2a - x)]^{-1}$ cm s$^{-1}$. Find how long it will take for the depth of the water to be at $\frac{1}{3}a$ cm and the rate at which the depth is decreasing at that instant. (Ans: 20.4 s, $\frac{1}{3}a$ cm)

**16**. **a)** Find in Cartesian form the equation of the line passing through the points A(1, 2, 5), B(1, 0, 4) and C(5, 2, 1).

**b)** Find the angle between the line $\frac{x+4}{8} = \frac{y-2}{2} = \frac{z+1}{-4}$ and the plane $4x + 3y - 3z + 1 = 0$ (Ans: $69.3^0$)

# 1999 PAPER TWO

## SECTION A

**1**. Given that A and $B$ are mutually exclusive events and $P(A) = \frac{2}{3}$ and $P(B) = \frac{1}{2}$, find:

i) $P(A \cup B)$, (Ans: 9/10)
ii) $P(A \cap B^1)$ (Ans: 2/5)
iii) $P(A^1 \cap B^1)$. (Ans: 1/10)

**2**. Four forces $a\mathbf{i} + (a-1)\mathbf{j}$, $3\mathrm{i} + 2a\mathbf{j}$, $5\mathbf{i} - 6\mathbf{j}$ and $-\mathbf{i} - 2\mathbf{j}$ act on a particle. The resultant of the forces makes an angle of $45^0$ with the horizontal. Find the value of $a$. Hence determine the magnitude of the resultant force.

(Ans: $a = 8$, 21.2N)

**3**. Show by means of a graph that the equation $x + \log_e x = 0.5$ has only one real root that lies between $\frac{1}{2}$ and 1.

**4.** An overloaded taxi travelling at a constant speed of 90 $\text{kmh}^{-1}$ overtakes a stationary traffic police car. Two seconds later, the police car sets off in pursuit of the taxi accelerating at a rate of 6 $\text{ms}^{-2}$. How long does the traffic car travel before catching up with the taxi?

(Ans: 300m)

**5**. The table below shows the variation of temperature with time in a certain experiment.

| Time(s) | 0 | 120 | 240 | 360 | 480 | 600 |
|---|---|---|---|---|---|---|
| Temperature($^0$C) | 100 | 80 | 75 | 65 | 56 | 48 |

use linear interpolation to find the

i) value of °C corresponding to 400 s, (Ans: 62°C)
ii) time at which the temperature is $77^0$C

(Ans: 192J)

**6**. A box of mass 4.5 kg rests on a rough horizontal plane inclined at an angle of 60° to the horizontal. If the coefficient of friction between the box and the plane is 0.35, determine the force acting parallel to the plane which will move the box up the plane. (Ans: 50 N)

**7**. At a bus park, 60% of the buses are of Isuzu make, 25% are styer type and the rest are of Tata make. Of the Isuzu type, 50% have radios while for the Styer and Tata make types only 5% and 1% have radios, respectively. If a bus is selected at random from the park, determine the probability that:

i) it has a radio. (Ans: 0.0315)
ii) a styer type is selected given that it has a radio

(Ans: 0.0398)

**8**. Given the variables $x$ and $y$ below,

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | 80 | 75 | 86 | 60 | 75 | 92 | 86 | 50 | 64 | 75 |
| $y$ | 62 | 58 | 60 | 45 | 68 | 68 | 81 | 48 | 50 | 70 |

Obtain a rank correlation coefficient between the variable $x$ and $y$. comment on you result (Ans: 0.7151)

## SECTION *B*

**9**.a) The area A of a parallelogram formed by vectors **a** and **b** is given by $A = |\mathbf{a}|\,|\mathbf{b}| \sin\theta$, where $\theta$ is the angle between the vectors. Find the percentage error made in the area if $|\mathbf{a}|$ and $|\mathbf{b}|$ are measured with errors ± 0.05 and the angle with an error of $\pm 0.5^0$, given that $|\mathbf{a}| = 2.5$ cm, $|\mathbf{b}| = 3.4$ cm and $\theta = 30^0$. (Ans: 5%)

b) Use the trapezium rule with six sub intervals to estimate $\int_0^{\pi} x \sin x dx$ correct to 3 decimal places.

Determine the error in your estimation and suggest how this error may be reduced. (Ans: 0.073)

**10**.a) A man buys 10 tickets from a total of 200 tickets in a lottery. There is only one prize ticket of sh. 10,000.

i) Find the probability that one of the tickets is a prize ticket. (Ans: 0.0478)

ii) If the price of each ticket is sh.100 and assuming that all tickets were sold, find the expected loss. (Ans: 522)

b). A man lives at a point which is 20 minutes' walk from the taxi stage. Taxis arrive at the stage punctually. If the probability density function for getting a taxi is given by

$$f(x) = \begin{cases} \frac{1}{20}, & 0 \le x \le 20, \\ 0, & Else\ where \end{cases}$$

Determine the:

i) expected time it takes to wait for a taxi (Ans: 10)

ii) variance of the time it takes to wait for the taxi (Ans: 33.33)

**11.** A particle is describing simple Harmonic motion in a straight line directed towards a fixed point *O*. When its distance from *O* is 3m, its velocity is 25 $ms^{-1}$ and its acceleration 75 $ms^{-2}$. Determine the

i) period and amplitude of oscillation, (Ans: $\frac{2\pi}{5}$ s and 5.83m)

ii) time taken by the particle to reach O, (Ans: $\frac{\pi}{10}$)

iii) velocity of the particle as it passes through O (Ans: 29.15$ms^{-1}$)

**12** (i) Show that the iterative formula for approximating the root of *f(x)* = 0 by the Newton Raphson process for the equation $xe^x + 5x - 10 = 0$ is:

$$x_{n+1} = \frac{x_n^2 e^{xn} + 10}{x_n e^{xn} + e^{xn} + 5}$$

(ii) Show that the root of the equation in (i) above lies between l and 2. Hence find the root of the equation correct your answer to 2decimal places. (Ans: 1.20)

**13**. A factory produces two types of bars of soap, A and *B*. Their lengths are normally distributed with type A having average length of 115 cm and standard deviation 3 cm. Type *B* has an average length 190 cm and standard deviation 5 cm

a) Determine the percentage of type

i) A bars of soap that have a length of more than 120 cm (Ans 4.78%)

ii) *B* bars of soap that have a length of more than 180 cm. (Ans: 97.72%)

b) Find the 95% confidence limits for the mean of length of type A bars of soap. (Ans: 109.12 < μ < 120.88)

**14**. A rod AB of length 0.6 m long and mass 10 kg is hinged at A. Its centre of mass is 0.5 m from A. a light inextensible string attached at B passes over a fixed smooth pulley 0.8 m above A and supports a mass M hanging freely. If a mass of 5 kg is attached at B so as to keep the rod in a horizontal, find the:

i) value of *M* (Ans: *M* = 16.7kg)

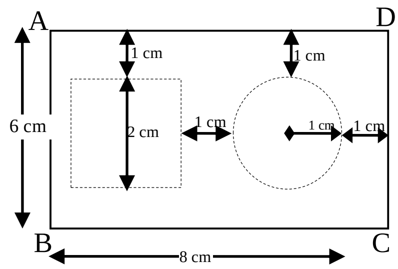
ii) reaction at the hinge. (Ans: 99.309 N)

**15** When a biased tetrahedron is tossed, the probability that any of its face shows up is proportional to the number of square of the number on the face that shows up.

i) Find the probability with which each of the numbers 1, 2, 3, and 4 on the face of the tetrahedron appear. (Ans: 8/9)

ii) If three independent tosses of the tetrahedron are made, what is the probability that the sum of the numbers on the faces that show up is a 3 or a 5? (Ans: 0.0028)

**16**




*ABCD* is uniform rectangular sheet of cardboard of length 8 cm and width 6 cm. A square and a circular hole are cut off from the cardboard as shown above. Calculate the position of the centre of gravity of the remaining sheet.

(Ans: 3.944cm from *AB* and 2.825cm *BC*)

# 2000 PAPER ONE

## SECTION A

**1**. Solve the simultaneous equations:

$x - 2y + 3z = 6$

$3x + 4y - z = 3$

$4x + 6y - 5z = 0$ (Ans: $x = 2$, $y = -1/2$, $z = 1$)

**2**. Solve $\cos\theta + \sqrt{3}\sin\theta = 2$ using the $t$-formulae,

(Ans: $\theta = 2\pi n + \frac{\pi}{3}$)

**3.** Differentiate $x10^{\sin x}$ with respect to $x$.

(Ans: $x10^{\sin x}[\frac{1}{x} + \cos x \log_e 10]$)

**4**. Show that $\log_8 x = \frac{2}{3}\log_4 x$. Hence without using tables or calculator, evaluate $\log_8 6$ correct to three decimal places, if $\log_4 3 = 0.7925$.

**5**. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^2 x}$. (Ans: 0.7854)

**6.** Show that the line $x - 2y + 10 = 0$ is a tangent to the ellipse $\frac{x^2}{64} + \frac{y^2}{9} = 1$.

**7.** In a culture of bacteria, the rate of growth is proportional to the population present at time, $t$. The population doubles every day. Given that the initial population $P_0$, is one million, determine the day when the population will be 100 million. (Ans: $7^{th}$ day)

**8.** Show that the equation of the line through the points (1, 2, 1) and (4, -2, 2) is given as $\frac{x-1}{3} = \frac{y-2}{-4} = z - 1$

## SECTION B

**9**. a) The $n^{th}$ term of a series is $U_n = a3^n + bn + c$. given that $U_1 = 4$, $U_2 = 13$ and $U_3 = 46$, find the values of $a$, $b$ and $c$. (Ans: $a = 2$, $b = -3$, $c = 1$)

**b**) If $\alpha$ and $\beta$ are the roots of the equation $x^2 - px + q = 0$, find the equation whose roots are $\frac{\alpha^3 - 1}{\alpha}$ and $\frac{\beta^3 - 1}{\beta}$.

(Ans: $qx^2 - (p^2q - 2q^2 - p)x + (q^3 - p^3 + 3pq) + 1 = 0$

**10**. a) Prove by induction that $2^n + 3^{2n-3}$ is always divisible by 7 for $n \geq 2$.

**b**) Expand $\left(1 - \frac{x}{3}\right)^{\frac{1}{2}}$ as far as the term in $x^2$. Hence evaluate $\sqrt{8}$, correct to three decimal places

(Ans: ; 2.829)

**11.a)** A point P is twice as far from the line $x + y = 5$ as from the point (3, 0). Find the locus of P.

(Ans: $7x^2 + 7y^2 - 38x + 10y + 47 = 0$)

**b)** A point $Q$ is given parametrically by $x = 2t$, $y = \frac{2}{t} + 1$. Determine the Cartesian equation of $Q$ and sketch it. (Ans: $y = \frac{4+x}{x}$)

**12**.a) Show that the equation of the plane through points A with position vector -2**i** + 4**k** perpendicular to the vector **i** + 3**j** – 2**k** is $x + 3y - 2z + 10 = 0$

**b)(i)** Show that the vector 2**i** – 5**j** + 3.5**k** is perpendicular to the line **r** = 2**i** – j + λ(4**i** + 3**j** + 2**k**).

**ii)** Calculate the angle between the vector 3**i** – 2**j** + **k** and the line in b (i) above. (Ans: 66.6°)

**13**. **a)** Solve $\cot^2\theta = 5(\text{cosec}\,\theta + 1)$ for $0° \leq \theta \leq 360°$

(Ans: θ = 9.6°, 170.4°, and 270°)

**b)** Use $\tan\frac{\theta}{2} = t$ to solve 5cos $\theta$ – 2sin = 2; $0 \leq \theta \leq 360°$. (Ans: θ = 46°, 270°for 0° < θ < 360°)

**14**. Express $f(x) = \frac{6x}{(x-2)(x+4)^2}$ into partial fractions.

Hence evaluate $\int f(x)dx$.

(Ans: $\ln\left(\frac{x-2}{x+4}\right)^{1/3} - \frac{4}{x+4} + c$)

**15**. Show that the tangent to the curve $4 - 2x - 2x^2$ at points (-1, 4) and $\left(\frac{1}{2},\ 2\frac{1}{2}\right)$ respectively passes through the point $\left(-\frac{1}{4},\ 5\frac{1}{2}\right)$. Calculate the area of the curve enclosed between the curve and the $x$-axis.

(Ans: 9 sq units)

**16**. An inverted cone with vertical angle of $60^0$ is collecting water leaking from a tap at a rate of $0.2cm^3s^{-1}$. If the height of water collected in the cone is 10cm, find the rate at which the surface area of water is increasing.

(Ans: $0.12\ cm^2s^{-1}$)

**b**) Given that $y = e^{\tan x}$, show that

$$\frac{d^2y}{dx^2} - (2\tan x + \sec^2 x)\frac{dy}{dx} = 0.$$

# 2000 PAPER TWO

## SECTION A

**1**. A family plans to have 3 children.

**i)** Write down the possible sample space and construct its probability distribution table.
(Ans: {BBB, BBG, BGB, BGG, GBB, GBG, GGB}

**ii)** Given that X is the number of boys in family, find the expected number of boys. (Ans: 1.5)

**2**. By the method of linear interpolation, use the table below to find the value of

i) ln(1.66) (correct to 3 decimal places) (Ans: 0.606)

ii) $x$ corresponding to ln($x$) = 0.4000. (Ans: 1.492)

| $x$ | 1.4 | 1.5 | 1.6 | 1.7 |
|---|---|---|---|---|
| ln$x$ | 0.3365 | 0.4055 | 0.4700 | 0.5306 |

**3**. Two balls are randomly drawn without replacement from a bag containing 10 white and 6 red balls. Find the probability that the second ball drawn is

i) red given that the first one was white. (Ans: 0.4)

ii) white (Ans: 0.375)

**4**. A boat travelling at 5ms $^{-1}$ in the direction $030^0$ in still water is blown by wind moving at 8ms$^{-1}$ from the bearing of $150^0$ . Calculate the speed and the course the boat will be steered. (Ans: 7 ms$^{-1}$)

**5**. Estimate the value of $\int_0^1 \frac{dx}{1+x^2}$ by the trapezium rule using five sub-intervals. (Give answer correct to 3 decimal places). (Ans: 0.784)

**6**. On a certain farm, 20% of the cows are infected by a tick disease. If a random sample of 50 cows is selected from the farm, find the probability that not more that 10% of the cows are infected. (Ans: 0.0558)

**7**. A force acting on a particle of mass 15 kg moves it along a straight line with a velocity of 10ms$^{-1}$. The rate at which work is done by the force is 50 watts. If the particle starts from rest, determine the time it takes to move a distance of 100m. (Ans: $10\sqrt{6}$ )

**8**. A particle executing simple harmonic motion about point O has a velocity of $3\sqrt{3}$ms$^{-1}$ and 3ms$^{-1}$ when at distances of 1m and 0.268m respectively, from the end point. Find the amplitude of the motion. (Ans: 2m)

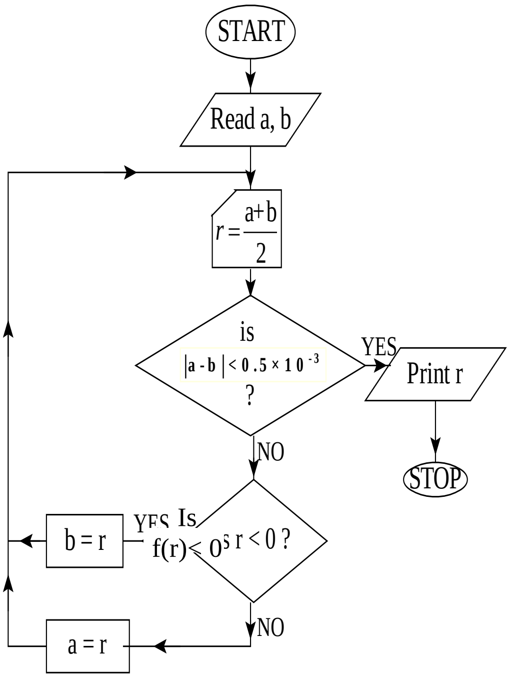
## SECTION B

**9**. Given that $x$ = 2.5, $y$ = 14.2 and $z$ = 8.1, all the values given correct to one decimal place, find the maximum value of

i) $\frac{x+y}{z^2}$, ii) $\frac{x-y}{z}$ iii) $\frac{1}{x}+\frac{1}{y}-\frac{1}{z}$, correct to 3 decimal places. (Ans: 0.259; -1.441; 0.356)

**b**) If the error in each of the values of $e^x$ and $e^{-x}$ is ± 0.0005, find the maximum and minimum values of the quotient $e^x/e^{-x}$, when $x$ = 0.04, giving your answer correct to 3 decimal places. (Ans: 0.694)

**10**. An interval bisection Algorithm that computes and prints the approximate value of the root, $r$ of the equation $f(x)$ = 0, in the interval [$a$, $b$], correct to 3 decimal places is given in the flow chart below:




By determining $f(x)$ and locating the approximate interval [$a$, $b$], perform a dry run for the flow chart to determine $\frac{1}{\sqrt[3]{3}}$ , correct to 3 decimal places. Tabulate values of $a$, $b$ and $r$ at each stage. (Ans: 0.694)

**11**. A particle moving with an acceleration given by **a** = $4e^{-3t}$**i** + 12sin$t$**j** – 7cos$t$**k** is located at the point (5, -6, 2) and has velocity, $v$ = 11**i** – 8**j** + 3**k** at time $t$ = 0. Find the

**i.** Magnitude of the acceleration when $t$ = 0. (Ans: 8.0623)

**ii.** Velocity at any time, $t$,
(Ans: $\frac{1}{3}\left((37-4e^{-3t})i + (4-12\cos t)j + (3-7\sin t)k\right)$)

**iii.** Displacement at any time, $t$.
(Ans: $r = \left[\frac{41}{a} + \frac{37}{3}t + \frac{ue^{-3t}}{a}\right]i + (-6+4t-12\sin t)j$
$+(-5+3t+7\cos t)k$ )

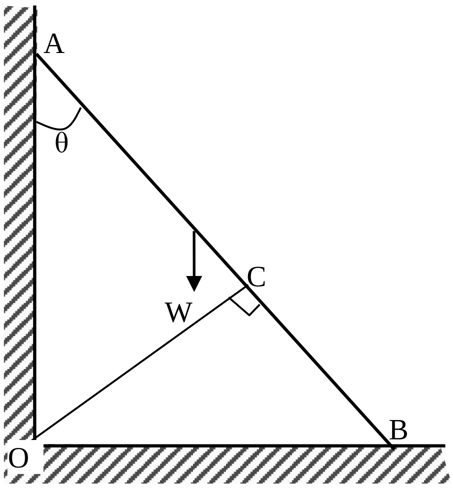
**12**. At 7:30am daily, a bus leaves Kampala for Masaka. The time (min) taken to cover the journey were recorded over a certain period of time and were grouped as shown in the table below:

| Time(min) | 80-84 | 85-89 | 90-94 | 95-99 | 100-104 |
|---|---|---|---|---|---|
| Freq.(f) | 10 | 15 | 35 | 40 | 28 |

| 105-109 | 110-114 | 115-119 | 120-124 |
|---|---|---|---|
| 15 | 4 | 2 | 1 |

a. Calculate the mean time of travel from Kampala to Masaka by the bus. (Ans: 96.6 minutes)
b. Draw a cumulative frequency curve for the data. Use your curve to estimate the :
   i. Median time for the journey, (Ans: 96.5)
   ii. Number of times the bus arrived in Masaka between 9:00 – 9: 25 am, (Ans: 119)
   iii. Semi-interquartile range of time of travel from Kampala to Masaka. (Ans: 5 minutes)

**13**. The diagram below shows a uniform rod **A*B*** of weight W and length $l$ resting at an angle $\theta$ against a smooth vertical wall at A. The other end *B* rests at a smooth horizontal table. The rod is prevented from slipping by an inelastic string **OC**, C being a point on **A*B*** such that **AC** is perpendicular to **A*B*** and O on the point of intersection of the wall and the table. Angle ***AOB*** is $90^0$




Find the i): tension in the string (Ans: $\frac{W}{2}\tan^2\theta$)

ii) reactions at A and *B* in terms of $\theta$ and W

(Ans: $\frac{W\tan^2\theta}{4}$)

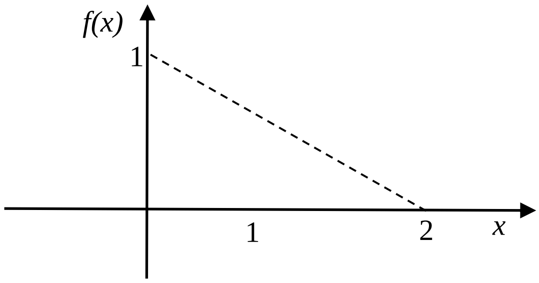
**14.** a) A random variable X takes on the values of the interval $0 < x < 2$ and has a probability density function given by

$$f(x)=\begin{cases} a & 0<x<1\frac{1}{2} \\ \frac{a}{2}(2-x) & 1\frac{1}{2}<x\le 2 \\ 0 & \text{elewhere} \end{cases}$$

Find i) value of $a$ (Ans: $\frac{16}{25}$)

ii) ***P***($x$ <16) (Ans: 0.9744)

b) the probability density function *f(x)* of the random variable *X* takes on the form shown in the diagram below




Determine the expression for *f(x)*. Hence obtain the

i. expression for the cumulative probability density function

(Ans: $f(x) = \begin{cases} 1-\frac{1}{2}x, & 0 < x < 2 \\ 0 & \text{, elsewhere} \end{cases}$

ii. mean and variance of *X*

(Ans: $f(x) = \begin{cases} 0 & , x < 0 \\ x-\frac{1}{5}x^2, & 0 < x \le 2 \\ 1 & , x > 2 \end{cases}$)

**15**. A sugar factory sells sugar in bags of mean weight 50kg and standard deviation 2.5kg. Given that the weight of the bags are normally distributed, find the

i) *P*robability that the weight of any bag of sugar randomly selected lies between 51.5 and 53kg. (Ans: 0.1592)

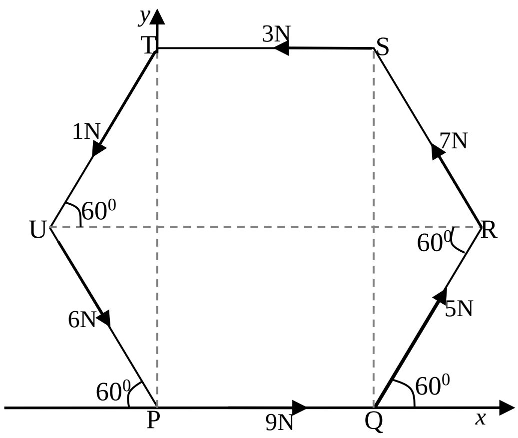
ii) *P*ercentage of bags whose weight exceeds 54kg. (Ans: 5.48%)

iii) Number of bags that will be rejected out of 1000 bags purchased for weighing below 45.0kg. (Ans: 23)

**16**. Six forces, 9N, 5N, 7N, 3N, 1N and 4N act along the sides *P*Q, QR, RS, ST, TU and U*P* of a rectangular hexagon of side 2m, their direction being indicated by the order of letters. Taking *PQ* as the reference axis, express each of the forces in vector form. Hence find the

i) magnitude and direction of the resultant of the forces. (Ans: 8.9 N, 43° with PQ)

ii) distance from *P*, where the line of action of the resultant cuts *P*Q. (Ans: 7.43m from P)



# 2001 PAPER ONE

## SECTION A

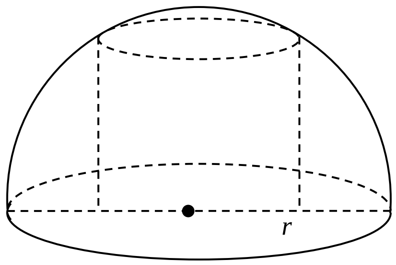
1. Solve the simultaneous equations:
$x^2 - 10x + y^2 = 25$
$y - x = 1$
(Ans: $x = 2, y = 7; x = -2, y = -1$)

**2**. If $y = \sqrt{x}$, show that $\frac{\delta y}{\delta x} = \frac{1}{\sqrt{(x+\delta x)} + \sqrt{x}}$. Hence deduce $\frac{dy}{dx}$.

**3.** Given that $\sin 2\theta = \cos 3\theta$, find the value of $\sin\theta$, $0 \le \theta \le \pi$. (Ans: 0.309)

**4.** Find the points of intersection of the line $\frac{x}{5} = \frac{y+2}{2} = \frac{z-1}{4}$ with the plane $3x + 4y + 2z - 25 = 0$ (Ans: (5, 0, 5))

**5.** A cylinder is inscribed in a semi-hemisphere of radius r as shown in the figure below




Find the maximum volume of the cylinder in terms of $r$ (Ans: $\frac{2\pi r^3}{3\sqrt{3}}$)

**6**. Expand $(1+x)^{-2}$ in descending powers of $x$ including the term $x^{-4}$. If $x = 9$, Find the percentage error in using the first two terms of the expression.

(Ans: 3.978%)

**7**. Find the locus of the point which is equidistant from the line $x = 2$ and the circle $x^2 + y^2 = 1$. Illustrate this with a sketch (Ans: $y^2 + 6x - 9 = 0$)

**8**. Solve the differential equation $\frac{dy}{dx} = \frac{y}{2x+1}$, given $x = 4$ when $y = 6$. Hence determine the value of $x$ when $y = 10$ (Ans: $x = 12$, $y^2 = 8x + 4$)

## SECTION B

**9.a**) Use De Moivres' theorem or otherwise to simplify
$$\frac{(\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta)}{\cos\theta/2 + i\sin\theta/2}$$
(Ans: $\cos(\frac{5\theta}{2}) + i\sin(\frac{5\theta}{2})$)

**b.** Express $\frac{i}{4+6i}$ in modulus-argument form
(Ans: $Z = 0.1387[\cos(0.187\pi) + i\sin(0.87\pi)]$)

**c.** Solve $(z + 2z^*)z = 5 + 2z$ where $z^*$ is the complex conjugate of z. (Ans: $Z^* = 1 + 2i$ )

**10.** It can be proved by induction that for all positive $n$,
$$1^3 + 2^3 + 3^3 + .... + n^3 = \frac{1}{4}n^2(n+1)^2.$$
From this result, deduce that
$$(n+1)^3 + (n+2)^3 + .... + (2n)^3 = \frac{1}{4}n^2(3n+1)(5n+3)$$

**b**) A man deposits sh. 800, 000 into his savings account on which interest is 15% per annum. If he makes no withdrawals, after how many years will his balance exceed sh. 8 million? (Ans: 16.5 years)

**11 a)** Using calculus of small increments or otherwise, find $\sqrt{98}$ correct to one decimal place. (Ans: 9.9)

**b**) Use Maclaurines theorem to expand $\ln(1 + ax)$, where a is a constant. Hence or otherwise expand $\ln\left(\frac{(1+x)}{\sqrt{(1-2x)}}\right)$ up to the term in $x^3$. For what values of $x$ is the expansion valid?
(Ans: $2x + \frac{x^2}{2} + \frac{5x^2}{3} + ...;\ x < \frac{1}{2}$ or $|x| < \frac{1}{2}$)

**12. a)(i)** Find the equation of the chord through the points $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ of the parabola $y^2 = 4ax$
(Ans: $(t_1 + t_2)y - 2x - 2at_1t_2 = 0$ )

**ii)** Show that the chord cuts the directrix when
$$y = \frac{2a(t_2t_1 - 1)}{t_1 + t_2}$$

**b)** Find the equation of the normal to the parabola $y^2 = 4ax$ at $(at^2, 2at)$ and determine its point of intersection with the directrix (Ans: $(a_1at(t^2 + 3))$)

**13a)** Show that $\tan\left(\frac{x+y}{2}\right) - \tan\left(\frac{x-y}{2}\right) = \frac{2\sin y}{\cos x + \cos y}$

**b**) Find in radians the solution of the equation $\cos x + \sin 2x = \cos 3x$, for $0 \le x \le 2\pi$
(Ans: $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ )

**14(a)** Find the Cartesian equation of the plane through $A(0, 3, -4)$, $B(2, -1, 2)$ and $C(7, 4, -1)$. Show that $Q(10, 13, -10)$ lies in the same plane**.**

**b)** Express the equation of the plane in (a) in the scalar product form. (Ans: $r \cdot \begin{pmatrix} 3 \\ -6 \\ -5 \end{pmatrix} = 2$ )

**c)** Find the area of ABC in (a)(Ans: 25.0998 sq. unts)

**15**. i) Find the Cartesian equation of the curve given parametrically by:
$$x = \frac{1+t}{1-t},\ y = \frac{(x-1)^2}{x+1}$$
(Ans: $y = \frac{(x-1)^2}{x+1}$ )

(ii) Sketch the curve

(iii) Find the area enclosed between the curve and the line $y = 1$. (Ans: 1.9548 sq units)

**16**) Integrate $\frac{2x}{\sqrt{x^2+4}}$ with respect to $x$

(Ans: $2(\sqrt{x^2+4}) + c$)

**b)** Evaluate $\int_0^{\pi/6} \sin x \sin 3x . dx$ (Ans: 0.1083)

**c)** Using the substitution $x = 3 \sin \theta$, evaluate $\int_0^3 \sqrt{\left(\frac{3+x}{3-x}\right)} dx$ (Ans: 7.7124)

# 2001 PAPER TWO

## SECTION A

**1**. The events A and $B$ are neither independent nor mutually exclusive. Given that $P(B) = 1/3$, $P(A) = ½$ and $P(A \cap B^1) = 1/3$, Find:

i) $P(A^1 \cup B^1)$, (Ans: 5/6)

ii) $P(A^1/B^1)$, (Ans: ½)

**2**. In an experiment to measure the rate of cooling of an object, the following temperatures, ($\theta^0$ C) against time T were recorded

| Temperature, $\theta^0$C | 80 | 70.2 | 65.8 | 61.9 | 54.2 |
|---|---|---|---|---|---|
| Time, T(s) | 0 | 10 | 15 | 20 | 30 |

Use linear interpolation to find

(i) The value of $\theta$ when T = 18s. (Ans: 63.5°C)

(ii) T when $\theta = 60^0$ (Ans: 22.5 seconds)

**3.** If $x = 4.5$, $y = 2.54$ and z = 26.4 all measured to the nearest number of decimal places of $x$, $y$ and $z$ respectively. Find the range within which the exact value of the expression $x - \frac{y}{xz}$ lies. (Ans: (4.42830, 4.52894))

**4**. A coin is biased so that a head is twice as likely to occur as a tail. If the coin is tossed 15 times, determine the,

**i)** expected number of heads (Ans: 10)

**ii)** probability of getting at most 2 tails (Ans: 0.0793)

**5**. A particle of mass 5 kg is placed on a smooth plane inclined at $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ to the horizontal. Find the magnitude of the force acting horizontally, required to keep the particle in equilibrium and the normal reaction to the plane. (Ans: 56.5803N)

**6**. A physics student measured the time required in seconds for A trolley to run down slopes of varying gradients and obtained the following results: 32.5, 34.5, 33.5, 29.3, 30.9, 31.8. Calculate the mean time and standard deviation. (Ans: 32.5333, 2.064)

**7.** A, $B$ and C are points on a straight road such that **A*B*** = ***B*C** = 20 m. A cyclist moving with uniform acceleration passes A and then notices that it takes him 10 seconds and 15 seconds to travel between A and $B$ , and A and C respectively.

Find:

i) his acceleration (Ans: 0.27 $ms^{-2}$)

ii) the velocity with which he passes A. (Ans: 0.65 $ms^{-1}$)

**8**. An inextensible string attached to two scale pans A and $B$, each of weight 20gm, passes over a smooth fixed pulley. Particles of weight 3.8N and 5.8N are placed in pans A and $B$ respectively. Find the reaction of the scale pan holding the 3.8N weight, if the system is released from rest. [Take g = 10 $ms^{-2}$].

(Ans: $R$ = 4.56 N)

## SECTION *B*

**9**. a) i) Round off 6.00213 (Ans: 6.00)
ii) Truncate 5415000, (Ans: 5410000)
to 3 significant figures. (2 marks)

b) Use the trapezium rule with eight su-intervals to estimate $\int_2^4 \frac{10}{2x+1}dx$ Correct to 4 decimal places.
Calculate the percentage error in your result. How may this error be reduced?

(Ans: 2.9418, 0.098%)

**10**. Bag A contains 2 green and 2 blue balls, while bag *B* contains 2 green and 3 blue balls. A bag is selected at random and 2 balls drawn from it without replacement. Find the probability that the balls are of different colours. (Ans: 19/30)

b) A fair die is drawn 6 times. Calculate the probability that
i) a 2 or 4 appears on the first throw, (Ans: 1/3)
ii) four 5s will appear in the 6 throws.

(Ans: 0.008)

**11**. Given two iterative formulae I and II (shown below) for calculating the positive root of the quadratic equation *f(x)* = 0

*I* $x_{n+1} = \frac{1}{2}\left(x_n^2 - 1\right)$ *II* $x_{n+1} = \frac{1}{2}\left(\frac{x_n^{\ 2}+1}{x_n - 1}\right)$

For $n$ = 1, 2, 3...

Taking $x_0$ = 2.5, use each formula thrice to two decimal places to decide which is the more suitable formula. Give a reason for your answer.

(Ans: $x_{n+1} = \frac{1}{2}\left(\frac{x_n^{\ 2}+1}{x_n - 1}\right)$)

**b**) If $\alpha$ is an approximate root of the equation $x^2 = n$, show that the iterative formula for finding the root reduces to $\frac{\frac{\alpha}{2}+\alpha}{2}$, Hence, taking $\alpha$ = 4, estimate $\sqrt{17}$ correct to 3 decimal places (Ans: 4.123)

**12.** The table below shows the weights to the nearest kg of 150 patients who visited a certain health unit during a certain week:

| Weight(kg) | No. of patients |
|---|---|
| 0 − 19 | 30 |
| 20 – 29 | 16 |
| 30 – 39 | 24 |
| 40 – 49 | 32 |
| 50 – 59 | 28 |
| 60 – 69 | 12 |
| 70 - 79 | 8 |

a) Calculate the approximate mean and modal weights of the patients. (Ans: 38.8333 kg)

b) *P*lot an ogive for the above data. Use the ogive to estimate the
i) median and semi interquartile range for the weights of the patients, (Ans: 41.5 kg, 14.75 kg)
ii) the probability that patients weighing between 13kg and 52.5kg visited the health unit.

(Ans: 43/75)

**13**. An object of mass 5kg is initially at rest at appoint whose position vector is -2i + **j**. If it is acted upon by a force, F = 2**i** + 3**j** − 4**k**, find
i) the acceleration. (Ans: $\frac{1}{5}$(2i + 3j – 4k))
ii) the velocity after 3 s. (Ans: $\frac{1}{5}$(6i + 9j – 12*k*))
iii) its distance from the origin after 3 s.

(Ans: 5.166 m)

**14. (a)** Amass oscillates with S.H.M of period one second. The amplitude of the oscillation is 5cm. Given that the particle begins from the centre of the motion, state the relationship between the displacement *x* of the mass at any time *t*. Hence find the first times when the mass is 3cm from its end position. (Ans: (0.066, 0.434))

**b**) A particle of mass *m* is attached by means of light strings *AP* and *BP* of the same natural length *a* m and moduli of elasticity *mg* and 2*mg* N respectively, to the points *A* and *B* on a smooth horizontal table. The particle is released from the mid –point of $\overline{AB}$, where $\overline{AB}$ = 3*a*m.

$$T = \left(\frac{4\pi^2 a}{3g}\right)^{1/2}$$

**15**. A continuous random variable *X* is defined by the p.d.f.

$$f(x) = \begin{cases} k\left(x - \frac{1}{a}\right), & 0 \le x \le 3 \\ 0, & \text{Else where} \end{cases}$$

Given that $P(x > 1)$ = 0.8, find the:
i) value of *a* and *k* (Ans: *a* = -1, *k* = 2/15)
ii) probability that *X* lies between 0.5 and 2.5

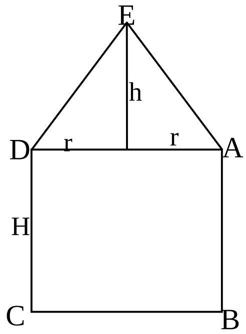
(Ans: 0.6667 (4 dp))

iii) mean of *X*. (Ans: 1.8)

**16. (a)** Prove that the centre of mass of a solid cone is ¼ of the vertical height from the base.

**(b)** The figure A*B*CDE below shows a solid cone of radius *r*, height (*h*), joined to a solid cylinder of the same material with the same radius and height *H*.

(Ans: 32.01°)




If the centre of mass of the whole solid lies in the plane of the cone where the two solids are joined, find $H$. If instead $H = h$ and $r = ½h$, find the angle A$B$ makes with the horizontal, if the body is hanged from $A$.

# 2002 PAPER ONE

## SECTION A

**1.** Solve the equation $2\cos\theta - \operatorname{cosec}\theta = 0$; $0^0 < \theta < 270^0$.
(Ans: θ = 45°, 225°)

**2**. The vertices of a triangle are $P(2, -1, 5)$, $Q(7, 1, -3)$ and $R(12, -2, 0)$. Show that $\angle PQR = 90^0$. Find the coordinates of $S$ if $PQRS$ is a rectangle.
(Ans: (8, -4, 8))

**3**. Show that $2 + \boldsymbol{i}$ is a root of the equation $2z^3 - 9z^2 + 14z - 5 = 0$. Hence find the other roots. (Ans: $2 - i$, ½)

**4**. The point $R(2, 0)$ and $P(3, 0)$ lie on the $x$-axis and $Q(0, -y)$ lies on the $y$-axis. The perpendicular from the origin to $RQ$ meets $PQ$ at point S($X$, -$Y$). Determine the locus of S in terms of $X$ and $Y$.
(Ans: $2x^2 + 3y^2 - 6x = 0$)

**5**. If $y = \sqrt{(5x^2 + 3)}$, show that $y\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^2 = 5$.

**6**. Given that $\log_3 x = p$ and $\log_{18} x = q$, show that

$$\log_6 3 = \frac{q}{p-q}.$$

**7.** Given $y = \ln\left(1 - \dfrac{1}{u}\right)^{1/2}$, $2u = \left(x - \dfrac{1}{x}\right)$, show that

$$\frac{dy}{dx} = \frac{(x+1)}{(x-1)(x^2+1)}.$$

**8**. Evaluate $\displaystyle\int_0^1 \frac{x^3}{x^2+1}dx$. (Ans: 0.15345)

## SECTION B

**9.(a)** The tenth term of an arithmetic progression (A.P) is 29 and the fifteenth term is 44. Find the value of the common difference and the first term. Hence find the sum of the first 60 terms. (Ans: $d = 3$, $a = 2$, 5430)

**b)** A cable 10 m long is divided into ten pieces whose lengths are in a geometric progression. The length of the longest piece is 8 times the length of the shortest piece. Calculate to the nearest centimeter the length of the third piece. (Ans: 45 cm)

**10**. $P$ is a variable point given by the parametric equations $x = \frac{a}{2}\left(t + \frac{1}{t}\right)$, $y = \frac{b}{2}\left(t - \frac{1}{t}\right)$. Show that the locus of $P$ is $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$.

State the asymptotes. Determine the coordinates of the point where the tangent from $P$ meets the asymptotes.

(Ans: $y = \dfrac{bx}{a}$; $y = \dfrac{-bx}{a}$; $(at, bt)$ and $\left(\frac{a}{t}, \frac{-b}{t}\right)$ )

**11(a)** Find the equation of the perpendicular line from point $\mathbf{A} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$ onto the line $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$. What is the distance from **A** to **r**?

(Ans: $p = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} + \mu\begin{pmatrix} -4/9 \\ 14/9 \\ -8/9 \end{pmatrix}$; 1.795 units)

**b).** Find the angle contained between the line $OR$ and the $x - y$ plane, where $\mathbf{OR} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$ (Ans: 41.81°)

**12(a)** Show that $\dfrac{\sin 3\theta \sin 6\theta + \sin\theta \sin 2\theta}{\sin 3\theta \cos 6\theta + \sin\theta \cos 2\theta} = \tan 5\theta$.

**(b)** Express $4\cos\theta - 5\sin\theta$ in the form $R\cos(\theta + \beta)$, where R is a constant and β an acute angle.

i) Determine the maximum value of the expression and the value of $\theta$ for which it occurs
(Ans: 6.403, when θ = -51.3° or 308.7°)

ii) Solve the equation $4\cos\theta - 5\sin\theta = 2.2$, for $0^0 < \theta < 360^0$. When θ = –51.3° OR 308.7°
(Ans: $\theta$ = 18.6° or 238.3°)

**13**.a)Find the equation whose roots are $-1 \pm \boldsymbol{i}$, where $\boldsymbol{i} = \sqrt{-1}$. (Ans: $Z^2 + 2Z + 2 = 0$)

b) Find the sum of the first 10 terms of the series $1 + 2\boldsymbol{i} - 4 - 8\boldsymbol{i} + 16 + \ldots$, in the form $a + b\boldsymbol{i}$ where $a$ and $b$ are constants and $\boldsymbol{i} = \sqrt{-1}$. (Ans: $205 + 410i$)

c) Prove by induction that
$(\cos\theta + \boldsymbol{i}\sin\theta)^n = \cos n\theta + \boldsymbol{i}\sin n\theta$.

**14**. **(a)**Use $t = \tan \theta/2$ to evaluate $\int_0^{\pi/2} \frac{d\theta}{3-\cos\theta}$.

b) Integrate the following with respect to $x$:

i) $\ln x$ ii) $x^2 \sin 2x$

(Ans: 0.6755); $x(\ln x - 1) + c$);

$\frac{-x^2}{2}\cos 2x + \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x + c$ )

**15**. Given the curve $y = \frac{x(x-3)}{(x-1)(x-4)}$,

i) Show that the curve does not have turning points

ii) Find the equations of the asymptotes. Hence sketch the curve. (Ans: $y = 1$, $x = 1$, $x = 4$)

**16(i)**. The volume of a water reservoir is generated by rotating the curve $y = kx^2$ about the $y$–axis. Show that when the central depth of the water in the reservoir is $h$ meters, the surface area, $A$ is proportional to $h$ and the volume $v$ is proportional to $h^2$.

ii) If the rate of loss of water from the reservoir due to evaporation is $\lambda A$ m$^2$ per day, obtain a differential equation for $h$ after $t$ days. Hence deduce that the depth of water decreases at a constant rate.

iii) Given that $\lambda = ½$, determine how long it will take for the depth of water to decrease from 20 m to 2 m.

(Ans: 36 days)

# 2002 PAPER TWO

## SECTION A

**1**. On a certain day, fresh fish from lakes: Kyoga, Victoria, Albert and George were supplied to one of the central markets of Kampala in the ratios 30%, 40%, 20% and 10% respectively. Each lake had an estimated ratio of poisoned fish of 2%, 3% and 1% respectively. If a health inspector picked a fish at random,

i. What is the probability that the fish was poisoned?

(Ans: 0.025)

ii. Given that the fish was poisoned, what is the probability that it was from Lake Albert?

(Ans: 0.24)

**2.** The table below shows how y varies with x in an experiment at different points

| $x$ | $y$ |
|---|---|
| -1.0 | -1.0 |
| -0.5 | -2.2 |
| -1.4 | -3.7 |

Use linear interpolation or extrapolation to find

a) $y$ when $x = 0.5$ (Ans: $y = -4.6$)

b) $x$ when $y = -4.5$ (Ans: $x = 0.456$)

**3**. A driver of a car travelling at 72 kmh$^{-1}$ notices a tree which has fallen across the road, 800 m ahead and suddenly reduces the speed to 36 kmh$^{-1}$ by applying the brakes. For how long did the driver apply the brakes?

(Ans: 53.33 s (2 dp))

**4.** The chance that a person picked from a Kampala street is 30 in every 48. The probability that that person is a university graduate given that he is employed is 0.6. Find the

a) probability that the person picked at random from the street is a university graduate and is employed

(Ans: 0.375)

b) number of people that are not university graduates and are employed from a group of 120 people.

(Ans: 30)

**5.** Use a suitable table of values to show that the function $x \to x^3 - \frac{8}{x}$ has two real roots in the interval (-3, 3), Hence use linear interpolation to determine the approximate value of the negative real roots of the function giving your answer correct to 1 decimal place**.** (Ans: -1.7)

**6.** The resistance to the motion of a lorry of mass m kg is 1/200 of its weight. When travelling at 108kmh$^{-1}$ on a level road and ascends a hill its engine fails to work. Find how far up the hill (in km) the lorry moves before it comes to rest. Give your answer close to one decimal place (Ans: $S = \frac{90000}{g(1 + 200\sin\alpha)}$ )

**7**. The table below shows the cumulative distribution of the age (in years) of 400 students of a girls' school

| Age(in years) | Cumulative frequency |
|---|---|
| <12 | 0 |
| <13 | 27 |
| <14 | 85 |
| <15 | 215 |
| <16 | 320 |
| <17 | 370 |
| <18 | 395 |
| <19 | 400 |

Plot an ogive for the data and use it to estimate the:

a) median age (Ans: 14.9)

b) 20$^{th}$ and 80$^{th}$ percentile age range

(Ans: 2.1)

**8**. A particle moves in the $x - y$ plane such that its position vector at any time t is given by

$\mathbf{r} = (3t^2 - 1)\mathbf{i} + (4t^3 + t - 1)\mathbf{j}$

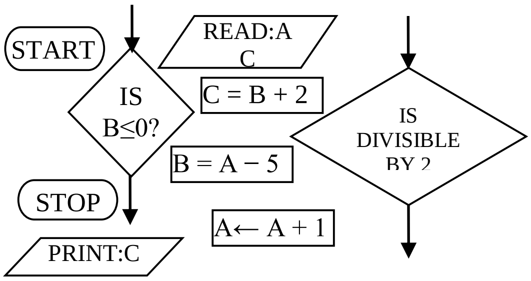
Find:

i) its speed, (Ans: $\sqrt{6t^2 + (12t^2 + 1)^2}$ ; = 48.3735)

ii) the magnitude of acceleration after *time* $t = 2$

## SECTION B

**9.** Given below are points of a flow chart not arranged in order.




Rearrange them and draw a complete logical flow chart.

(a) State the purpose of the flow chart

(b) Perform a dry run of your rearranged flow chart by copying and completing the table below

**10**. A pair of dice is tossed 180 times. Determine the probability that a sum of 7 appears

**i)** Exactly 40 times (Ans: 0.0108)

**ii)** Between 25 and 35 inclusive times

(Ans: 0.7286)

**11.** A random variable X has the probability distribution function

$$f(X) = \begin{cases} \frac{1}{b-a}, a \le x \le b \\ 0, \quad \text{Else where.} \end{cases},$$

Show that the variance of $X$ is $\frac{(b-a)^2}{12}$.

**b)** During rush hours, it was observed that the number of vehicles departing for Entebbe from Kampala old taxi park take on a random variable $X$ with a uniform distribution over the interval $[x_1, x_2]$. If in 1 hour, the expected number of vehicles leaving the stage is 12, with variance of 3, calculate the:

i) values of $x_1$ and $x_2$, (Ans: $x_1 = 9$, $x_2 = 15$)

ii) probability that at least 11 vehicles leave the stage

(Ans: 2/3)

**12(a)** A particle of mass 3Kg is attached to the lower end B of an inextensible string. The upper end A of the string is fixed to a point on the ceiling of a roof. A horizontal force of 22N and an upward vertical force of 4.9 N act upon the particle making it to be in equilibrium, with the string making an angle α with the vertical. Find the value of α and the tension in the string.

(Ans: 33N, $\alpha = 45°$)

(b) A non – uniform rod of mass 9Kg rests horizontally in equilibrium supported by two light inextensible strings tied to the ends of the rod. The strings make angles of $50^0$ and $60^0$ with the rod. Calculate the tensions in the strings. (Ans: 46.92N and 60.34 N)

**13.** Show graphically that there is only one positive real root of the equation $x^3 + 2x - 2 = 0$.

Using the Newton Raphson formula thrice estimate the root of the equation, give your answer correct to 2 decimal points. (Ans: 0.77 (2 dp))

**14**. (a) The times taken by a group of students to solve a mathematical problem are given below

| Time(min) | 5-9 | 10-14 | 15-19 | 20-24 | 25-29 | 30-34 |
|---|---|---|---|---|---|---|
| No. of students | 5 | 14 | 30 | 17 | 11 | 3 |

a) Draw a histogram for the data. Use it to estimate the modal time for solving a problem

**b**) Calculate a mean time and standard deviation of solving a problem

(Ans: 18.5 minutes; 5.9896 minutes)

**15 (a)** The velocities of two ships P and Q are **i** + 6**j** and –**i** + 3**j** $\text{kmh}^{-1}$ respectively.

At a certain instant the displacement between the two ships is 7**i** +4**j** km

Find the:

(i) relative velocity of ship $P$ to $Q$. (Ans: $2i + 3j$)

(ii) magnitude of displacement between ships P and Q after 2hours. (Ans: 14.87 km)

(b) The position vector of two particles are:-

$\mathbf{r}_1 = (4\mathbf{i} - 2\mathbf{j})t + (3\mathbf{i} + \mathbf{j})t^2$ and

$\mathbf{r}_2 = 10\mathbf{i} + 4\mathbf{j} + (5\mathbf{i} - 2\mathbf{j})t$ respectively .

Show that the two particles will collide. Find their speeds at the time of collision. (Ans: 5.3852 units)

**16.** A particle is projected from level ground towards a vertical pole, 4m high and 30m away from the point of projection. It just passes the pole in one second. Find:

a) its initial speed and angle of projection.

(Ans: 31.2933; $\alpha = 16.5°$)

b) the distance beyond the pole where the particle will fall. (Ans: 24.42m)

# 2003 PAPER ONE
## SECTION A

**1.** Show that z = 1 is a root of the equation $z^3 - 5z^2 + 9z - 5 = 0$. Hence solve the equation for the other roots.

**2.** Given the position vectors ***OA*** = (3, -2, 5), and ***OB*** = (9, 1, -1), find the position vector of point *C* such that *C* divides $\overline{AB}$ internally in the ratio 5: -3

(Ans: $18i + \frac{11}{2}j - 10k$)

**3.** Solve the equation
$\cos 2\theta + \cos 3\theta + \cos\theta = 0;\ 0^0 \le \theta \le 180^0$.

(Ans: θ = 45°, 120°, and 135°)

**4.** Find $\int x\ln x dx$. (Ans: $\frac{x^2}{2}\ln x = \frac{x^2}{4} + c$)

**5.** Solve for *x* in the equation $\log_4(6 - x) = \log_2 x$

(Ans: *x* = 2)

**6.** If $y = e^{-t}\cos(t+\beta)$, show that $\frac{d^2y}{dx^2} + \frac{2dy}{dx} + 2y = 0$

**7**. The points *A* (2, 1), *P* (α, β) and point *B*(1, 2) lie in the same plane. *PA*, meets the *x*-axis at the point (*h*, 0) and *PB* meets the *y*-axis at the point (0, *k*). Find h and k in terms of α and β.

(Ans: $n = \frac{2\beta - \alpha}{\beta - 1}$, $k = \frac{2\alpha - \beta}{\alpha - 1}$)

**8**. Determine $\frac{d}{dx}\left(\ln\frac{x}{\sqrt{(1+x^2)}}\right)$ when *x* = 2. (Ans: 1/10)

## SECTION B

**9.** (a) Show that cot *A* + tan 2*A* = cot *A* sec 2*A*

(b) Show that $\tan 3\theta = \frac{3t - t^3}{1 - 3t^2}$, where *t* = tan θ

Hence or otherwise show that $\tan\frac{\pi}{12} = 2 - \sqrt{3}$

**10(a) .** Given the inequalities $y > x - 5$ and $0 < y < \frac{6}{x}$, illustrate graphically by shading out the unwanted regions.

**b**) Solve the simultaneous equations

$xy + 2x = 5$

$9x = y + 6$.

Illustrate your solutions on a graph.

(Ans: $x = 1, x = \frac{-5}{9}$; $y = -1, y = 3$)

**11(a).** In a triangle *ABC*, the altitudes from *B* and *C* meet the opposite sides at *E* and *F* respectively. *BE* and *CF* intersects at *O*. taking *O* as the origin, use the dot product to prove that *AO* is perpendicular to *BC*

**b**) Prove that $\angle ABC = 90°$ given that A is (0, 5, -3), B(2, 3, -4) and C(1, -1, 2). Find the coordinates of *D* if *ABCD* is a rectangle. (Ans: D(-1, 1, 3))

**12(a) .** Use De Moivres theorem to express tan 5θ in terms of tan θ

(Ans: $\tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$)

**b)** Solve the equation $z^3 + 1 = 0$

(Ans: $Z = -1$, $Z = \frac{1}{2} + \frac{i\sqrt{3}}{2}$, $Z = \frac{1 - i\sqrt{3}}{2}$)

**13**. Determine the nature of the turning points of the curve $y = \frac{x^2 - 6x + 5}{(2x - 1)}$. Sketch the graph of the curve for *x* = -2 to *x* = 7. State any asymptotes.

(Ans: (-1, -4) maximum; (12, -1) minimum)

**14**. A conic section is given by $x = 4\cos\theta$; $y = 3\sin\theta$. Show that the conic section is an ellipse and determine its eccentricity.

b) Given that the line *y* = *mx* + *c* is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, show that $c^2 = a^2m^2 + b^2$. Hence determine the equations of the tangents at the point (-3, 3) to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

(Ans: *y* = 3 and $y = \frac{18}{7}x + \frac{75}{7}$)

**15(a).** Find $\int x^3 e^{x^4} dx$. (Ans: $\frac{1}{4}e^{x^4} + c$)

**b)** Use the substitution t = tan *x* to find

$\int \frac{1}{1 + \sin^2 x}dx$ (Ans: $\frac{1}{\sqrt{2}}\tan^{-1}(\sqrt{2}\tan x) + c$)

**16.a)** Solve the differential equation $\frac{dR}{dt} = e^{2t} + t$, given that *R*(0) = 3 (Ans: $R = \frac{1}{2}e^{2t} + \frac{1}{2}x^2 + \frac{5}{2}$)

**(b)** The acceleration of a particle after time *t* seconds is given by *a* = 5 + cos½*t*. If initially the particle is moving at $1ms^{-1}$, find its velocity after 2π seconds and the distance it would have covered by then.

(Ans: $v = 10\pi + 1$; $d = 10\pi^2 + 2\pi + 4$)

# 2003 PAPER TWO

## SECTION A

**1**. Events A and B are such that P(A) = ½, P(B) = 3/8 and P(A/B) = 7/12, find

i) $P(A \cap B)$ (Ans:7/32 )

ii) $P(B/\bar{A})$ (Ans: 5/16)

**2**. Two decimal numbers $x$ and y are recorded off to give $X$ and $Y$ with the errors $E_1$ and $E_2$ respectively. Show that the maximum relative error recorded in approximating $x^2y$ by $X^2Y$ is given by: $2\left|\frac{E_1}{X}\right| + \left|\frac{E_2}{Y}\right|$

**3.** ABCD is a square of side a. Forces of magnitude 2N, 1N, $\sqrt{2}$ N and 4N act along AB, BC, AC and DA respectively. The directions being in the order of letters. Find the magnitude and direction of the resultant force. (Ans: 3.6056N, 33.69°)

**4.** In an examination, scaling is done such that candidate A who had originally scored 35% gets 50% and candidate B with 40% gets 65%. Determine the original mark for candidate C whose new mark is 80% (Ans: 45%)

**5.** Find the approximate value of $\int_0^1 \frac{1}{x^2+1}dx$ using five sub-intervals. (Ans: 0.784)

**6**. A spring $AB$ of natural length 1.5m and modulus λN is fixed at A and hangs in a vertical position. The other end is joined to a second spring BC of natural length 1m and modulus 2λN. A particle of weight 15 N is then attached to end C of the second spring. When the system is hanging freely in equilibrium the distance AC is 4m. Find the value of λ. (Ans: $\lambda = \frac{25}{3}$)

**7**. The table below shows the marks scored in a mathematics examination by students in a certain school

| Marks | Number students |
|---|---|
| 30 – 39 | 12 |
| 40 – 49 | 16 |
| 50 – 59 | 14 |
| 60 – 69 | 10 |
| 70 – 79 | 8 |
| 80 – 89 | 4 |

Draw a histogram and use it to estimate the mode.
Calculate the mean score. (Ans: 46; mean, 54.1875)

**8**. A train starts from station A with a uniform acceleration of 0.2 ms$^{-2}$ for 2 minutes and attains a maximum speed and moves uniformly for 15 minutes. It is then brought to rest at constant retardation of 5/3 ms$^{-2}$ at station B. Find the distance between stations A and B. (Ans:23212.8 m )

## SECTION B

**9. (i)** Show that the equation $x = 1n(8 - x)$ has a root between 1 and 2.

**(ii)** Use Newton Raphson method to find the approximate root of $x = 1n(8 - x)$. Correct to 3 decimal places. (Ans: 1.821 (3 dp))

**10**. Given the cumulative distribution function,

$$F(x) = \begin{cases} \frac{x^2-1}{2} - x, 1 \le x < 2; \\ 3x - \frac{x^2}{2}, 2 \le x < 3 \\ 1, \quad x \ge 3; \end{cases}$$

a) Find:

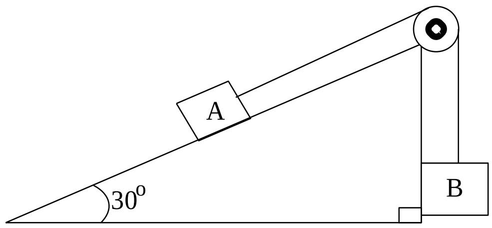
i) the p.d.f. (Ans: $f(x) = \begin{cases} x-1 : 1 \le x < 2 \\ 3-x : 2 \le x < 3 \\ 0 \quad : \text{elsewhere} \end{cases}$)

ii) P(1.2 < $x$ < 2.4 (Ans: 0.8)

iii) the mean of $x$ (Ans: 2)

b) Sketch $f(x)$

**11.** Blocks A and B of masses 2 and 3 kg respectively are connected by a light inextensible string passing over a smooth pulley as shown below




Block $A$ is resting on a rough plane inclined at at 30° to the horizontal while block $B$ hangs freely. When the system is released from rest, block $B$ travels a distance of 0.75m before it attains a speed of 2.25ms$^{-1}$.

Calculate the

(i) acceleration of the blocks (Ans: 3.375 ms$^{-1}$)

(ii) coefficient of friction between the plane and block $A$, (Ans: 0.16)

(iii) reaction of the pulley on the string. (Ans: 33.4N) (Ans: 33.4N)

**12**. **i)** Determine the iterative formula for finding the fourth root of a given number N.

(Ans: $\frac{3}{4}\left(x_n + \frac{N}{3x_n^3}\right)$ for $n$ = 0, 1, 2, … )

ii) Draw a floor chart that reads N and the initial approximation, $X_0$, computes and prints the fourth root of N correct to 3decimal places and N. (Ans: $x$ = 3.500)

iii) Perform a dry run for $N$ =150.10 and $x_0$ =3.200.

**13**. In a school of 800 students their average weight is 54.5 kg and standard deviation 6.8 kg. If the weights of all the students in the school assume a normal distribution, find the

i) probability that a student picked at random weighs 52.8 or less kg, (Ans: 0.4013)

ii) number of students who weigh over 75 kg (Ans: 0.8)

iii) weight range of the middle 56% of the students of the school (Ans: $49.2504 < x < 59.7496$ kg)

**14**. A particle of weight 24N is suspended by a light inextensible string from a light ring. The ring can slide along a rough horizontal rod. The coefficient of friction between the rod and the ring is ⅓. A force of *P* Newtons acting upwards on a particle at 45° to the horizontal, keeps the system in equilibrium with the ring at a point of sliding. Find the:

i) value of *P* (Ans: $6\sqrt{2}$ N)

ii) tension in the string (Ans: $6\sqrt{10}$ N)

**15. (a)** The table below shows the percentage sand, *y* in the soil at different depths, *x*, (in cm);

| Soil depth($x$) in cm | 35 | 65 | 55 | 25 | 45 | 75 | 20 | 90 | 51 | 60 |
|---|---|---|---|---|---|---|---|---|---|---|
| % of sand, $y$ | 86 | 70 | 84 | 92 | 79 | 68 | 96 | 58 | 86 | 77 |

a) **i)** Plot a scatter diagram for the data. Comment on the relationship between the depth of the soil and the percentage of sand in the soil

**ii)** Draw a line of best fit through the points of the scatter diagram. Use your result to estimate the

- percentage of the sand in the soil at depth of 31 cm.
- depth of the soil with 54 % sand

(Ans: 92%; 96 cm)

**b)** Calculate a rank correlation coefficient between the percentage of sand in soil and depth of the soil

(Ans: -0.9485)

**16** Two particles P and Q initially at positions 3**i** + 2**j** and 13**i** + 2**j** respectively begin moving. Particle P moves with a constant velocity of 2**i** + 6**j** while particle Q moves with a constant velocity of 5**j**, the units being in meters and metres per second respectively.

a) Find the:

i) time the two particles are nearest to each other (Ans: 43)

ii) bearing of particle P from Q when they are nearest to each other. (Ans: 333.43)

b) Given that after half the time, the two particles are moving closest to each other, particle P reduces its speed to half its original speed, in the direction to approach particle Q and the velocity of Q remains unchanged , find the direction of particle P

(Ans: N50.8°E)

# 2004 PAPER ONE

## SECTION A

**1**. Solve the inequality $(0.8)^{-3x} > 4.0$, correct to 2 decimal places. (Ans: $x > 2.07$)

**2**. A right circular cone of radius *r* cm has a maximum volume, the sum of its vertical height *h*, and the circumference is 15 cm. If the radius varies, show that the maximum volume of the cone is $\frac{625}{\pi}cm^3$.

**3**. Solve $\cos\theta + \sin 2\theta = 0$ for $0^0 \le \theta \le 360^0$.

(Ans: θ = 90°, 210°, 270°, 330°)

**4.** Given that: $y = \sqrt{\frac{1+\sin x}{1-\sin x}}$, show that

$$\frac{dy}{dx} = \frac{1}{1-\sin x}.$$

**5**. *A* is the point (1, 3) and *B* the point (4, 6). *P* is a variable point which moves in such a way that $(\overline{AP})^2 + (\overline{PB})^2 = 34$. Show that the locus of *P* describes a circle. Find the centre and radius of the circle. (Ans: centre $\left(\frac{5}{2}, \frac{9}{2}\right)$, r = $\frac{5\sqrt{2}}{2}$ units)

**6**. Find the equation of the plane through the point (1, 2, 3) and perpendicular to the vector ***r*** = 4***i*** + 5***j*** + ***k***.

(Ans: $4x + 5y + z = 17$)

**7.** Prove by induction that: $\sum_{r=1}^{n} 3^{r-1} = \frac{3^n - 1}{2}$ , where *n* is a whole number.

**8**. Use the substitution $t = \tan\frac{x}{2}$ to evaluate

$\int_0^{\pi} \frac{dx}{3+5\cos x}$. (Ans: 0.2747)

## SECTION B

**9**. Find *n* if ${}^nP_4 = 30\,{}^nC_5$, (Ans: 8)

**b**) How many arrangements can be made from the letters of the name *MISSISSIPPI*,

When all the letters are taken at a time

(Ans: 34650 ways)

If the two letters P begin every word?

(Ans: 630 ways)

**c**) Find the number of ways in which a senior six mathematics student can choose one or more of the four girls in the mathematics class to join a discussion group. (Ans: 15 ways)

**10** a) Find all the values of $\theta$, $0^0 \le \theta \le 360^0$, which satisfy the equation $\sin^2\theta - \sin 2\theta - 3\cos^2\theta = 0$.

(Ans: θ = 71.6°, 135°, 251.6°, 315°)

b) Show that $\frac{\cos A}{1+\sin A} = \cot\left(\frac{A}{2} + 45^0\right)$. Hence or otherwise solve $\frac{\cos A}{1+\sin A} = \frac{1}{2}$; $0^0 \le A \le 360^0$

**11. a)** Find the equation of the line through $A(2, 2, 5)$ and $B(1, 2, 3)$. (Ans: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix} + t\begin{pmatrix} -1 \\ 0 \\ -2 \end{pmatrix}$)

**b)** If the line in (a) above meets the line $\frac{x-1}{1} = \frac{y-2}{0} = \frac{z-1}{3}$ at P, find the:

i) coordinates of P (Ans: (3, 2, 7))
ii) angle between the two lines. (12 *marks*)

(Ans: $\theta$ = 8.1° or 171.9°)

**12**. Express $f(x) = \frac{x^2-4}{(x+1)^2(x-5)}$ in partial fractions.

Hence evaluate $\int_6^7 f(x)dx$ correct to 4 decimal places. (Ans: 04689)

**13. a)** Show that the line $5y = 4x + 25$ is a tangent to the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$

**b)** Find the equation of the normal to the ellipse at the point of contact. (Ans: $y = \frac{-5}{4}x - \frac{16}{5}$)

**c)** Determine the eccentricity of the ellipse.

(Ans: $\pm\frac{4}{5}$)

**14.** a) Differentiate the following with respect to $x$

i) $(\sin x)^x$ (Ans: $(\sin x)^x(x \cot x + \ln \sin x)$)

ii) $\frac{(x+1)^2}{(x+4)^3}$ Giving your answers in their simplest forms. (Ans: $\frac{(5-x)(x+1)}{(x+4)^4}$)

b) The distance of a particle moving in a straight line from a fixed point after time t is given by

$x = e^{-t}\sin t$.

Show that the particle is instantaneously at rest at time $t = \frac{\pi}{4}$ seconds. Find its acceleration at $t = \frac{\pi}{4}$ seconds. (Ans: -0.6447s)

**15. a)**Without using table or calculators, simplify

$\frac{\left(\cos\frac{\pi}{17} + i\sin\frac{\pi}{17}\right)^8}{\left(\cos\frac{\pi}{17} - i\sin\frac{\pi}{17}\right)^9}$ (Ans: -1)

**b**) Given that $x$ and $y$ are real, find the values of $x$ and $y$ which satisfy the equation: $\frac{2y+4i}{2x+y} - \frac{y}{x-i} = 0$.

(Ans: $x$ = -1, $y$ = -2; $x$ = 1, $y$ = 2)

**16.** Solve the differential equation

$\tan x\frac{dy}{dx} - y = \sin^2 x$. (Ans: $y = \sin^2 x + c \sin x$)

**b**) An athlete runs at a speed proportional to the square root of the distance he still has to cover. If the athlete starts running at 10 $ms^{-1}$ and has a distance of 1600 m to cover, find how long he will take to cover the distance. (Ans: 320 s)

# 2004 PAPER TWO

## SECTION A

**1**. A particle is performing simple Harmonic motion with center *O*, amplitude 6m and period 2π. Points *B* and C lie between *O* and *A* with $\overline{OB}$ = 1m, $\overline{OC}$ = 3m and $\overline{OA}$ = 6m. Find the least time taken while traveling from

a) *A* to *B* (Ans: 0.4467 π s)
b) *A* to *C*. (Ans: 0.3333π)

**2.** The probability of two independent events *P* and *Q* occurring together is 1/8. The probability that either or both events occur is 5/8. Find

a) Prob(*P*) (Ans: ½, ¼)
b) Prob(*Q*) (Ans: ¼, ½)

**3**. In the table below is an extract of part of log $x$ to base 10, $\log_{10} x$

| $x$ | 80.00 | 80.20 | 80.50 | 80.80 |
|---|---|---|---|---|
| $\log_{10} x$ | 1.9031 | 1.9042 | 1.9058 | 1.9074 |

Use linear interpolation to estimate:

a) $\log_{10} 80.759$ (Ans: 1.9072)
b) the number whose logarithm is 1.90388.

(Ans: 80.14)

**4**. A particle is projected at an angle of $60^0$ to the horizontal with a velocity of 20 $ms^{-1}$. Calculate the greatest height the particle attains. [Use g = 10 $ms^{-2}$]

(Ans: 15m)

**5**. Twenty percent of eggs supplied by a certain farm have cracks on them. Determine the probability that a sample of 900 eggs supplied by the farm will have more than 200 eggs with cracks. (Ans: 0.0439)

**6**. Two forces of magnitude 12N and 9N act on a particle producing an acceleration of 3.65 $ms^{-2}$. The forces act at an angle of $60^0$ to each other. Find the mass of the particle. (Ans: 5)

**8.** Given the numbers; $x = 2.678$ and $y = 0.8765$, measured to the nearest number of decimal places indicated,

a) State the maximum possible error in $x$ and $y$. (Ans: 0.0005); 0.00005)

b) Determine the absolute error in $xy$ (Ans: 0.00057215)

c) Find the limit within which the product $xy$ lies, correct to 4 decimal places. (Ans: (2.3467, 2.3478))

# SECTION B

**9.** a) Abel, Bob and Charles applied for the same job in a certain company. The probability that Abel will take the job is ¾. The probability that Bob will take it is ½ while the probability that Charles won't take the job is 1/3. What is the probability that:

i) None of them will take the job? (Ans: 1/224)

ii) One of them will take the job? (Ans: ¼)

b) Two events $A$ and $B$ are independent. Given that $P(A \cap B^1) = ¼$ and $P(A^1/B) = 1/6$, use a Venn diagram to find the probabilities

i) $P(A)$ (Ans: 5/6)

ii) $P(B)$ (Ans: 7/10)

iii) $P(A \cap B)$ (Ans: 7/12)

iv) $P(A \cup B)^1$. (Ans: 7/20)

**10(a).** Use the trapezium rule to estimate the area of $y = 5^{2x}$ between the $x$-axis, $x = 0$ and $x = 1$, using five sub intervals. Give your answer correct to 3 decimal places (Ans: 7.712)

b) Find the exact value of: $\int_0^1 5^{2x}\,dx$. (Ans: 7.456)

c) Determine the percentage error in the two calculations in (a) and (b) above (Ans: 3.43%)

**11.** The probability density function of a random variable is given by

$$f(x) = \begin{cases} k(x+2) & ; \quad -1 < x < 0, \\ 2k & ; \quad 1 < x \le 1, \\ \frac{k}{2}(5-x) & ; \quad 1 < x \le 3, \\ 0 & ; \quad Else\ where \end{cases}$$

Sketch the function *f(x)*.

Find the:

(i) value of $k$, (Ans: $k = 13$)

(ii) mean of $X$, (Ans: 12/13)

(iii) $P(0 < x < 1/x > 0)$. (Ans: 7/13)

**12(a)**. Use a graphical method to find the approximation to the real root of $x^3 - 3x + 4 = 0$.

**(b)** Use the Newton-Raphson method to find the root of the equation correct to 2 decimal places.

(Ans: -2.20)

**13** A car of mass $m$ kg has an engine which works at a constant rate of $2H$ kW. The car has a constant speed of $V$ ms$^{-1}$ along a horizontal road

a) Find in terms of $m$, $H$, $V$ $g$ and $\theta$ the acceleration of the car when traveling:

i) up a road of inclination $\theta$ with a speed of $3/4V$ ms$^{-1}$,

$$a = \frac{2000H - 3mvg\sin\theta}{3mv}$$

ii) down the same road with a speed of 3/5 $V$ms$^{-1}$, the resistance to the motion of the car apart from the gravitational force, being constant.

(Ans: $a_1 = \dfrac{4000H + 3mug\sin\theta}{3mu}$)

**b)** If an acceleration in (a) (ii) above is 3 times that of (a) (i) above, find the angle of inclination $\theta$ of the road.

**(Ans:** $\theta = \sin^{-1}\left(\dfrac{2000H}{12mug}\right)$**)**

**c)** If the car continues directly up the road, in case (a) (i) above, show that its maximum speed is 12/13 $V$ ms$^{-1}$.

(Ans: $v_1 = \dfrac{12v}{13}$)

**14.** The heights (in cm) of senior six candidates in a certain school were recorded as in the frequency table below.

| Height(cm) | Frequency($f$) |
|---|---|
| 149 – 152 | 5 |
| 153 – 156 | 17 |
| 157 -160 | 20 |
| 161 – 164 | 25 |
| 165 – 168 | 15 |
| 169 – 172 | 6 |
| 173 - 176 | 2 |

**a)** Estimate the mean height and standard deviation of the candidates. (Ans: 160.9 cm, 5.5873)

**b)** Plot a cumulative frequency curve (Ogive)

**c)** Use your Ogive in (b) above to estimate the:

i. Median height (Ans: 161)

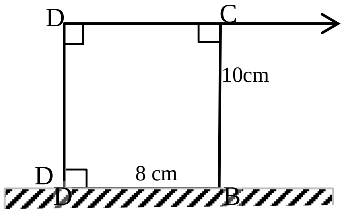
ii. Range of the height of the middle 60% of the candidates. (Ans: 54)

**15. a)** A non-uniform ladder A$B$, 10m long and mass 8 kg, lies in limiting equilibrium with its lower end A resting on a rough horizontal ground and the upper end $B$ resting against a smooth vertical wall. If the centre of gravity of the ladder is 3 m from the foot of the ladder, and the ladder makes an angle of $30^0$ with the horizontal, find the:

i) coefficient of friction between the ladder and the ground. (Ans: $\frac{3\sqrt{3}}{10}$)

ii) reaction at the wall. (Ans: $\frac{12g\sqrt{3}}{5}$)

**b)** The diagram below shows a cross section *ABCD* of a uniform rectangular block of base 8 cm and height, 10 cm resting on a rough horizontal table




An increasing force F parallel to the table is applied on the upper edge. If the coefficient of friction between the block and the table is 0.7, show that the table will tilt before sliding.

**16**. Two planes *A* and *B* are both flying above the Pacific Ocean. *P*lane *A* is flying on a course of $010^0$ at a speed of 300kmh$^{-1}$ *P*lane *B* is flying on a course of $340^0$ at 200 kmh$^{-1.}$ At a certain time, plane *B* is 40 km from plane *A*. *P*lane *A* is then on a bearing $060^0$. After what time will they come closest together and what will be their minimum distance apart? Give your answers correct to 1 decimal place) (Ans: 14.4 minutes; 8.1km)

# 2005 PAPER ONE

## SECTION A

**1**. Given the complex number

$z = \frac{(3i+1)(i-2)^2}{i-3}$, determine:

i) $z$ in the form $a + b$i, where $a$ and $b$ are constants, (Ans: -4 – 3i)

ii) Arg($z$) (Ans: -143.13°)

**2**. Find $\int_0^{\sqrt{\frac{\pi}{2}}} 2x\cos(x)^2\,dx$. (Ans: 1)

**3**. Solve the inequality $(0.6)^{-2x} < 3.6$, correct to two decimal places. (Ans: $x < 1.252$)

**4**. Given that the vectors $a\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $2a\mathbf{i} + a\mathbf{j} - 4\mathbf{k}$ are perpendicular, find the values of $a$.

(Ans: $a = 2$, or -1)

**5.** A spherical balloon is inflated such that the rate at which its radius is increasing is 0.5 cms$^{-1.}$ Find the rate at which

i) the volume is increasing at the instant

(Ans: 157.08 cm$^3$ s$^{-1}$)

ii) the surface area is increasing when r = 8.5 cm.

(Ans: 106.814 cm$^2$ s$^{-1}$ (3 dp))

**6**. Solve the equation $2\sin^2\theta + 3\cos\theta = 0$, $0^0 \le \theta \le 360^0$. (Ans: $\theta$ = 120°, 240°)

**7.** Sketch the parabola $y^2 = 12(x - 4)$. State the focus and equation of the directrix.

**8**. Determine $\frac{d}{dx}\left\{\ln\left(\frac{x}{\sqrt{1+x^2}}\right)\right\}$, when $x = 2$.(Ans: 1/10)

## SECTION B (60 *marks*)

**9**. (a) Given that *X*, *Y* and *Z* are angles of a triangle *XYZ*. Prove that $\tan(X - Y) = \frac{x-y}{x+y}\cot\frac{z}{2}$. Hence solve the triangle if $x = 9$ cm, $y = 5.7$ cm and Z = $57^0$.

(07 *marks*)

**b**) Use the substitution $t = \tan \theta/2$ to solve the equation $3\cos\theta - 5\sin\theta = -1$ for $0^0 < \theta < 360^0$.

(Ans: θ = 40.84°, 201.08°)

**10**. **i)** Determine the coordinates of the point of intersection of the line $\frac{x+1}{2} = \frac{y-3}{5} = \frac{z+2}{-1}$ and the plane $x + y + z = 12$. (Ans: (3, 13, -4))

**ii)** Find the angle between the line $\frac{x+1}{2} = \frac{y-3}{5} = \frac{z+1}{-1}$ and the plane $x + y + z = 12$.

(Ans: 39.2315°)

**11. a**) Determine the Binomial expansion of $\left(1+\frac{x}{2}\right)^4$

Hence evaluate $(2.1)^4$ correct to 2 decimal places.

(Ans: $1+2x+\frac{3x^2}{2}+\frac{x^3}{2}+\frac{x^4}{16}$; = 19.45)

**b**) A geometric progression (G.P) has a common ratio $r < 1$, $u_1 = 15$, and $S_\infty = 22.5$, where $S_\infty$ is its sum to infinity and $u_1$, the first term.
Find the:

i) value of $r$, (Ans: 1/3)
ii) ratio of $u_1 : u_2$. (Ans: 3:1)

**12. a)** Find the equation of a circle which passes through the points (5, 7), (1, 3) and (2, 2).
(Ans: $x^2 + y^2 - 7x - 9y + 24 = 0$)

**b**) i) If $x = 0$ and $y = 0$ are tangents to the circle, $x^2 + y^2 + 2gx + 2fy + c = 0$, show that $c = g^2 = f^2$.

**ii**) Given that the line $3x - 4y + 6 = 0$ is also a tangent to the circle in (b) (i) above, determine the equation of the circle lying in the first quadrant.
(Ans: $x^2 + y^2 - 2x - 2y + 1 = 0$)

**13**. Given the curve $y = \sin 3x$, find the

**a**) **i)** value of $\frac{dy}{dx}$ at the point $\left(\frac{\pi}{2},\ 0\right)$. (Ans: -3)

**ii)** equation of the tangent to the curve at this point
(Ans: $y = -3x + \pi$)

**b**) **i)** Sketch the curve $y = \sin 3x$.

**ii**) Calculate the area bounded by the tangent in **a** (i) above, the curve and the $y$ - axis.
(Ans: 0.9783 sq. units)

**14**. **a)** Solve the equation $\frac{4x-3y}{4} = \frac{2y-x}{3} = \frac{z+4y}{2}$ and $6x + 6y + 2z = 6$. (Ans: $x = \frac{17}{15}$, $y = \frac{16}{15}$, $z = \frac{-18}{5}$)

**b**) Given the polynomial $f(x) = Q(x)\,g(x) + R(x)$, where $Q(x)$ is the quotient, $g(x) = (x - \alpha)(x - \beta)$ and $R(x)$ the remainder, show that

$$R(x) = \frac{(x-\beta)f(\alpha) + (\alpha - x)f(\beta)}{\alpha - \beta}$$

when $f(x)$ is divided by $g(x)$.
Hence find the remainder when $f(x)$ is divided by $x^2 - 9$, given that $f(x)$ divided by $x - 3$ is 2 and when divided by $x + 3$ is -3. (07 *marks*)

**15**. Express $\frac{3x^2+x+1}{(x-2)(x+1)^3}$ into partial fractions. Hence evaluate $\int_3^4 \frac{3x^2+x+1}{(x-2)(x+1)^3}\,dx$. Give your answer correct to 3 decimal places. (Ans: 0.317)

**16. a**) Solve the differential equation
$\frac{1}{x}\frac{dy}{dx} = \sin x \sec^2 3y$ (Ans: $\sin x - x\cos x + c$)

**b)** A hot body at a temperature of $100^0$C is placed in a room of temperature of $20^0$C. Ten minutes later, its temperature is $60^0$C.

**i)** Write down a differential equation to represent the rate of change of temperature, $\theta$ of the body with time, $t$.
(Ans: $\frac{dt}{d\theta} = -k(\theta - R)$)

**ii)** Determine the temperature of the body after another 10 minutes. (Ans: 40°)

# 2005 PAPER TWO

## SECTION A

**1.** A particle moves in a straight line with S.H.M. of period 5 seconds. The greatest speed is $4ms^{-1}$. Find the

i) Amplitude (Ans: $\frac{10}{2\pi}$ m)

**ii)** speed when it is $\frac{6}{\pi}$ m from the centre.
(Ans: 3.2 $ms^{-1}$)

**2**. In the table below is part of an extract from sec $x^0$.

| $x = 60^0$ | 0′ | 12′ | 24′ | 36′ | 48′ |
|---|---|---|---|---|---|
| Sec $x$ | 2.0000 | 2.0122 | 2.0245 | 2.0371 | 2.0498 |

Use linear interpolation to estimate the

i) value of sec $60^0 15'$, (Ans: 2.03065)
iii) angle whose secant is 2.0436. (Ans: 42")

**3**. A good football striker is nursing his injury in the leg. The probability that his team will win the next match when he is playing is 4/5, otherwise it is 2/3. The probability that he will have recovered by the time of the match is ¼. Find the probability that his team will win the match. (Ans: 7/10)

**4**. On the average 15% of all boiled eggs sold in a restaurant have cracks. Find the probability that a sample of 300 boiled eggs will have more than 50 cracked eggs. (Ans: 0.215)

**5**. A particle is projected with a velocity of 40 $ms^{-1}$ at an angle of $60^0$ to the horizontal from the foot of a plane inclined at an angle of $30^0$ from the horizontal. Find the time at which the particle hits the plane. [Use g = 10 $ms^{-1}$] (Ans: 4.6188 s)

**6**. The table below shows the marks scored by ten students in Mathematics and Fine Art tests.

| | A | *B* | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| Mathematics | 40 | 48 | 79 | 26 | 55 | 35 | 37 | 70 | 60 | 40 |
| Fine Art | 59 | 62 | 68 | 47 | 46 | 39 | 63 | 29 | 55 | 67 |

Calculate the rank correlation coefficient for the students performance in the two subjects. Comment on your result. (Ans: 0.1121)

**7**. The forces 3N, 4N, 5N and 6N act along the sides *AB*, *BC*, *CD* and *DA* of a rectangle. Their direction is in the order of the letters. ***BC*** is the horizontal. Find the resultant force and the couple at the centre of a rectangle of sides 2 m and 4 m.
(Ans: $2\sqrt{2}$, 26m anticlockwise)

**8**. Given the numbers $a = 23.037$ and $b = 8.4658$, measured to their nearest number of decimal places indicated,

i) state the maximum possible errors in $a$ and $b$
(Ans: $e_A = 0.0005$)

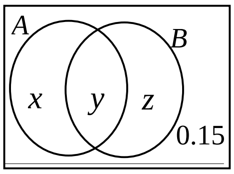
ii) determine the absolute errors in $\frac{a}{b}$

(Ans: 0.00007513)

iii) find the limits within which $\frac{a}{b}$ lies. Correct to 4 decimal places. (Ans: 2.7211 and 2.7213)

**SECTION *B***

**9**. a) A and *B* are intersecting sets as shown in the Venn diagram below.




Given that $P(A) = 0.6$, $P\left(A^1/B\right) = \frac{5}{7}$, and $P(A \cup B) = 0.85$, find

i) the value of *x*, y and z

(Ans: $x = 0.5$, $y = 0.1$. $z = 0.25$)

ii) $P\left(A/B\right)$

b) A bag contains 4 white balls and 1 black ball. A second bag contains 1 white ball and 4 black balls. A ball is drawn at random from the first bag and put into the second bag, then a ball is taken from the second bag and put into the first bag. Find the probability that a white ball will be picked when a ball is selected from the first bag. (Ans: 7/10)

**10. i)** Use the trapezium rule to estimate the area of $y = 3^x$ between the $x$ – axis, $x = 1$ and $x = 2$, using five sub-intervals. Give your answer correct to four significant figures. (Ans: 5.48338)

ii) Find the exact value of $\int_1^2 3^x\, dx.$ (Ans: 5.4614)

**iii)** Find the percentage error in calculations (i) and (ii) above. (Ans: 0.4028%)

**11**. The probability density function of a random variable *x* is given by $f(x) = \begin{cases} 2kx & ; \quad 0 \le x \le 1 \\ k(3-x) & ; \quad 1 \le x \le 2 \\ 0 & ; \quad Else\ where \end{cases}$

a) Sketch the function *f*(*x*)

b) Find the i) value of *k* (Ans: $k = 2/5$)

ii) Mean of *x* (Ans: 1.1333)

iii) $P(1 < x < \frac{2}{x} > 0)$.

**12**. Use a graphical method to show that the equation $e^x + x - 4 = 0$ has only one real root. Use the Newton-Raphson method to find the root of the equation correct to 3 significant figures. (Ans: 1.07)

**13**. a) A pump raises 2 $m^3$ of water through a vertical distance of 10 meters in one and a half minutes, discharging it at a speed of 2.5 $ms^{-1}$. Show that the power it develops is approximately 2.25 kW (to 3 significant figures).

b) A car of mass 1000 kg has a maximum speed of 150 km $h^{-1}$ on a level rough road and the engine is working at 60 kW.

**i)** Calculate the coefficient of friction between the car and the road if all the resistance is due to friction.

(Ans: 0.1469)

**ii)** Given that the tractive force remains unaltered and the non-gravitational resistance in both cases varies as the square of the speed, find the greatest slope on which a speed of 120 km $h^{-1}$ could be maintained.

(Ans: 3.0322°)

**14**. A particle *P* moving with a constant velocity $2\boldsymbol{i} + 3\boldsymbol{j} + 8\boldsymbol{k}$, passes through a point with position vector $6\boldsymbol{i} - 11\boldsymbol{j} + 4\boldsymbol{k}$. At the same instant, a particle *Q* passes through a point with position vector $\boldsymbol{i} - 2\boldsymbol{j} + 5\boldsymbol{k}$, moving with constant velocity $3\boldsymbol{i} + 4\boldsymbol{j} - 7\boldsymbol{k}$. Find the

i) position and velocity of *Q* relative to *P* at that instant (Ans: $\begin{pmatrix} -5 \\ 9 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ -15 \end{pmatrix} t$)

ii) shortest distance between *P* and *Q* in the subsequent motion. (Ans: 10.3183 units)

iii) time that elapses before the particle are nearest to one another. (Ans: $\frac{11}{227}$ S)

**15**. The weights of senior five science class in a certain school were recorded as in the frequency table below.

| Weight (kg) | Frequency |
|---|---|
| 50 – 53 | 3 |
| 54 – 57 | 8 |
| 58 – 61 | 12 |
| 62 − 65 | 18 |
| 66 – 69 | 11 |
| 70 – 73 | 5 |
| 74 – 77 | 2 |
| 78 − 81 | 1 |

a. Estimate the mean and standard deviation of the students' weights (Ans: 63.1, 6)

b. Plot an ogive

c. Use your ogive to estimate the:

i) median weight (Ans: 63.1)

ii) number of students who weigh between 58.9 kg and 66.7 kg. (Ans: 29 students)

**16**. A uniform ladder of length 2*l* and weight W rests in a vertical plane with one end against a rough vertical wall and the other against a rough horizontal surface, the angles of friction at each end being $\tan^{-1}\left(\frac{1}{3}\right)$ and $\tan^{-1}\left(\frac{1}{2}\right)$ respectively.

**a) If** the ladder is in limiting equilibrium at either end, find $\theta$, the angle of inclination of the ladder to the horizontal (Ans: 39.8°)

**b)** A man of weight 10 times that of the ladder begins to ascend it, how far will he climb before the ladder slips? (Ans: ½ of the ladder)

# 2006 PAPER ONE

## SECTION A

**1.** Prove that $\tan\left(\frac{\pi}{4}+\theta\right)-\tan\left(\frac{\pi}{4}-\theta\right)=2\tan 2\theta$

**2**. Differentiate $\frac{3x+4}{\sqrt{2x^2+3x-2}}$ with respect to $x$

(Ans: $\frac{-7x-24}{2(2x^2+3x-2)^{3/2}}$ )

**3**. Show that when the quadratic expressions $x^2 + bx + c = 0$ and $x^2 + px + q = 0$ have a common root, then, $(c - q)^2 = (b - p)(cp - bq)$

**4**. Prove that $y = -3x + 6$ is a tangent to a rectangular hyperbola whose parametric co- coordinates are of the form $\left(\sqrt{3}t, \frac{\sqrt{3}}{t}\right)$

**5**.Find the point of intersection of the plane $11x - 3y + 7z = 8$ and the line $\mathbf{r}=\begin{pmatrix}-3\\1\\5\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\-2\end{pmatrix}$ where $\lambda$ is a scalar. (Ans: (-4, -1, 7))

**6**. A group of nine has to be selected from ten boys and eight girls. It can consist of either five boys and four girls or four boys and five girls. How many different groups can be chosen? (Ans: 29400)

**7**. Solve the differential equation $\frac{dy}{dx}+3y=e^{2x}$ given that when $x = 0, y = 1$. (Ans: $y=\frac{1}{5}(e^{2x}+4e^{-3x})$)

**8**. Evaluate $\int_0^2 \frac{8x}{x^2-4x-12}dx$ correct to **2** decimal places.

(Ans: -1.05 )

## SECTION B

**9**. Express the complex numbers $Z_1 = 4\mathbf{i}$ and $Z_2 = 2 - 2\mathbf{i}$ in the trigonometric form $r(\cos\theta + i\sin\theta)$.

Hence or otherwise evaluate $\frac{Z_1}{Z_2}$.

(Ans: $Z_1 = 4(\cos 90° + i\sin 90°)$;
$Z_2 = 2\sqrt{2}(\cos(-45°) + \sin(-45°)$; $\frac{Z_1}{Z_2}=\frac{1}{2}(-1+0i)$)

Find the values of $x$ and $y$ in $\frac{x}{(2+3i)}-\frac{y}{(3-2i)}=\frac{(6+2i)}{(1+8i)}$

(Ans: $x = 2.8, y = 0.4$)

**10(a)**. Differentiate from first principles $y=\frac{x}{x^2+1}$ with respect to $x$ (Ans: $\frac{dy}{dx}=\frac{1-x^2}{(x^2+1)^2}$ )

**(b) (i)** Determine the turning points of the curve

$y = x^2(x-4)$ (Ans: (0, 0) $\left(\frac{8}{3}, \frac{-256}{27}\right)$)

**(ii)** Sketch the curve in (i) above for $-1 \le x \le 5$.

**(iii)** Find the area enclosed by the curve above and the $x$ – axis. (Ans: $\frac{64}{3}$ sq. units)

**11**. a) Given the vectors $\boldsymbol{a} = 3\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{b} = \boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}$, Find:

i) the acute angle between the vectors (Ans: 36.7°)

ii) vector c such that it is perpendicular to both vectors ***a*** and ***b***. (Ans: $\begin{pmatrix}2\\5\\4\end{pmatrix}$)

**b**) Given that **OA** = **a** and **OB** = **b**, point $R$ is on $\overline{OB}$ such that $\overline{OR}:\overline{RB} = 4:1$. Point $P$ is on $\overline{BA}$ such that $\overline{BP}:\overline{PA} = 2:3$ and when $\overline{RP}$ and $\overline{OA}$ are both produced they meet at point $Q$.

Find:

i) **OR** and **OP** in terms of **a** and **b**.

(Ans: $\frac{4}{5}b$, $OP = \frac{1}{5}(2a+3b)$ )

ii) **OQ** in terms of **a**. (Ans: $\frac{8}{5}a$ )

**12**. **a**) Solve the equation 3 cos $x$ + 4 sin $x$ = 2 for $0 \le x \le 360^0$. (06 marks)

**b**) If $A$, $B$, $C$ are angles of the triangle, show that cos $2A$ + cos $2B$ + cos $2C$ = -1 − 4 cos$A$ cos $B$ cos $C$.

(Ans: 119.6°, 346.7°)

**13**. a) Form the equation of a circle that passes through the points $A$ (-1, 4), $B$ (2, 5) and $C$ (0, 1)

(Ans: $x^2 + y^2 - 2x - 6y + 5 = 0$)

**b**) The line $x + y = c$ is a tangent to the circle $x^2 + y^2 - 4y + 2 = 0$. Find the coordinates of the point of contact of the tangent for each value of $c$. (Ans: (1, 3))

**14**. **a**) Find the first three terms of the expansion of $\frac{1}{1+x}$, using Maclaurin's theorem. (Ans: $1 - x + x^2$)

**b**) Use Maclaurin's theorem to expand tan $x$ in ascending powers of $x$ up to the term in $x^3$.

(Ans: $x + \frac{x^3}{3}$ )

**15**. **(a)** Expand $(a + b)^4$. Hence find $(1.996)^4$ correct to 3 decimal places. (Ans: 15.872)

**(b)** A credit society gives out a compound interest of 4.5% per annum. Muggaga deposits Shs 300,000 at the beginning of each year. How much money will he have at the beginning of four yeas If there are no withdrawals during this period? (Ans: 1341212917)

**16.** Find: a) $\int \ln x^2 dx$ (Ans: $2x(\ln x - 1) + c$)

b) $\int \frac{dx}{e^x - 1}$ (Ans: $\ln(1 - e^{-x}) + c$)

# 2006 PAPER TWO

## SECTION A

1. A and B are two independent events with A twice as likely to occur as B. If $P(A) = \frac{1}{2}$, find:

i) $P(A \cup B)$

ii) $p[(A \cap B)/A]$

**2.** A cylindrical pipe has a radius of 2.5 cm measured to the nearest unit. If the relative absolute error made in calculating its volume is 0.125, find the relative absolute error made in measuring its height. (05 marks)

**3**. Joan played 12 chess games. The probability that she wins a game is $\frac{3}{4}$. Find the probability that she will win:

i) exactly 8 games

ii) more than 10 games

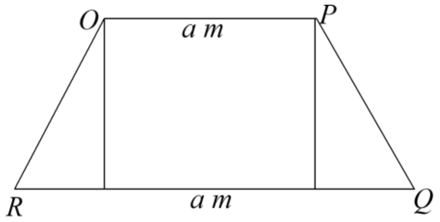
**4.** The resultant of the forces $F_1 = 3\mathbf{i} + (a - c)\mathbf{j}$, $F_2 = (2a + 3c)\mathbf{i} + 5\mathbf{j}$, $F_3 = 4\mathbf{i} + 6\mathbf{j}$ acting on a particle is $10\mathbf{i} + 12\mathbf{j}$.

Find the:

i) values of $a$ and $c$

ii) magnitude of force $F_2$.

**5**. The figure below shows a uniform lamina OPQR in the form of a trapezium. $\overline{OP} = a$ m, $\overline{RQ} = 3a$ m. The vertical height of P from $RQ = a$ m. Calculate the centre of mass of $OPQR$.




**6**. A vehicle of mass 2.5 metric tonnes is drawn up on a slope of 1 in 10 from rest with an acceleration of 1.2 ms-2 against a constant frictional resistance of $\frac{1}{100}$ of the weight of the vehicle, using a cable. Find the tension in the cable.

**7.** The probability distribution for the number of heads that show up when a coin is tossed 3 times is given by

$$P(X = x) = \begin{cases}\frac{1}{k}\binom{3}{x}, & x = 0, 1, 2, 3.\end{cases}$$

Find:

i) the value of $k$

ii) $E(X)$.

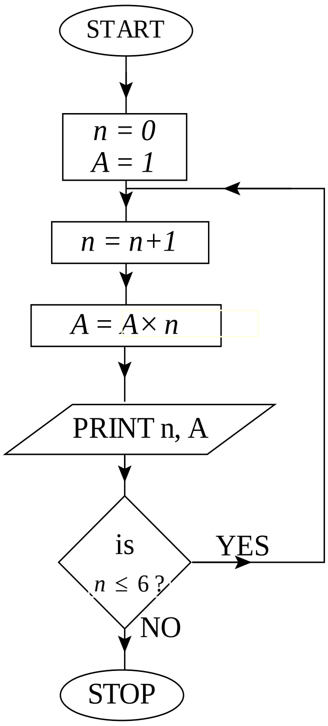
**8**. Use the trapezium rule with 7 ordinates to estimate $\int_0^3 \frac{1}{1+x}dx$, correct to 3 decimal places

**SECTION B**

**9**. **a**) Among the spectators watching a football match, 80% were the home team's supporters while the rest were the visitor team's supporters. If 2500 of the spectators are selected at random, what is the probability that there were more than 540 visitors in this sample? (06 *marks*)

**b**) The times a factory takes to make a unit of a product are approximately normally distributed. A sample of 49 units of the product were taken and found to take an average of 54 minutes with a standard deviation of 2 minutes.
Calculate the 99% confidence limits of the mean time of making all the units of the product.

**10. a)** Study the flow chart below and answer the questions that follow:




i) Perform a dry run for the flow chart
ii) State the purpose of the flow chart
iii) Write down the relationship between $n$ and $A$

**b**) Draw a flow chart that reads and prints the mean of the first twenty numbers and perform a dry run of your chart. (07 *marks*)

**11**. **a**) A light inextensible string of length 5*a* meters has one end attached to end *A* and the other end to point *B* which is vertically below *A* and 3*a* meters from it. A particle *P* of mass *m* kg is fastened to the mid-point of the string and moves with speed *u* in a circle whose centre is the mid-point of *AB*.
Show that the tensions in the upper and lower strings are: $(15mu^2 + 40\ mga)/48a$ and $(15mu^2 - 40\ mga)/48a$ respectively.
Hence deduce that the motion is possible if $15u^2 \geq 40ga$. (07 *marks*)

**b)** A particle is placed on the lowest point of a smooth spherical shell of radius 3a m and is given a horizontal velocity of $\sqrt{13ag}$ m/s. How high above the point of projection does the particle rise? (05 *marks*)

**12**. The table below is the distribution of weights of a group of animals.

| Mass(kg) | Frequency |
|---|---|
| 21 – 25 | 10 |
| 26 – 30 | 20 |
| 31 – 35 | 15 |
| 36 – 40 | 10 |
| 41 – 50 | 30 |
| 51 – 65 | 45 |
| 66 – 74 | 5 |

**(a)** Draw a cumulative frequency curve to estimate the semi-inter-quartile range.
**b)** Find the:
i) mode
ii) standard deviation of the weights. (12 *marks*)

**13**. A light elastic string of natural length *l* has one end fastened to a fixed point *O*. The other end of the string is attached to a particle of mass *m*. When the particle hangs in equilibrium so that it moves in equilibrium, the length of the string is $\frac{7l}{2}$. The particle is displaced from equilibrium so that it moves vertically with simple harmonic motion when the string is taut.

**a**) Show that its period is $\pi\sqrt{\left(\frac{3l}{g}\right)}$.

**b**) At $t = 0$, the particle is released from rest at appoint *A*, at a distance $\frac{3l}{2}$ vertically below O. Find the:
i) depth below O of the lowest point L,
ii) time taken to move from A to L
iii) Depth below O of the particle at time $t = \frac{1}{3}\pi\sqrt{\left(\frac{3l}{g}\right)}$

.

**14.** Show that the Newton-Raphson formula for approximating the $K^{th}$ root of a number N is given by:

$$x_{n+1} = \frac{1}{K}\left[(K-1)x_n + \frac{N}{x_n^{k-1}}\right]$$

Use your formula to find the positive square root of 67 correct to four significant figures.

**b**) Show that one of the roots of the equation $x^2 = 3x - 1$ lies between 2 and 3. By use of linear interpolation, find the root to two decimal places

**15.** A continuous random variable $X$ has a probability density function given by:

$$f(x) = \begin{cases} \beta, & 2 < x < 3 \\ \beta(x-2), & 3 < x < 4 \\ 0, & \text{Otherwise} \end{cases}$$

i) Sketch $f(x)$
ii) Find the value of $\beta$, hence f($x$).
iii) Median, $m$.
iv) P($2.5 < x < 3.5$) (12. *marks*)

**16 (a)** A particle is projected at an angle of elevation of $30^0$ with a speed of 21 m/s. If the point of projection is 5m above the horizontal ground, find the horizontal distance that the particle travels before striking the ground. (Take g = 10 $ms^{-s}$).

**(b)** A boy throws a ball at an initial speed of 40 m/s at an angle of elevation, α. Show , taking g to be 10 $m/s^2$, that the times of flight corresponding to a horizontal range of 80m are positive roots of the equation $T^4 - 64T^2 + 256 = 0$

# 2007 PAPER ONE
## SECTION A

**1**. The 5th term of an Arithmetic Progression (A. P) is 12 and the sum of the first 5 terms is 80. Determine the first term and the common difference. (Ans: 20, -2)

**2.** Given that $\int_0^a (x^2 + 2x - 6)\,dx = 0$, find the value of $a$

(Ans: $a$ = -6)

**3.** Solve the equation $\log_2 x - \log_x 8 = 2$.

(Ans: $x$ = 8 or $x$ = ½)

4. Show that $\frac{\sin\theta - 2\sin 2\theta + \sin 3\theta}{\sin\theta + 2\sin 2\theta + \sin 3\theta} = -\tan^2\frac{\theta}{2}$

5. A point $P$ has co-ordinates (1, -2, 3) and a certain plane has the equation $x + 2y + 2z = 8$. The line through $P$ parallel to the line $\frac{x}{3} = \frac{y+1}{-1} = z+1$ meets the plane at a point **Q.**

Find the co-ordinates of **Q.** (Ans: $(6, \frac{-11}{3}, \frac{14}{3})$ )

6. A hemispherical bowl of internal radius, $r$ is fixed with its rim horizontal and contains a liquid to a depth, $h$. Show by integration that the volume of the liquid in the bowl is $\frac{1}{3}\pi h^2(3r - h)$.

7. Find the locus of the point $P(x$, y) which moves such that its distance from the point S( -3, 0) is equal to its distance from a fixed line $x$ = 3. (Ans: $y^2 = -12x$)

**8.** Differentiate: $\log_e\left(\frac{1+x}{1-x}\right)^{1/2}$ Simplify your answer.

(Ans: $\frac{dy}{dx} = \frac{1}{1-x^2}$ )

## SECTION B

9 (a) The function f(x) = $x^3 + px^2 - 5x + q$ has a factor ($x - 2$) and has a value of 5 when x = –3. Find p and $q$.

(Ans: $p$ = 3, $q$ = -10)

**(b)** The roots of the equation $ax^2 + bx + c = 0$ are $\alpha$ *and* $\beta$. Form the equation whose roots are $\frac{\alpha}{\beta}$ *and* $\frac{\beta}{\alpha}$ (Ans: $acx^2 - (b^2 - 2ac)x + ac = 0$)

(c) Simplify: $\frac{\sqrt{3-2}}{(2\sqrt{3+3})}$ in the form p + q $\sqrt{3}$ where p, q are rational numbers. (Ans: $p$ = 4, $q$ = -7/3)

**10**. Sketch the curve: $y = \frac{4(x-3)}{x(x+2)}$.

**11**. (i) Show that the equation of the tangent to the hyperbola ($a\sec\theta$, $b\tan\theta$ ) is $bx - ay\sin\theta - ab\cos\theta = 0$.

(ii) Find the equations of the tangents to $\frac{x^2}{4} - \frac{y^2}{9} = 1$, at the points where $\theta = 45^0$ and where $\theta = -135°$.

(Ans: $y=\left(\frac{3}{2}\sqrt{2}\right)x-3$; $y=\left(\frac{3}{2}\sqrt{2}\right)x+3$)

(iii) Find the asymptotes. (Ans: $y=\pm\frac{3}{2}x$)

**12.** (a) Solve 2 sin 2$x$ = 3 cos $x$, for $-180^0 \le x \le 180°$.

(Ans: $x$ = (-90°, 48.6°, 90°, 131.4°)

(b) Solve sin$x$ – sin 4$x$ = sin2$x$ – sin3$x$ for $-\pi \le x \le \pi$

(Ans: $x = \frac{-\pi}{5}°, \frac{-\pi}{2}°, \frac{3\pi}{5}°, 0°, \frac{\pi}{5}°, \frac{\pi}{2}°, \frac{3\pi}{5}°, \frac{3\pi}{5}°$)

**13.** (a) $\int \frac{x^2}{\left(1+x^2\right)^{1/2}dx}$ (Ans: $\frac{1}{3}(1+x^2)^{1/2}(x^2-2)+c$ )

(b) Use the substitution $t = \tan\left(\frac{1}{2}x\right)$ to evaluate

$$\int_0^{\pi/2}\frac{dx}{1+\sin x+\cos x}$$ (Ans: ln 2)

**14.** (a) What is the smallest number of terms of the Geometric Progression (G.P.) 5, 10, 20, ... that can give a sum greater than 500,000? (Ans: 17)

(b) Prove by induction $\sum_{r=1}^{n}\frac{1}{r(r+1)}=\frac{n}{n+1}$

(c) Solve simultaneously $a^3 + b^3 = 26$ and $a + b = 2$.

(Ans: $a$ = -1, or 3; $b$ = 3 or -1)

**15.** Given that the position vectors of $A$, $B$ and $C$ are

$\boldsymbol{OA} = \begin{pmatrix}1\\-2\\2\end{pmatrix}$, $\boldsymbol{0B} = \begin{pmatrix}3\\2\\-1\end{pmatrix}$ and $\boldsymbol{OC} = \begin{pmatrix}7\\-2\\2\end{pmatrix}$,

(i) Prove that $A$, $B$ and $C$ are collinear.

(ii) Find the acute angle between $OA$ and $OB$.

(Ans: 106.1°)

(iii) If $OABD$ is a parallelogram, find the position vectors of $E$ and $F$ such that $E$ divides $DA$ in the ratio 1: 2 and $\boldsymbol{F}$ divides it externally in the ratio 1: 2.

(Ans: $\begin{pmatrix}5/3\\2\\-4/3\end{pmatrix}$, OF = $\begin{pmatrix}3\\10\\-8\end{pmatrix}$)

**16**. (a) Given $x = r\cos\theta$ and $y = r\sin\theta$, show that $\frac{d^2y}{dx^2}=-\frac{1}{r}\operatorname{cosec}^3\theta$.

(b) Solve: $\frac{dy}{dx}$ + 2y tan $x$ = cos$^2x$ given that $y$ = 2, when $x$ = 0. (Ans: $y = \cos^2x(2 + x)$)

# 2007 PAPER TWO

## SECTION A

1. A die is tossed 40 times and the probability of getting a six on any one toss is 0.122. Estimate the probability of getting between 6 to 10 sixes. (Ans: 0.2048)
2. Find the position vector of the centre of gravity of a uniform lamina in the form of a triangle whose vertices are; (2, 2), (4, 6 ) and (0,3). (Ans: (2, 11/3))
3. Use the Trapezium rule with 7 ordinates to find the value of $\int_0^{\pi}(1+\sin x)^{\frac{1}{2}}\,dx$, correct to two decimal places. (Ans: 3.98)
4. A particle of mass 2 kg moving with speed 10 ms$^{-1}$ collides with a stationary particle of mass 7 kg. Immediately after impact the particles move with the same speed but in opposite directions. Find the loss in Kinetic Energy during collision. (Ans: 283)
5. The table below shows the likelihood of where A and B spend a Saturday evening.

| | A | B |
|---|---|---|
| Goes to dance | $1/2$ | $2/3$ |
| Visits a neighbour | $1/3$ | $1/6$ |
| Stays at home | $1/6$ | $1/6$ |

(i) Find the probability that they both go out. (Ans: 25/36)

(ii) If we know that they both go out, what is the probability that they both went to dance? (Ans: 12/25)

6. Show that the equation $f(x) = x^3 + 3x - 9$ has a root between $x$ = l and $x$ = 2. Using the Newton Raphson formula once, estimate the root of the equation, rounded off to two significant figures. (Ans: 1.6)
7. The heights, in centimetres, of children in a Senior-One class were:

| Heights (cm) | 151-153 | 154-156 | 157-159 | 160-162 | 163-165 | 166-168 |
|---|---|---|---|---|---|---|
| Frequency | 2 | 14 | 13 | 13 | 2 | 1 |

Calculate the:

(i) mean height, (Ans: 158.133)

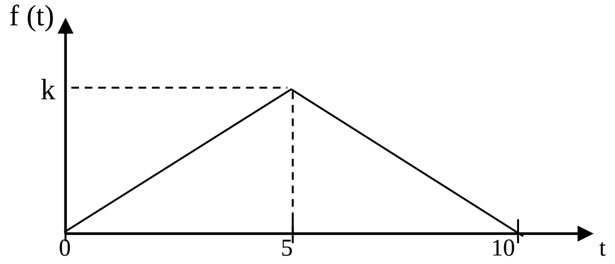
(ii) standard deviation. (Ans: 3.222)

8. The initial velocity of a particle moving with constant acceleration is (3**i** − 5**j**) ms$^{-1}$. After 2 seconds the velocity of the particle is of magnitude 6 ms$^{-1}$ and parallel to (**i** + **j**). Find the acceleration of the particle.

(Ans: $a=\left(\frac{3}{2}\sqrt{2}-\frac{3}{2}\right)i+\left(\frac{3}{2}\sqrt{2}+\frac{3}{2}\right)j$)

## SECTION B

9. The departure time, T of pupils from a certain day primary school can be modeled as in the diagram below, where t is the time in minutes after the final bell at 5.00 pm.




Determine the:

(a) value of $k$, (Ans: $k = \frac{1}{5}$)

(b) equations of the p.d.f.

(Ans: $f(t) = \begin{cases} \frac{1}{25}t & ,0 \le t \le 5 \\ \frac{1}{25}(10-t), & 5 \le t \le 10 \\ 0 & \text{, otherwise} \end{cases}$ )

(c) E(T). (Ans: 5)

(d) probability that a pupil leaves between 4 and 7 minutes after the bell. (Ans: 0.5)

**10.** A car started from rest, accelerated uniformly for 2 minutes and then maintained a speed of 50 $kmh^{-1}$. Another car started 2 minutes later from the same spot, and this car too accelerated uniformly for 2 minutes and it then maintained a speed of 75 $kmh^{-1}$.

(i) Draw a Velocity – Time graph and find when and where the second car overtook the first.

(ii) The first car maintained the speed of 50$kmh^{-1}$ for 10 minutes. It then decelerated uniformly for a further 2½ minutes before coming to rest. How far has the car travelled from the start? (Ans: 10.2 km)

**11. (a)** The table below gives the values of x and their corresponding values of f(x):

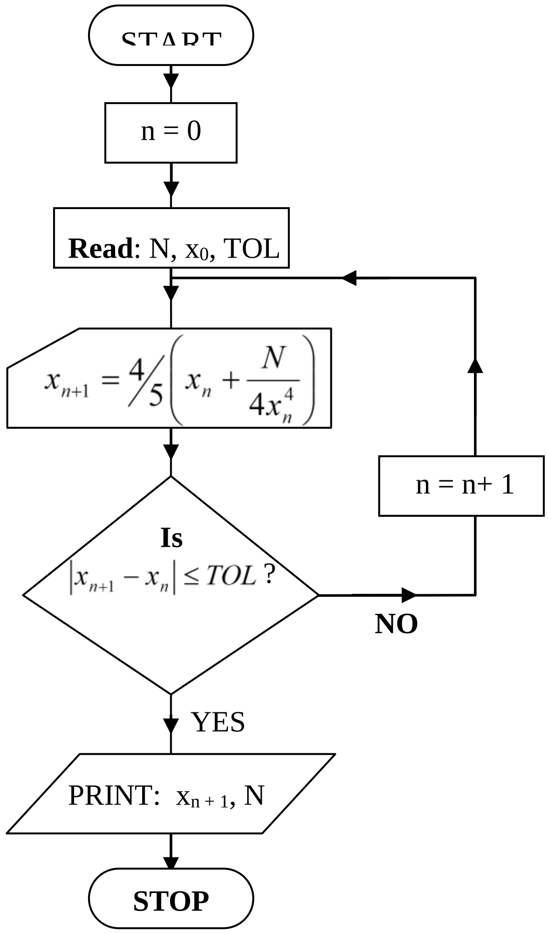
| $x$ | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $f(x)$ | 3.88 | 5.11 | 8.14 | 11.94 |

Use linear interpolation to determine the value of:

(i) $f(x)$ when $x$ = 2.15, (Ans: 4.06)

(ii) $x$ when $f(x)$ = 10.72. (Ans: 4.68)

(b) Study the flow chart below:




(i) Perform a dry run for $x_0$ = 2 and N= 65, TOL = 0.0005.

(ii) State the purpose of the flow chart.

**12.** Below are marks scored by 8 students A, B, C, D, E, F, G and H in Mathematics, Economics and Geography in the end of term examinations.

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| Maths | 52 | 75 | 41 | 60 | 81 | 31 | 65 | 52 |
| Econ | 50 | 60 | 35 | 65 | 66 | 45 | 69 | 48 |
| Geog | 35 | 40 | 60 | 54 | 63 | 40 | 55 | 72 |

Calculate the Rank Correlation Coefficients between the performances of the students in:

(i) Mathematics and Economics. (Ans: 0.8512)

(ii) Geography and Mathematics. (Ans: 0.1905)

Comment on the significance of Mathematics in the performance of Economics and Geography. [Spearman, $\rho$ = 0.86, Kendall's, $\tau$ 0.79 based on 8 observations at 1% level of significance.]

(Ans: Not significant at 1% level)

13. (a) A rod AB, 1m long, has a weight of 20 N and it's centre of gravity is 60 cm from A. It rests horizontally with A against a rough vertical wall. A string BC is fastened to the wall at C, 75 cm vertically above A.

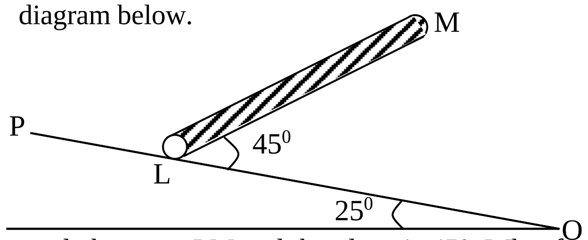
Find the:

(i) normal and frictional forces at A. If friction is limiting, find the coefficient of friction.

(Ans: 0.5)

(ii) tension in the string. (Ans: 20N)

(b) A uniform rod LM of weight W rests with L on a smooth plane PO of inclination 25° as shown in the diagram below.




The angle between LM and the plane is 45°. What force parallel to PU applied at M will keep the rod in equilibrium?
(Give your answer in terms of W.)? (Ans: 0.66W)

**14** The numbers A and B are rounded off to $a$ and $b$ with errors $e_1$ and $e_2$, respectively.

(i) Show that the maximum relative error made in the approximation of $\frac{A}{b}$ by $\frac{a}{b}$ is $\left|\frac{e_1}{a}\right| + \left|\frac{e_2}{b}\right|$.

(ii) If also the number C is rounded off to c with error $e_3$, deduce the expression for the maximum relative error in taking the approximation of $\frac{A}{B+C}$ as $\frac{a}{b+c}$ in terms of $e_1$, $e_2$, $e_3$, $a$, $b$ and $c$.

(Ans: $\left|\frac{e_1}{a}\right| + \left|\frac{|e_2|+|e_3|}{b+c}\right| + \left|\frac{e_3}{b+c}\right|$)

(iii) Given that a = 42.326, b = 27.26 and C = -12.93 are rounded off to the given decimal places, find the range within which the exact value of the expression, B lies.

(Ans: $\pm 0.0021 = 2.9516 \le \frac{A}{B+C} \le 2.9558$ )

15 (a) A box contains 7 red balls and 6 blue balls. Three balls are selected at random without replacement. Find the probability that:
(i) they are of the same colour. (Ans: 0.1923)
(ii) at most two are blue. (Ans: 0.9301)

(b) Two boxes P and Q contain white and brown cards. P contains 6 white cards and 4 brown cards. Q contains 2 white cards and 3 brown cards. A box is selected at random and a card is selected.
Find the probability that:
(i) a brown card is selected. (Ans: 0.5)
(ii) box Q is selected given that the card is white.
(Ans: 0.4)

**16** (a) A particle of mass, m kg is projected with a velocity of 10 $ms^{-1}$ up a rough plane of inclination $30^0$ to the horizontal. If the coefficient of friction between the particle and the plane is $1/4$ calculate how far up the plane the particle travels. (Ans: 7.121 m)

(b) A car is working at 5kW and is travelling at a constant speed of 72$kmh^{-1}$. Find the resistance to motion. (Ans: 250 N)

# 2008 PAPER ONE

## SECTION A

**1.** Find the fourth root of 4 + 3$i$
(Ans: $\pm(1.4760 + 0.23971i)$; $\pm(0.2397 - 1.4760i)$)

**2.** Without using tables or calculators, show that
$\tan 15^0 = 2 - \sqrt{3}$

**3.** Evaluate $\int_0^{\frac{\pi}{2}} \sin 2\theta \, Cos\theta \, d\theta$ (Ans: $\frac{2}{3}$ )`

**4.** Given the vectors $\mathbf{a} = \mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = -\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ find the:
(i) acute angle between vectors **a** and **b**
(Ans: 30.86°)
(ii) equation of the plane containing **a** and **b**
(Ans: $-3x + 5y + 6z = 0$)

**5.** Given the points **O**(0, 0) and **P**(4, 2), **A** is the locus of the points such that **OA** : **AP** = 1 : 2. **Q** is the midpoint of **AP**. Find the locus of **Q** in its simplest form. (Ans: $3x^2 + 3y^2 - 8x - 4y = 0$)

**6.** Given that $\alpha$ and $\beta$ are the roots of the equation $x^2 + px + q = 0$, express ($\alpha^2 - \beta^2$) and ($\alpha^3 - \beta^3$) in terms of $p$ and $q$ (Ans: $-\sqrt{p^2 - 4q}$ ; $p(3q - p^2)$);

**7.** Differentiate $\tan^{-1}\left(\frac{x^2}{2} + 2x^3\right)$ with respect to $x$

(Ans: $\frac{4x(1+6x)}{4+(x^2+4x^3)^2}$ )

***8.*** Find the volume of the solid of revolution formed by rotating the area enclosed by the curve $y = x(1+x)$, the $x$ – axis, the lines $x = 2$ and $x = 3$ through four right angles about the $x$= axis
(Ans: 81.033$\pi$ cubic units)

## SECTION B:

**9.** A circle cuts the $y$ – axis at two points A and B. It touches the $x$- axis at a distance 4 units from the origin and distance AB is 6 units. A is the point (0,1)
Find the;
(a) equation of the circle
(Ans: $x^2 + y^2 + \frac{23}{4}x - 8y + 7 = 0$ , centre ($\frac{23}{8}$, 4) )
(b) equations of the tangents to the circle at A and B
(Ans: $24y = 23x + 168$; $24y = -23x + 24$)

**10.** **(a)** Solve the equation Cos $x$ + Cos 2$x$ = 1 for values of $x$ from $0^0$ to $360^0$ inclusive
(Ans: $x$ = 38.67°, 321.33°)

**(b) (i)** Prove that $\frac{\cos A + \cos B}{\sin A + \sin B} = \cot\frac{A+B}{2}$

**(ii)** Deduce that $\frac{\cos A+\cos B}{\sin A+\sin B}=\tan\frac{C}{2}$ where A, B and C are angles of a triangle.

**11.** (a) Given that $y = \frac{1+\sin^2 x}{\cos^2 x+1}$, show that

$\frac{dy}{dx}=\frac{3\sin 2x}{(\cos^2 x+1)^2}$ (Ans: $\frac{3\sin 2x}{(\cos^2 x+1)^2}$, = -1.6628 )

Hence, find $\frac{dy}{dx}$ when $x=\frac{2\pi}{3}$

**(b)** A curve is represented by the parametric equations $x = 3t$ and $y=\frac{4}{t^2+1}$. Find the general equation of the tangent to the curve in terms of *x*, *y* and *t*. hence determine the equation of the tangent at the point (3, 2).

(Ans: $3y(t^2 + 1)^2 + 8 + x = 24t^2 + 12(t^2 + 1)$; $3y + 2x = 12$)

**12.** The position vectors of points A and B are **OA** = 2**i** – 4**j** – **k** and **OB** = 5**i** – 2**j** + 3**k** respectively. The line AB is produced to meet the plane $2x + 6y – 3z = -5$ at a point C.

Find the;

(a) coordinates of C (Ans: (8, 0, 7))

(b) angle between AB and the plane (Ans: (80.8414°)

**13.** (a) Use partial fractions to evaluate $\int_4^6 \frac{dx}{x^2-2x-3}$

(Ans: 0.1905)

(b) Evaluate $\int_0^{\frac{\pi}{2}} x\sin^2 2x\,dx$ (Ans: $\frac{\pi^2}{16}$)

**14.** On the same axes sketch the curves

$f(x) = x^2 (x+2)$ and $g(x) = \frac{1}{f(x)}$

Show the asymptotes and turning points.

**15.** (a) Find the binomial expansion of $\left(1-\frac{x}{2}\right)^2$. Use your expansion to estimate $(0.875)^5$ to four decimal places. (Ans: 0.5129)

**(b)** A financial credit society gives a 2%compound interest per annum to its members. If Ochola deposits shs 100,000 at the beginning of every year starting with 2004, how much would he collect at the end of 2008 if there are no withdraws within this period?

(Ans: 530, 810.0563)

**16.** (a) Solve the differential equation:

$x\frac{dy}{dx}-y=x^3e^{x^2}$ (Ans: $y=\frac{x}{2}(e^{x^2}+A)$)

(b) The number of car accidents *x* in a year on a high way was found to approximate the differential equation $\frac{dx}{dt}=kx$, where *t* is the time in years and *K* a constant.

At the beginning of 2000 the number of recorded accidents was 50.

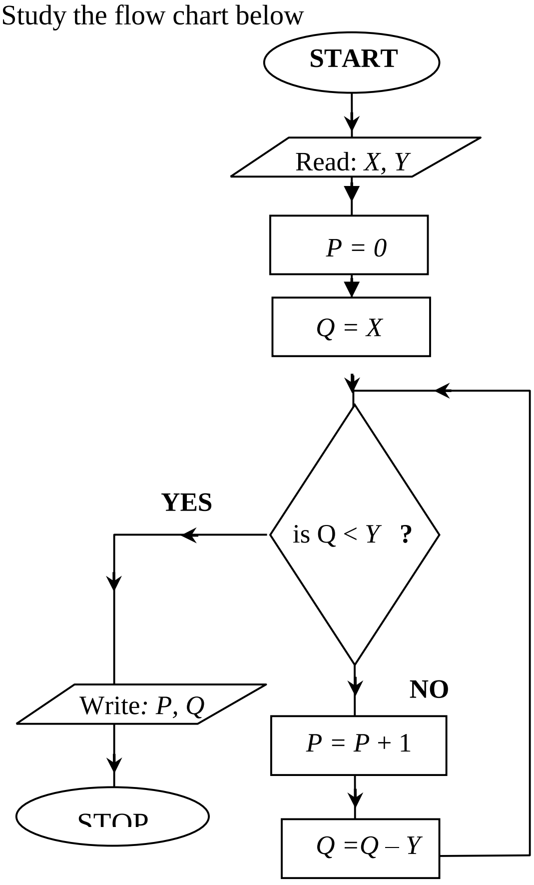
If the number of accidents increased to 60 at the beginning of 2002, estimate the number that was expected at the beginning of 2005. (Ans: 79)

# 2008 PAPER TWO

## SECTION A

1. The probability that Anne reads the *New Vision* is 0.75 and the probability that she reads the *New Vision* and not the *Daily Monitor* is 0.65. The probability that she reads neither of the papers is 0.15. Find the probability that she reads the *Daily Monitor*. (Ans: 0.2)

2. Study the flow chart below




Using the flow chart, perform a dry run and complete the table below for X = 22 and Y = 7

| *P* | *Q* |
|---|---|
| *O* | 22 |
| ………………… | …………………… |
| ………………… | …………………… |
| ………………… | …………………… |

Thus record: *P* = …………………
*Q* = …………………

What is the purpose of the flow chart? (05 marks)

3. The force **A** of magnitude 5N acts in the direction with unit vector $\frac{1}{5}(3i+4j)$ And force **B** of magnitude 13N acts in direction with unit vector $\frac{1}{5}i+\frac{12}{13}j$ Find the resultant of forces **A** and **B.** (Ans: $8\sqrt{2}$ N, 315°

4. Sugar packed in 500g packets is observed to be approximately normally distributed with a standard deviation of 4g. If only 2% of the packets contained less than 500g of sugar, calculate the mean weight of the sugar in the packets. (Ans: 508.216 g)

5. Use the trapezium rule with 6 ordinates to evaluate $\int_0^1 e^{-x^2}$ correct to 2 decimal places. (Ans: 0.74)

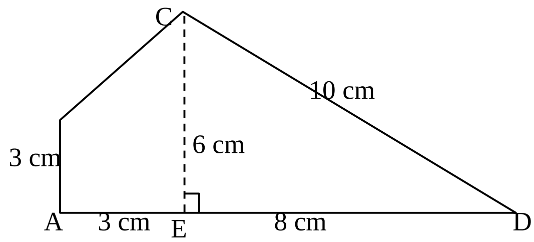
6. The engine of a train exerts a force of 35, 000 N on a train of mass 240 tonnes and draws it up a slope of 1 in 120 against resistance totaling to 60N/ tonne. Find the acceleration of the train.
(Ans: 0.004167 $ms^{-2}$)

7. A discrete random variable $X$ has the following probability distribution.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X=x)$ | 0.09 | 0.15 | 0.40 | 0.25 | 0.11 |

Find the mean and standard deviation of the distribution. (Ans: 2.14; 1.0865)

8. Find the coordinates of the centre of mass of the lamina shown below. Take A as the origin and AD, AB as $x$ – and $y$ – axes respectively.
(Ans: (4.227, 2.12))




## SECTION B

9. The table below shows the amount of money (in thousands of shillings) that was paid out as allowance to participants during a certain workshop.

| Amount (sh'000s) | No. of participants |
|---|---|
| 110 – 114 | 13 |
| 115 – 119 | 20 |
| 120 – 129 | 32 |
| 130 – 134 | 17 |
| 135 – 144 | 16 |
| 145 – 159 | 12 |

(a) Draw a histogram and use it to estimate the modal allowance. (Ans: 11800)
(b) Calculate: (i) median allowances (Ans: 126375)
(ii) mean allowances (Ans: 128,000)

10. (a) The numbers $X$ and $Y$ were estimated with maximum possible errors of $\Delta X$ and $\Delta Y$ respectively. Show that the percentage relative error in XY is $\left[\frac{\Delta X}{X}+\frac{\Delta Y}{Y}\right]\times 100.$
(b) Obtain the range of values within which the exact value of 3.551 × 2.71635 lies.
(Ans: (9.6444, 9.6471).)
(c) Locate each of the three roots of the equation $x^3 - 5x^2 + 5 = 0$ (Ans: (-1, 0), (1, 2), (4, 5))

**11.** (a) Derive the equation of the path of a projectile projected from origin $O$ at angle $\alpha$ to the horizontal with initial speed $U$ms$^{-1}$. (05 marks)
(b) A particle projected from a point on a horizontal ground moves freely under gravity and hits the ground again at $A$. Taking $O$ as the origin, the equation of the path of the particle is $60y = 20\sqrt{3x - x^2}$, where $x$ and $y$ are measured in metres. Determine the:
(i) initial direction and speed of projection (Ans: 20 ms$^{-1}$, 30°)
(ii) distance $OA$ (Take g as 10ms$^{-2}$). (Ans: $20\sqrt{3}$)

**12.** A continuous random variable X has the probability density function

$$f(x) = \begin{cases} \lambda(1-\cos x); & 0 \le x \le \frac{\pi}{2}, \\ \lambda \sin x; & \frac{\pi}{2} < x \le \pi \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Find:
(i) the value of $\lambda$ (Ans: $\lambda = \frac{2}{\pi}$)
(ii) $P\left(\frac{\pi}{3} < X < \frac{3\pi}{4}\right)$ (Ans: 0.6982)
(b) Show that the mean, $\mu$ of the distribution is $1+\frac{\pi}{4}$

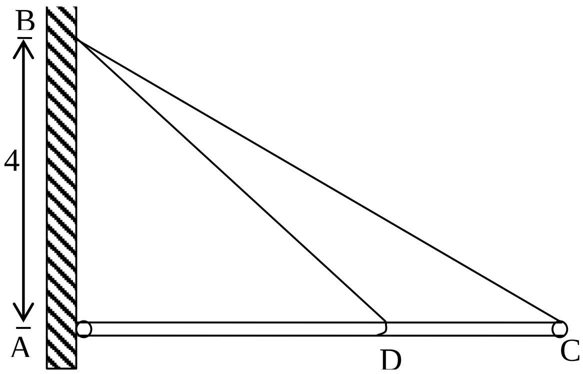
**13.** A particle of mass 1.5 kg lies on a smooth horizontal table and is attached to two light elastic strings fixed at points P and Q 12m apart. The strings are of natural length 4m and 5m and their moduli λ and 2.5λ respectively.
(a) Show that the particle stays in equilibrium at a point $R$ midway between $P$ and $Q$.
(b) If the particle is held at some point S in the line PQ with PS = 4.8m and then released, show that the particle performs simple harmonic motion and find the:
(i) period of oscillation
(ii) velocity when the particle is 5.5m from P

**14.** (a) Show graphically that there is only one positive real root of the equation
$e^x - 2x - 1 = 0$, between 1 and 2. (05 marks)
(b) Use the Newton Raphson method to calculate the root of the equation in (a), correct to 2 decimal places. (Ans: 1.26)

**15.** (a) Sixty students sat for a mathematics contest whose pass mark was 40 marks. Their scores in the contest were approximately normally distributed. 9 students scored less than 20 marks while 3 scored more than 70 marks. Find the:
(i) Mean score and standard deviation of the contest, (Ans: $\mu$ = 39.32, σ = 18.65)
(ii) Probability that a student chosen at random passed the contest

(b) The times a machine takes to print each of the 10 documents were recorded in minutes as given below:
16.5, 18.3, 18.5, 16.6, 19.4, 16.8, 18.6, 16.0, 20.1, 18.2
If the times of printing the documents are approximately normally distributed with variance of 2.56 minutes, find the 80% confidence interval for the mean time of printing the documents
(Ans: (17.25, 28.55))

**16.** A uniform beam AC of mass 8kg and length 8m is hinged at end A and maintained in equilibrium by two strings attached to it at points C and D as shown below.
The tension in BC is twice that in BD; $\overline{AB} = 4m, \overline{AD} = \frac{3}{4}\overline{AC}$




Find the:
(i) tension in string BC, (Ans: 59.8339 N)
(ii) magnitude and direction of the resultant force at the hinge (Ans: 85.8937N, 24.084°)

# 2009 PAPER ONE
## SECTION A

1. Solve the simultaneous equations
$p + 2q - r = -1$
$3p - q + 2r = 16$
$2p + 3q + r = 3$

**2.** Given that $\sin(\theta - 45°) = 3\cos(\theta + 45°)$, show that $\tan\theta = 1$. Hence find $\theta$ if $0° \le \theta \le 360°$.

**3**. Differentiate $e^{ax^2}$

**4.** If $y = \frac{3-2x}{4+x^2}$, find the range of possible values of $y$ for real $x$

**5.** The points P(2,3), Q(−11, 8) and R(-4, -5) are vertices of a parallelogram $PQRS$ which has $PR$ as a diagonal. Find the co-ordinates of vertex S.

**6.** Find $\int \frac{dx}{1-\cos x}$

**7.** Find the equation of a line through the point (1, 3, -2) and perpendicular to the plane whose equation is $4x + 3y - 2z - 16 = 0$

**8.** Solve the differential equation $x(1-y)\frac{dy}{dx} + y = 0$, given that $y = 1$ when $x = e$

## SECTION B

**9.** (a) By using the binomial theorem, expand $(8-24x)^{2/3}$ as far as the 4th term. Hence evaluate $4^{2/3}$ to one decimal place.

b) Find the coefficient of $x$ in the expansion of $\left(x + \frac{2}{x^2}\right)^{10}$

**10** a) Differentiate $\left(1-2x^2\right)^{-1/2}$ with respect to $x$

b) Integrate $\frac{x^4 - x^3 + x^2 + 1}{x^3 + x}$ with respect to $x$.

**11**. a) Use the factor formula to show that

$$\frac{\sin(A+2B) + \sin A}{\cos(A+2B) + \cos A} = \tan(A+B)$$

**b)** Express $y = 8\cos x + 6\sin x$ in the form $R\cos(x - \alpha)$ where $R$ is positive and $\alpha$ is acute .

Hence find the maximum and minimum values of

$$\frac{1}{8\cos x + 6\sin x + 15}$$

**12.** a) Given that $\frac{ix}{1+iy} = \frac{3x+i4}{x+3y}$, find the values of $x$ and $y$

b) If $Z = x + iy$, find the equation of the locus $\left|\frac{Z+3}{Z-1}\right| = 4$

**13**. a) Find the angle between the planes $x - 2y + z = 0$ and $x - y = 1$.

b) Two lines are given by the parametric equations;

$-\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k} + t(\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k})$ and

$-3\boldsymbol{i} + p\boldsymbol{j} + 7\boldsymbol{k} + s(\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k})$

If the lines intersect, find the

i) values of $t$, $s$ and $p$.

ii) coordinates of the points of intersection

**14. a)** Use Maclaurin's theorem to expand $\frac{1}{\sqrt{(1+x)}}$ up to the term in $x^3$

b) Given that $e^x = \tan 2y$, show that

$$\frac{d^2y}{dx^2} = \frac{e^x - e^{3x}}{2\left(1+e^{2x}\right)^2}$$

**15. a)** Find the equation of the tangent and normal to the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ at the point $P(2\cos\theta, \sin\theta)$

b) If the tangent in (a) cuts the $y$ – axis at point A and the $x$– axis at point B, and the normal cuts the $x$-axis at point C, find the co-ordinates of the points A, B and C.

**16.** In a certain process the rate of production of yeast is $kx$ grammes per minute, where $x$ grammes is the amount produced and $k = 0.003$.

a) Show that the amount of yeast is doubled in about 230 minutes.

b) If in addition yeast is removed at a constant rate of m grammes per minute, find the

i) amount of yeast at time $t$ minutes, given that when $t = 0$, $x = p$ grammes

ii) Value of m if $p = 20{,}000$ grammes and the supply of yeast is exhausted in 100 minutes.

# 2009 PAPER TWO

## SECTION A

1. If A and B are independent events;
   (i) show that events A and B' are also independent
   (ii) find P(B) given that P(A) = 0.4 and $P(A \cup B)$ = 0.8

(Ans: 0.66)

**2.** A car moves from Kampala to Jinja and then back. Its average speed on the return journey is $4km^{-1}$ greater than that on the outward journey and it takes 12 minutes less. Given that Kampala and Jinja are 80km apart, find the average speed on the outward journey. (Ans: 38.05 $ms^{-1}$)

**3.** The table below shows the distance in kilometers (km) a truck can run with a given amount of fuel in litres (*l*).

| | | | | |
|---|---|---|---|---|
| Distance(km) | 20 | 28 | 33 | 42 |
| Fuel (*l*) | 10 | 13 | 21 | 24 |

Estimate:
(a) how far the truck can move on 27.5*l* of fuel, (Ans: (Ans: 52.5 km)
(b) the amount of fuel required to cover 29.8km

(Ans: 18.88 *l*)

**4.** The random variable $X$ has a probability function

$$f(x) = \begin{cases} k2^2; x = 0, 1, 2, 3 \\ 0 \;\; ; \;\; \text{elsewhere} \end{cases}$$

Find:
(a) the value of the constant $k$ (Ans: $k = \frac{1}{5}$)
(b) $E(X)$ (Ans: $\frac{34}{15}$)

5. A body of mass 8kg rests on a rough plane inclined at $\theta$ to the horizontal. If the coefficient of friction is $\mu$, find the least horizontal force in terms of $\mu$, $\theta$ and $g$ which will hold the body in equilibrium. (Ans: $P = \dfrac{8g(\sin\theta - \mu\cos\theta)}{(\cos\theta + \mu\sin\theta)}$)

6. Use the trapezium rule with 6 ordinates to estimate $\int_1^2 \frac{\ln x}{x} dx$. Give your answer correct to three decimal places (Ans: 0.237)

7. The following information relates to three products sold by a company in the year 2001 and 2004

| Product | 2001 | | 2004 | |
|---|---|---|---|---|
| | Quantity in thousands | Selling price per unit | Quantity in thousands | Selling price per unit |
| A | 76 | 0.60 | 72 | 0.18 |
| B | 52 | 0.75 | 60 | 1.00 |
| C | 28 | 1.10 | 50 | 1.32 |

Calculate:
(a) percentage increase in sales over the period

(Ans: 10.26%)

(b) corresponding percentage increase in income of the period (Ans: 8.98%)

8. The velocity of a particle at any time $t$ is given by an equation;

$$v(t) = -a\omega\sin\omega t + b\omega\cos\omega t$$

(a) find the expression for the displacement $x$ at any time given that $x$ = 0 when time $t$ = 0 (Ans: $x = 9\cos\omega t + b\omega\sin\omega t$)
(b) show that the motion of the particle is Simple Harmonic

## SECTION B

**9.** The dimensions of a rectangle are 6.2cm and 5.36cm
(i) State the maximum possible error in each dimension (Ans: 0.05, 0.005)
(ii) Find the range within which the area of the triangle lies (Ans: (32.93325, 33.53125))

b) The numbers $a$ = 26.23, $b$ = 13.18 and $c$ = 5.1 are calculated with percentage errors of 4, 3 and 2 respectively. Find the limits to two decimal places within which the exact error of the expression $ab - \frac{b}{c}$ lies. (Ans: (319.21, 367.87))

**10.** A pile driver of mass 1200kg falls freely from a height of 3.6m and strikes without rebounding, a pile of mass 800kg. The blow drives the pile a distance of 36cm into the ground. Find the
(a) resistance of the ground (Ans: 90160N)
(b) time for which the pile is in motion (Ans: 0.14 s)
[*Assume the resistance of the ground to be uniform*]

**11.** The table below shows the income of 40 factory workers in millions of shillings per annum.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1.0 | 1.1 | 1.0 | 1.2 | 5.4 | 1.6 | 2.0 | 2.5 |
| 2.1 | 2.2 | 1.3 | 1.7 | 1.8 | 2.4 | 3.0 | 2.2 |
| 2.7 | 3.5 | 4.0 | 4.4 | 3.9 | 5.0 | 5.4 | 5.3 |
| 4.4 | 3.7 | 3.6 | 3.9 | 5.2 | 5.1 | 5.7 | 1.5 |
| 1.6 | 1.9 | 3.4 | 4.3 | 2.6 | 3.8 | 5.3 | 4.0 |

(a) Form a frequency distribution table with class intervals of 0.5 million shillings starting with the lowest limit of 1 million shillings.
(b) Calculate the
  i. mean income (Ans: 3175,000)
  ii. standard deviation (Ans: 1413992.574)
(c) Draw a histogram to represent the above data. Use it to estimate the modal income.

**12.** Forces of magnitude 3N, 4N, 4N, 3N and 5N act along the lines *AB*, *BC*, *CD*, *DA* and *AC* respectively of the square ABCD whose side has a length of $a$ units. The direction of the forces are indicated by the order of the letters.
(a) find the magnitude and direction of the resultant force

(Ans: 5.196 N, 60.8°)

(b) if the line of action of the resultant force cuts *AB* produced at *E*, find the length *AE* (Ans: 1.76*a*)

**13. (a)** A box contains two types of balls, red and black. When a ball is picked from the box, the probability that it is red is $7/_{12}$. Two balls are selected at random from the box without replacement. Find the probability that

(i) the second ball is black (Ans: $\frac{5}{2}$)

(ii) the first ball is red, given that the second one is black

(Ans: $\frac{7}{11}$)

b) An interview involves written, oral and practical tests. The probability that an interviewee passes a written test is 0.8, the oral test is 0.6 and the practical test is 0.7. What is the probability that the interviewee will pass

(i) the entire interview? (Ans: 0.336)

(ii) exactly two of the interview tests? (Ans: 0.452)

**14. (a)** Show that the root of the equation $2x - 3\cos(x/2) = 0$ lies between 1 and 2

**(b)** Use Newton Raphson's method to find the root of the equation in (a) above. Give your answer correct to two decimal places. (Ans: 1.23 (2 dp))

**15.** The masses of soap powder in certain packets is normally distributed with mean 842 grams and variance 225 (grams)$^2$.

Find the probability that a random sample of 120 packets has sample mean with mass

(i) between 844 grams and 846 grams (Ans: 0.0702)

(ii) less than 843 grams (Ans: 0.7673)

b) A random sample of size 76 electrical components produced by a certain manufacturer has resistances $r_1, r_2, \ldots r_{76}$ ohms where $\Sigma r_i$ = 740 and $\Sigma r_i^2$ = 8,216

Calculate the

(i) unbiased estimate for the population variance (Ans: 13.476)

(ii) 91.86% confidence interval for the mean resistance of the electrical components produced. [*Give answers correct to* 3 *decimal places*]

(Ans: (9.003, 10.470))

**16.** The particles P and Q move with constant velocities of $(4\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k})\text{ms}^{-1}$ and $(6\boldsymbol{i} + 3\boldsymbol{k})\text{ms}^{-1}$ respectively. Initially P is at the point with position vector $(-\boldsymbol{i} + 20\boldsymbol{j} + 21\boldsymbol{k})$m and Q is at the point with position vector $(\boldsymbol{i} + 3\boldsymbol{k})$m.

Find the

(a) time for which the distance between P and Q is least

(Ans: 2.2 s)

(b) distance of P from the origin at the time when the distance between P and Q is least (Ans: 28.8 m)

(c) least distance between P and Q. (Ans: 24.14 m)

# 2011 PAPER ONE

## SECTION A

**1.** Solve the equation $\log_{25} 4x^2 = \log_5 (3 - x^2)$.

(Ans: $x$ = -3, $x$ = 1)

**2.** Find the equation of a line through the point (2, 3) and perpendicular to the line $x + 2y + 5 = 0$.

(Ans: $y = 2x + 1$)

**3.** Evaluate $\int_1^3 \left(\frac{3x^2 + 4x + 1}{x^3 + 2x^2 + x}\right) dx$ (Ans: ln 12)

**4.** A committee of 4 men and 3 women is to be formed from 10 men and 8 women. In how many ways can the committee be formed? (Ans: 11760 ways)

**5.** Show that $\tan^{-1}(\frac{1}{2}) + \tan^{-1}(\frac{1}{5}) = \tan^{-1}(\frac{7}{9})$

**6.** Given that $R = q\sqrt{(1000 - q^2)}$, find

**(a)** $\frac{dR}{dq}$, (Ans: $\frac{1000 - 2q^2}{\sqrt{1000 - q^2}}$)

**(b)** the value of $q$ when $R$ is maximum. (Ans: $\sqrt{500}$)

**7.** Show that the points *A*, B and C with position vectors $3i + 3j + k$, $8i + 7j + 4k$ and ll$i + 4j + 5k$ respectively, are vertices of a triangle.

**8. (a)** Form a differential equation by eliminating the constants $a$ and $b$ from $x = a \cos t + b \sin t$. (Ans: -$x$)

**(b)** State the order of the differential equation formed in (a) above. (Ans: second order)

***SECTION B***

**9. (a)** The first term of an Arithmetic Progression (A.P.) is ½. The sixth term of the A.P is four times the fourth term. Find the common difference of the A.P.

(Ans: $\frac{-3}{14}$)

**(b)** The roots of a quadratic equation $x^2 + px + q = 0$ are $\alpha$ and $\beta$. Show that the quadratic equation whose roots are $\alpha^2 - q\alpha$ and $\beta^2 - q$ is given by $x^2 - (p^2 + pq - 2q)x + q^2(q + P + 1) = 0$.

**10. (a)** Form a quadratic equation having $-3 + 4i$ as one of its roots. $(Z^2 + 6Z + 25) = 0$

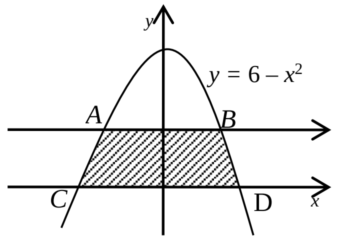
**(b)** Given that $Z_1 = -1 + i\sqrt{3}$ and $Z_2 = -1 - i\sqrt{3}$

**(i)** express $\frac{Z_1}{Z_2}$ in the form $a + i\sqrt{b}$, where $a$ and $b$ are real numbers. (Ans: $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$)

**(ii)** represent $\frac{Z_1}{Z_2}$ on an Argand diagram.

**(iii)** find $\left|\frac{Z_1}{Z_2}\right|$ (Ans: 1)

**11.** In the diagram below, the curve $y = 6 - x^2$ meets the line $y = 2$ at $A$ and $B$, and the $x$ - axis at C and $D$.




Find the (a) coordinates of A, B, C and D

(Ans: A(-2, 2), B(2, 2), C$(-\sqrt{6},\ 0)$, D$(\sqrt{6},\ 0)$)

(b) area of the shaded region, correct to **one** decimal place. (Ans: 8.9293 sq. units)

**12. (a)** Find the angle between the lines

$$x = \frac{y-1}{2} = \frac{z-2}{3} \text{ and } \frac{x}{2} = \frac{y+1}{3} = \frac{z+2}{4}.$$

(Ans: 8.53°)

**(b)** Find in vector form the equation of the line of intersection of the two planes $2x + 3y - z = 4$ and $x - y + 2z = 5$. **(**Ans: $\mathbf{r} = \begin{pmatrix} 19/5 \\ -6/5 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$)

**13. (a)** Find the equation of the tangent to the parabola $y^2 = \frac{x}{16}$ at the point $\left(t^2,\ \frac{t}{4}\right)$.

(Ans: $x - 8ty + t^2 = 0$)

**(b)** If the tangents to the parabola in (a) above at the points $P\left(p^2,\ \frac{p}{4}\right)$ and $Q\left(q^2,\ \frac{q}{4}\right)$ meet on the line $y = 2$,

(i) show that $p + q = 16$,

(ii) deduce that the mid-point of $PQ$ lies on the line $y = 2$.

**14. (a)** Solve $3 \sin x + 4 \cos x = 2$ for $-180° \le x \le 180°$. (Ans: $x = -29.55°, 103.29°$)

**(b)** Show that in any triangle $ABC$,

$$\frac{a^2 - b^2}{c^2} = \frac{\sin(A-B)}{\sin(A+B)}$$

**15.(a)** Differentiate the following with respect to $x$:

**(i)** $(x+1)^{\frac{1}{2}}(x+2)^2$ (Ans: $\frac{(5x + 6)(x + 2)}{2(x + 1)^{1/2}}$)

**(ii)** $\frac{2x^2 + 3x}{(x-4)^2}$ (Ans: $\frac{(4x + 3(x - 4) - 2(2x^2 + 3x)}{(x - 4)^3}$)

**(b)** The base radius of a right circular cone increases and the volume changes by 2%. If the height of the cone remains constant, find the percentage increase in the circumference of the base.

(Ans: 1%)

**16.(a)** Solve the differential equation $\frac{dy}{dx} = \frac{\sin^2 x}{y^2}$, given that $y = 1$ when $x = 0$.

(Ans: $4y^3 = 6x - 3 \sin 2x + 4$)

**(b)** It is observed that the rate at which a body cools is proportional to the amount by which its temperature exceeds that of its surroundings. A body at 78°C is placed in a room at 20°C and after 5 minutes the body has cooled to 65°C. What will be its temperature after a further 5 minutes? (Ans: $\theta = 54.9°$)

# 2011 PAPER TWO

## SECTION A

**1.** The data below represents the lengths of the leaves in centimeters.

4.4, 6.2, 9.4, 12.6, 10.0, 8.8, 3.8 and 13.6

Find the: (a) mean length, (Ans: 8.575 cm)

(b) variance. (Ans: 11.224 cm)

**2.** A particle of mass 2 kg moves under the action of three forces, $\boldsymbol{F}_1$, $\boldsymbol{F}_2$, and $\boldsymbol{F}_3$, At a time, $t$,

$\boldsymbol{F}_1 = (\frac{1}{4}t - 1)\boldsymbol{i} + (t - 3)\boldsymbol{j}$ N,

$\boldsymbol{F}_2 = (\frac{1}{2}t + 2)\boldsymbol{i} + (\frac{1}{2}t - 4)\boldsymbol{j}$ N and

$\boldsymbol{F}_3 = (\frac{1}{4}t - 4)\boldsymbol{i} + (\frac{3}{2}t + 1)\boldsymbol{j}$ N

Find the acceleration of the particle when $t = 2$ seconds. (Ans: $a = -½$ ms$^{-2-}$)

**3.** The table below shows delivery charges by a courier company.

| Mass (gm) | 200 | 400 | 600 |
|---|---|---|---|
| Charges( shs) | 700 | 1200 | 3000 |

Using linear interpolation or extrapolation, find the:

(a) delivery charge of a parcel weighing 352gm, (Ans: 1080)

(b) mass of a parcel whose delivery charge is shs3,300. (Ans: $633\frac{1}{3}$ kg)

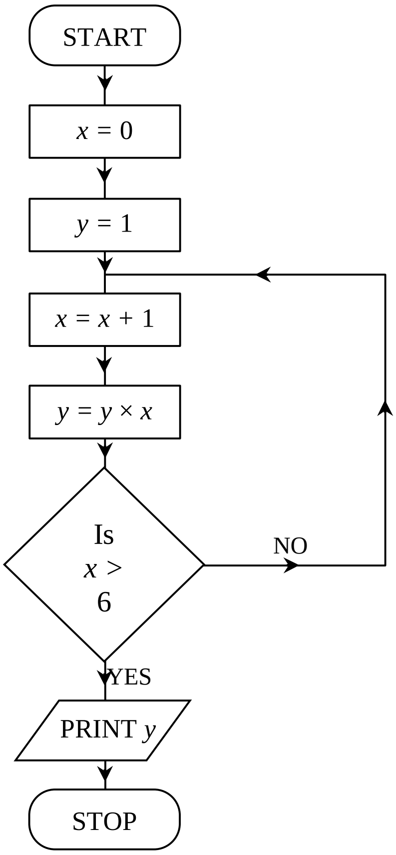
**4.** Two events $A$ and $B$ are such that $P(A' \cap B) = 3x$, $P(A \cap B') = 2x$, $P(A' \cap B') = x$, and $P(B) = \frac{4}{7}$. Using a Venn diagram, find the values of

(a) $x$, (Ans: 1/7)

(b) $P(A \cap B)$ (Ans: 1/7)

**5.** A man can row a boat in still water at 6 kmh$^{-1}$. He wishes to cross a river to a point directly opposite his starting point. The river flows at 4 kmh$^{-1}$ and has a width of 250 m. Find the time the man would take to cross the river. (Ans: 3.354 min)

**6.** Study the flow chart given below.




(a) Perform a dry run.
(b) What is the purpose of the flow chart?

**7.** Given that $X \sim N(2, 2.89)$, find $P(X < 0)$.
(Ans: 0.1198)

**8.** Particles of weights 12 N, 8 N and 4 N act at points (1, -3), (0, 2) and (1, 0) respectively. Find the centre of gravity of the particles. (Ans: $(\frac{2}{3}, \frac{-5}{6})$)

## SECTION B

**9.** The continuous random variable $X$ has the probability density function (p.d.f.) given by

$$f(x) = \begin{cases} k_1 x, & 1 \le x \le 3, \\ k_2(4-x) & 3 < x \le 4, \\ 0 & \text{otherwise} \end{cases}$$

where $k_1$ and $k_2$ are constants.

(a) Show that $k_2 = 3k_1$
(b) Find:
(i) The value of $k_1$ and $k_2$
(Ans: $k_1 = 2/11$, $k_2 = 6/11$)
(ii) $E(X)$, the expectation of $X$ (Ans: 2.485)

**10.** An elastic string of length $a$ metres is fixed at one end $P$ and carries a particle of mass 3 kg at its other end $Q$. The particle is describing a horizontal circle of radius 80 cm with an angular speed of 5 rads$^{-1}$. Determine the:
(a) (i) angle the string makes with the horizontal,
(Ans: 26.1°)
(ii) tension in the string. (Ans: 66.816N)
(b) value of $a$. (Ans: (0.891 - $x$)
(c) linear speed of the particle. (Ans: 4 ms$^{-1}$)

**11.** (a) Use the trapezium rule with five sub-intervals to estimate $\int_0^{\frac{\pi}{3}} \tan x\, dx$ correct to **three** decimal places.
(Ans: 0.704 )
(b) **(i)** Find the value of $\int_0^{\frac{\pi}{3}} \tan x\, dx$ to **three** decimal places. (Ans: 0.693)
**(ii)** Calculate the percentage error in your estimation in (a) above. (Ans: 1.587%)
**(iii)** Suggest how the percentage error may be reduced.
(Ans: Increasing number of sub-intervals)

**12.** The heights and masses of ten students are given in the table below.

| Height (cm) | Mass (kg) |
|---|---|
| 156 | 62 |
| 151 | 58 |
| 152 | 63 |
| 146 | 58 |
| 160 | 70 |
| 157 | 6O |
| 149 | 55 |
| 142 | 57 |
| 158 | 68 |
| 141 | 56 |

(a) (i) Plot the data on a scatter diagram.
(ii) Draw the line of best fit. Hence estimate the mass corresponding to a height of 155cm.
(Ans: 65 kg)
(b) (i) Calculate the rank correlation coefficient for the data. (Ans: 0.87 (2 dp))
(ii) Comment on the significance of the heights on masses of the students. [Spearman's $\ell$ = 0.79 and Kendall's $\tau$ = 0.64 at 1% level of significance based on 10 observations.]
(Ans: 1%)

**13.** A football player projects a ball at a speed of 8ms$^{-1}$ at an angle of 30° with the ground. The ball strikes the ground at a point which is level with the point of projection. After impact with the ground, the ball bounces and the horizontal component of the velocity of the ball remains the same but the vertical component is reversed in direction and halved in magnitude. The player running after the ball kicks it again at a point which is at a horizontal distance of 1.0 m from the point where it bounced, so that the ball continues in the same direction. Find the:
(a) horizontal distance between the point of projection and the point at which the ball first strikes the ground. [Take g = 10 ms$^{-2}$ ].
(Ans: 5.543m)
(b) (i) the time interval between the ball striking the ground and the player kicking it again.
(Ans: 0.1443 s)

(ii) the height of the ball above the ground when it is kicked again. [Take g = 10 ms$^{-2}$]

(Ans: 0.1845 m)

**14.** (a) (i) On the same axes, draw graphs of $y = x^2$ and $y = \cos x$ for $0 \le x \le \frac{\pi}{2}$ at intervals of $\frac{\pi}{8}$ .

(ii) From your graphs, obtain to **one** decimal place, an approximate root of the equation $x^2 - \cos x = 0$. (Ans: 0.8 (1 dp))

(b) Using Newton-Raphson method, find the root of the equation $x^2 - \cos x = 0$, taking the approximate root in (a) as an initial approximation. Give your answer correct to **three** decimal places. (Ans: 0.824 (3 dp))

**15.** Box *A* contains 4 red sweets and 3 green sweets. Box *B* contains 5 red sweets and 6 green sweets. Box *A* is twice as likely to be picked as box *B.* If a box is chosen at random and two sweets are removed from it, one at a time without replacement;

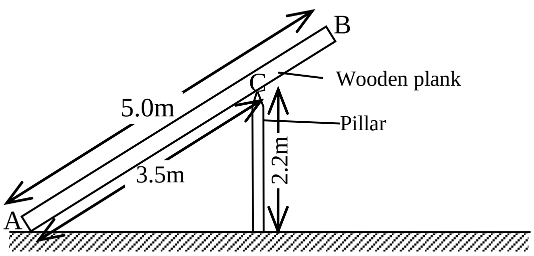
(a) find the probability that the two sweets removed are of the same colour. (Ans: 0.4372)

(b) (i) construct a probability distribution table for the number of red sweets removed.

(ii) find the mean number of red sweets removed.

(Ans: 1.065)

**16.** The diagram below shows a uniform wooden plank *AB* of mass 70 kg and length 5m. The end *A* rests on a rough horizontal ground. The plank is in contact with the top of a rough pillar at C. The height of the pillar is 2.2m and $\overline{AC}$ = 3.5 m.




Given that the coefficient of friction at the ground is 0.6 and the plank is just about to slip, find the:

(a) angle the plank makes with the ground at *A.*

(Ans: 38.945°)

(b) normal reaction at

(i) *A* (Ans: 380.52 N)

(ii) *C* (ANs: 381.097 N)

(c) coefficient of friction at *C*. (Ans: 0.0379)

# 2012 PAPER ONE

**1**. Solve the simultaneous equations

$$3x - y + z = 4,$$
$$x - 2y + 4z = 3,$$
$$2x + 3y - z = 4.$$

(Ans: $x = 1, y = 1, z = 1$)

**2**. (a) Prove that $\frac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$

(b) Solve $\sin 2\theta = \cos\theta$ for $0° \le \theta < 90°$

3. Differentiate $\frac{3x-1}{\sqrt{x^2+1}}$ with respect to $x$

(Ans: $\frac{x+3}{(x^2+1)^{3/2}}$ )

**4.** A line passes through the points $A(4,6,3)$ and $B(1,3,3)$.

**(a)** Find the vector equation of the line.

(Ans: $\begin{pmatrix}4\\6\\3\end{pmatrix} + \lambda\begin{pmatrix}-3\\-3\\0\end{pmatrix}$)

**(b)** Show that the point $C(2, 4, 3)$ lies on the line in (a) above.

**5.** The sum of the first $n$ terms of a Geometric Progression (G.P) is $\frac{4}{3}(4^n - 1)$ . Find its $n^{th}$ term as an integral power of 2. (Ans: $2^{2n}$)

**6**. The line $y = mx + c$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ when $c = \pm\sqrt{a^2m^2 + b^2}$ . Find the equations of the tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ from the point $(0, \sqrt{5})$ (Ans: $y = x + \sqrt{5}$ , $y = -x + \sqrt{5}$ )

**7.** Using a suitable substitution, find $\int \frac{\sin^{-1} 2x}{\sqrt{(1-4x^2)}}\,dx$.

(Ans: $\left(\frac{\sin^{-1} 2x}{2}\right)^2 + c$)

**8.** Find the equation of the normal to the curve $x^2y + 3y^2 - 4x - 12 = 0$ at the point (1, 2). (Ans: $-3x + 2$)

**9.** If $z = \frac{(2-i)(5+12i)}{(1+2i)^2}$

(a) Find: (i) modulus of $z$ (Ans: 5.814)

(ii) argument of $z$ (Ans: -86.055°)

(b) represent $z$ on a complex plane.

(c) Write $z$ in the polar form.

(Ans: $5.814(\cos 0.478\pi - i \sin 0478\pi)$)

**10. (a)** Solve the equation $8\cos^4 x - 10\cos^2 x + 3 = 0$ for $x$ in the range $0° \le x \le 180°$

(Ans: $x$ = 30°, 45, 135°, 150°)

**(b)** Prove that $\cos 4A - \cos 4B - \cos 4C = 4\sin 2B \sin 2C \cos 2A$, given that $A$, $B$, $C$ are angles of a triangle.

**11.(a)** Find the derivatives with respect to $x$ of the following:

i) $\frac{\cos 2x}{1+\sin 2x}$ (Ans: $\frac{-2}{1+\sin 2x}$)

(ii) ln(sec $x$ + tan $x$ (Ans: sec $x$)

**(b)** $\int_0^{\pi/2} x^2 \sin x$ (Ans: $\pi - 2$)

**12.** Triangle $OAB$ has $OA = a$ and $OB = b$. $C$ is a point on $\overline{OA}$ such that $\overline{OC} = \frac{2a}{3}$. $D$ is the midpoint of $\overline{AB}$. When $\overline{CD}$ is produced, it meets $\overline{OB}$ produced at $E$, such that ***DE = nCD*** and ***BE = kb***. Express ***DE*** in terms of:

(a) $n$, ***a***, and ***b*** (Ans: $\frac{5n}{6}\mathbf{a} - \frac{n}{2}\mathbf{b}$)

(b) $k$, ***a***, and ***b***. Hence find the values of $n$ and $k$.

(Ans: $\frac{1}{2}\mathbf{a} + \left(\frac{2k-1}{2}\right)\mathbf{b}$, $n = 3/5$, $k = 1/5$)

**13. (a)** Find the equation of the locus of a point which moves such that its distance from $D(4, 5)$ is thrice its distance from the origin.

(Ans: $x^2 + y^2 + x + 1.25y - 5.125 = 0$)

**(b)** The line $y = mx$ intersects the curve $y = 2x^2 - x$ at the points $A$ and $B$. Find the equation of the locus of the point $P$ which divides $AB$ in the ratio 2:5.

(Ans: $y = 7x^2 - x$)

**14**. (a) On the same axes, sketch the curves

$y = x(x + 2)$ and $y = x(4 - x)$

**(b)** Find the area enclosed by the two curves in (a).

(Ans: 1/3 sq. units)

**(c)** Determine the volume of the solid generated when the area enclosed by the two curves in (a) is rotated about the $x$-axis. (Ans: $\pi$ cubic units)

**15.** Solve for $x$ in the following equations:

(a) $9^x - 3^{(x+1)} = 10$ (Ans: $x = 1.465$)

(b) $\log_4 x^2 - 6\log_x 4 - 1 = 0$ (Ans: $x = 16$, $x = 1/8$)

**16.** At 3:00 pm, the temperature of a hot metal was 80°C and that of the surroundings 20°C. At 3:03 pm, the temperature of the metal had dropped to 42°C. The rate of cooling of the metal was directly proportional to the difference between its temperature $\theta$ and that of the surroundings.

**(a)(i)** Write a differential equation to represent the rate of cooling of the metal

(Ans: $\theta = 20 + Ae^{-kt}$)

**(ii)** Solve the differential equation using the given conditions. (Anss: $\theta = 20 + 60e^{\frac{t}{3}\ln(\frac{60}{22})}$)

**(b)** Find the temperature of the metal at 3:05 pm.

(Ans: $\theta = 31.27°$)

# 2012 PAPER TWO

**1**. The forces $\begin{pmatrix}0\\2\end{pmatrix}$, $\begin{pmatrix}0\\4\end{pmatrix}$, $\begin{pmatrix}0\\3\end{pmatrix}$ and $\begin{pmatrix}0\\q\end{pmatrix}$ N act at points ($p$, 1), (2, 3), (4, 5) and (6, 1) respectively. The resultant is $\begin{pmatrix}0\\3\end{pmatrix}$ N acting at (1, 1). Find the values of $a$ and $p$.

(Ans: $a$ = -6, $p$ = 9.5)

**2.** Two points $A$ and $B$ are such that $P(A) = \frac{1}{5}$ and $P(B) = \frac{1}{2}$. Find P(A∪B) when $A$ and $B$ are:

(a) independent events (Ans: 0.6)

(b) mutually exclusive events (Ans: 0.7)

**3.** Use the trapezium rule with four sub-intervals to estimate $\int_0^{\pi/2} \frac{1}{1+\sin x}\,dx$ (Give your answer correct to three decimal places). (Ans: $x = 1.013$)

**4. (a)** Show that the velocity $v$ of a body which starts with an initial velocity $u$ and moves with uniform acceleration $a$ consequently covering a distance $x$, is given by $v = \left[u^2 + 2ax\right]^{1/2}$.

(b) Find the value of $x$ in (a) if $v$ = 30 m/s, $u$ = 10 m/s and $a$ = 5 m/s$^2$. (Ans: $x$ = 80m)

**5.** A teacher gave two tests in chemistry. Five students were graded as follows.

| | GRADE | | | | |
|---|---|---|---|---|---|
| Test 1 | *A* | *B* | *C* | *D* | *E* |
| Test 2 | *B* | *A* | *C* | *D* | *E* |

Determine the rank correlation coefficient between the two tests. Comment on your result.

(Ans: 0.9, high positive)

**6.** A light extensible string passes over a smooth pulley fixed at the top of a smooth plane inclined at 30° to the horizontal. A mass, $m$ is attached to the other end of the string and rests on the inclined plane. If the system is in equilibrium, find $m$. (Ans: $m$ = 8 kg)

**7.** The table below shows the cost $y$ shillings for hiring a motor cycle for a distance $x$ kilometres.

| Distance ($x$ km) | 10 | 20 | 30 | 40 |
|---|---|---|---|---|
| Cost (Shs. $Y$) | 2800 | 3600 | 4400 | 5200 |

Use linear interpolation or extrapolation to calculate the:

(a) cost of hiring the motor cycle for a distance of 45 km. (Ans: 5600)

(b) Distance Mukasa travelled if he paid Shs 4000

(Ans: 25 km)

**8.** A random variable $X$ has the following probability, distribution: $P(X=0)\ \frac{1}{8}$, $P(X=1) = P(X = 2) = \frac{3}{8}$ and $P(X = 3) = \frac{1}{8}$. Find the:

(a) mean of $X$, (Ans: 1.5)

(b) variance of $X$ (Ans: 0.75)

**9.** The table below shows the marks obtained in an examination by 200 candidates.

| Marks (%) | Number of candidates |
|---|---|
| 10 – 19 | 18 |

| | |
|---|---|
| 20 – 29 | 34 |
| 30 – 39 | 58 |
| 40 – 49 | 42 |
| 50 – 59 | 24 |
| 60 – 69 | 10 |
| 70 – 79 | 6 |
| 80 – 89 | 8 |

(a) Calculate the:
(i) mean mark, (Ans: 40.2%)
(ii) modal mark (Ans: 35.5%)

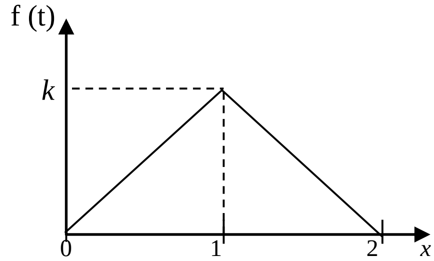
(b) Draw a cumulative frequency curve for the data.
Hence estimate the lowest mark for a distinction one if the top 5% of the candidates qualify for the distinction. (Ans: 75%)

**10.** At 11:45 a.m, ship $A$ has position vector $\begin{pmatrix}5\\1\end{pmatrix}$ km and moving at 8 kmh$^{-1}$ in the direction N30°E. At 12 noon, another ship $B$ has position vector $\begin{pmatrix}8\\7\end{pmatrix}$ km and moving at 3 kmh$^{-1}$ in the direction South East.
(a) Find the position vector of ship $A$ at 12 noon (Ans: $\begin{pmatrix}6\\2.73205\end{pmatrix}$ km)
(b) If the ships after 12 noon maintain their courses, find the:
(i) time when they are closest (Ans: 0.49612s)
(ii) least distance between them. (Ans: 1.091 km)

**11. (a)** (i) Show that the equation $e^x - 2x - 1 = 0$ has a root between $x = 1$ and $x = 1.5$
(ii) Use linear interpolation to obtain an estimation of the root (Ans: 1.18 (2dp))

(b) (i) Solve the equation in (a)(i), using each formula below twice. Take the approximation in (a)(ii) as the initial value.
Formula 1: $x_{n+1} = \frac{1}{2}(e^{x_n} + 1)$ (Ans: 1.2642)
Formula 2: $x_{n+1} = \frac{e^{x_n}(e_n - 1) + 1}{e^{x_n} - 2}$ (Ans: 1.2565)
(ii) Deduce with a reason which of the two formulae is appropriate for solving the given equation in (a)(i). Hence write down a better approximate root, correct to 2 decimal places. (Ans: 1.26 (2 dp))

**12.** A continuous random variable $X$ has a probability density function (p.d.f) $f(x)$ as shown in the graph below.

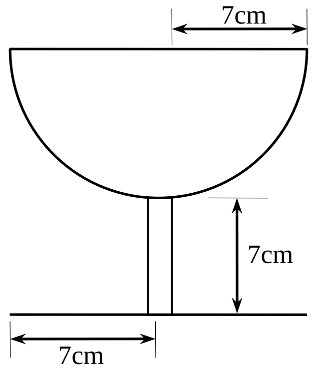



(a) Find the:
(i) value of $k$, ($k = 1$)
(ii) expression for the probability density function (p.d.f) of $X$

(Ans: $f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2 - x, & 1 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$)

(b) Calculate the:
(i) mean of $X$, (Ans: 1)
(ii) $P(X < 1.5 / X > 0.5)$ (Ans: 08751)

**13.** (a) Show that the centre of gravity of a uniform thin hemispherical cup of radius, $r$ is at a distance $r/2$ from the base.
(b) The figure below is made up of a thin hemispherical cup of radius 7 cm. It is welded to a stem of length 7 cm and then to a circular base of the same material and of radius 7 cm. The mass of the stem is one quarter than of the cup.




Find the distance from the base, of the centre of gravity of the figure. (Ans: 6.5)

**14.(a)** The length, width and height of a water tank were all rounded off to 3.65m, 2.14m and 2.5m respectively. Determine in m$^3$ the least and greatest amount of water the tank can contain (Ans: 19.992 m$^3$)
(b) A shop offered 25% discount on all items in its store and a second discount of 5% to any customer who paid cash.
(i) Construct a flow chat which shows the amount paid for each item.
(ii) Using your flow chart in (i), compute the amount paid for the following items: (Ans: 225000/-)

| **Item** | **Price** | **Mode of payment** |
|---|---|---|
| Mattress | 125,000 | Cash |
| Television set | 340,000 | Credit |

**15.(a)** A box of oranges contains 20 good and 4 bad oranges. If 5 oranges are picked at random, determine the probability that 4 are good and the other is bad. (Ans: 0.456)

**(b)** An examination has 100 questions. A student has 60% chance of getting each question correct. A student fails the examination for a mark less than 55. A student gets a distinction for a mark of 68 or more. Calculate the probability that a student:

**(i)** fails the examination (Ans: 0.1308)

**(ii)** gets a distinction (Ans: 0.0629)

**16.** A gun of mass 3000 kg fires horizontally a shell at an initial velocity of 300 $ms^{-1}$. If the recoil of the gun is brought to rest by a constant opposing force of 9000N in 2 seconds, find the:

(a) (i) initial velocity of the recoil (Ans: $6ms^{-1}$)

(ii) mass of the shell (Ans: 60 kg)

(iii) gain in kinetic energy of the shell just after firing (Ans: 2700 kJ)

(b) (i) displacement of the gun (Ans: 6m)

(ii) work done against the opposing force (Ans: 54 kJ)

# 2013 PAPER ONE

### SECTION A

**1.** Solve $\log_x 5 + 4\log_5 x = 4$ (Ans: $x = \sqrt{5}$)

**2.** In a Geometric Progression (G.P), the difference between the second and fifth term is 156. The difference between the seventh and fourth term is 1404. Find the possible values of the common ratio. (Ans: $r = 3$, $r = -3$)

**3.** Given that $r = 3\cos\theta$ is the equation of a circle, find its Cartesian form. (Ans: $x^2 + y^2 - 3x = 0$)

**4.** The position vector of point $A$ is $2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$, of B is $5\mathbf{j} + 4\mathbf{k}$ and of $C$ is $\mathbf{i} + 2\mathbf{j} + 12\mathbf{k}$. Show that $ABC$ is a triangle.

**5.** Solve $5\cos^2 3\theta = 3(1 + \sin 3\theta)$ for $0° < \theta < 90°$. (Ans: θ = 7.859°, 52.141°, 90°)

**6.** If $y = (x - 0.5)e^{2x}$, find $\frac{dy}{dx}$. (Ans: $2xe^{2x}$,

Hence determine $\int_0^1 xe^{2x}dx$, (Ans: 2.0973)

**7.** The region bounded by the curve $y = \cos x$, the $y$-axis and $x$-axis from $x = 0$ to $x = \frac{\pi}{2}$ is rotated about the $x$-axis. Find the volume of the solid formed. (Ans: $\pi^2/4$ cubic units)

**8.** Solve $(1 - x^2)\frac{dy}{dx} - xy^2 = 0$, given that $y = 1$ when $x = 0$ (Ans: $y = \frac{1}{\ln(1 - x^2)^{1/2} + 1}$)

**9.** (a) The complex number is the conjugate of $Z$.

(i) Express $Z$ in the modulus argument form (Ans: $Z = 2(\cos 30° + i\sin 30°)$)

(ii) On the same Argand diagram, plot $\bar{Z}$ and $2\bar{Z} + 3i$

(b) What are the greatest and least values of $|Z|$ if $|Z - 4| \leq 3$. (Ans: 7, 1 respectively)

**10.** Given the equation $x^3 + x - 10 = 0$,

(a) Show that $x = 2$ is a root of the equation.

(b) deduce the values of $\alpha + \beta$ and $\alpha\beta$ where $\alpha$ and $\beta$ are the roots of the equation. Hence form a quadratic equation whose roots are $\alpha^2$ and $\beta^2$. (Ans: $x^2 + 6x + 25 = 0$)

**11.** (a) Find the point of intersection of the lines

$\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$. (Ans: (1, 3, 2)

(b) The equations of a line and a plane are $\frac{x-2}{1} = \frac{y-2}{2} = \frac{z-3}{2}$ and $2x + y + 4z = 9$ respectively. $P$ is a point on the line where $x = 3$. $N$ is the foot of the perpendicular from point $P$ to the plane. Find the coordinates of $N$. (Ans: N(1, 3, 1)

**12.** (a) Find the equation of the tangent to the hyperbola whose points are of the parametric form $(2t, 2/t)$. (Ans: $t^2y = -x + 4t$)

(b)(i) Find the equation of the tangents in (a) which are parallel to $y + 4x = 0$ (Ans: $y = -4x - 8$, $y = -4x + 8$)

(ii) Determine the distance between the tangents in (i). (Ans: 3.881 units)

**13.** A curve has the equation $y = \frac{2}{1+x^2}$.

(a) Determine the nature of the turning point on the curve. (Ans: (0, 2), maximum)

(b) Find the equation of the asymptote. Hence sketch the curve. (Ans: $y = 0$, horizontal asymptote)

**14.** (a) Prove that $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Hence show that $\frac{1 - \tan 15°}{1 + \tan 15°} = \frac{1}{\sqrt{3}}$

(b) Given that $\cos A = 3/5$ and $\cos B = 12/13$ where $A$ and $B$ are acute, find the value of:

(i) $\tan(A + B)$ (Ans: 3.9375)

(ii) $\cos(A + B)$ (Ans: 1.0317)

**15.** Resolve $y = \frac{x^3 + 5x^2 - 6x + 6}{(x-1)^2(x^2+2)}$ into partial fractions. Hence find $\int y\,dx$ and $\frac{dy}{dx}$.

Ans: $\frac{1}{x-1}+\frac{2}{(x-1)^2}+\frac{4}{x^2+1}$,

$\ln(x-1)-\frac{2}{x-1}+\frac{4}{\sqrt{2}}\tan^{-1}(\frac{x}{\sqrt{2}})+c$,

$\frac{-1}{(x-1)^2}-\frac{4}{(x-1)^3}-\frac{8x}{(x^2+2)^2}$

**16.** (a) The differential equation $\frac{dp}{dt}=kp(c-p)$ shows the rate at which information flows in a student population $c$. $p$ represents the number who have heard the information in $t$ days and $k$ is a constant.

(a) Solve the differential equation

(Ans: $\frac{1}{c}\ln\left(\frac{p}{c-p}\right)=kt+a$ )

(b) A school has a population of 1000 students. Initially, 20 students had heard the information. A day later, 50 students had heard the information. How many students heard the information by the tenth day? (Ans: 990)

# 2013 PAPER II

**1.** A class performed an experiment to estimate the diameter of a circular object. A sample of five students had the following results in centimetres; 3.12, 3.16, 2.94 and 3.0. Determine the sample:

(a) mean\` (Ans: 3.11)

(b) standard deviation (Ans: 0.1356)

**2.** The table below shows the values of a function $f(x)$

| $x$ | 1.8 | 2.0 | 2.2 | 2.4 |
|---|---|---|---|---|
| $f(x)$ | 0.532 | 0.484 | 0.436 | 0.384 |

Use linear interpolation to find the value of:

(a) $f(2.08)$. (Ans: 0.465 (3 dp))

(b) $x$ corresponding to $f(x) = 0.5$ (Ans: 1.933)

**3.** The speed of a taxi increased from 90 $kmh^{-1}$ to 18$kmh^{-1}$ in a distance of 120 metres. Find the speed of the taxi when it had covered a distance of 50 metres. (Ans: 69.7137 $kmh^{-1}$)

**4.** Events $A$ and $B$ are such that $P(A\cap B)=\frac{1}{2}$ and $P(A/B)=\frac{1}{3}$. Find $P(B\cap A')$. (Ans: 1/6)

**5.** Find the approximate value of $\int_0^2 \frac{1}{1+x^2}dx$ using thr trapezium rule with 6 ordinates. Give your answer to 3 decimal places. (Ans: 1.105 (3 dp))

**6.** Forces of 7N and 4N act away from a common point and make an angle of $\theta°$ with each other. Given that the magnitude of their resultant is 10.75N, find the:

(a) value of $\theta$ (Ans: 25.4578°)

(b) direction of the resultant (Ans: 9.2056°)

**7.** An industry manufactures iron sheets of mean length 3.0 m and standard deviation of 0.05 m. Given that the lengths are normally distributed, find the probability that the length of any iron sheet picked at random will be between 2.95 m and 3.15m.

(Ans: 0.8400)

**8.** A particle of mass $m$ kg is released at rest from the highest point of a solid spherical object of radius $a$ metres. Find the angle to the vertical at which the particle leaves the sphere. (Ans: 48.19°)

**9.** The heights (cm) and ages (years) of a random sample of 10 farmers are given in the table below.

| Height(cm) | 156 | 151 | 152 | 160 | 146 | 157 | 149 | 142 | 158 | 140 |
|---|---|---|---|---|---|---|---|---|---|---|
| Age(years) | 47 | 38 | 44 | 55 | 46 | 49 | 45 | 30 | 45 | 30 |

(a) (i) Calculate the rank correlation coefficient

(Ans: 0.7515)

(ii) comment on your result (Ans: high positive)

(b) Plot a scatter diagram for the data.

(c) Use your diagram in (b) to find:

(i) $y$ when $x = 147$ (Ans: $y = 37$)

(ii) $x$ when $y = 43$. (Ans: $x = 151$)

**10.** A mass of 12 kg rests on a smooth inclined plane which is 6 m long and 1 m high. The mass is connected by a light inextensible string, which passes over a smooth pulley fixed at the top of the plane, to a mass of 4 kg which is hanging freely. With the string taut, the system is released from rest.

(a) Find the

(i) acceleration of the system.

(Ans: 1.225 $ms^{-2}$)

(ii) (Ans: 38.2N)

(b) Determine the:

(i) velocity with which the 4 kg mass hits the ground (Ans: 1.5652 $ms^{-1}$)

(ii) time the 4kg mass takes to hit the ground.

(Ans: 1.3 s)

**11.** The probability density function (pdf) of a continuus random variable $X$ is given by:

$$f(x)=\begin{cases} kx(16-x^2, & 0\le x\le 4 \\ 0 \quad , & \text{else where}\end{cases}$$

Where $k$ is a constant.

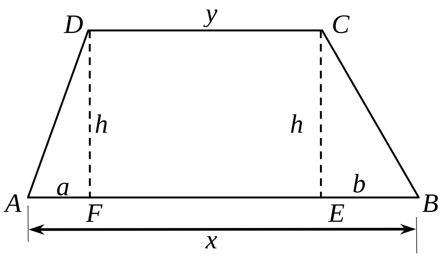
Find: (a) value of $k$ (Ans: $k = 1/64$)

(b) mode of $X$. (Ans: 2.3099)

(c) mean of $X$ (Ans: 2.1333)

**12.** (a) Particles of masses 3 kg, 2 kg, 3 kg and 2 kg act at points with position vectors $3\boldsymbol{i}-\boldsymbol{j}$, $2\boldsymbol{i}+3\boldsymbol{j}$, $-2\boldsymbol{i}+5\boldsymbol{j}$ and $-\boldsymbol{i}-2\boldsymbol{j}$ respectively. Find the position vector of their centre of gravity. (Ans: $0.917i+j$)

(b) The figure $ABCD$ below shows a metal sheet of uniform material cut in the shape of a trapezium. $\overline{AB}=x$, $\overline{CD}=y$, $\overline{AF}=a$, $\overline{EB}=b$ and $h$ is the vertical distance between $AB$ and $CD$.



Show that the centre of gravity of the sheet is at a distance $\frac{h}{3}\left[\frac{3y+a+b}{x+y}\right]$ from side *AB*.

**13.** The numbers $x$ and $y$ are measured with possible errors $\Delta x$ and $\Delta y$ respectively.

(a) Show that the maximum absolute error in the quotient $\frac{x}{y}$ is given by $\frac{|y||\Delta x|+|x||\Delta y|}{y^2}$

(b) Find the interval within which the exact value of $\frac{2.58}{3.4}$ is expected to lie. (Ans: (0.7464, 0.7716)

**14.** A particle is projected with a speed of $36ms^{-1}$ at an angle of 40° to the horizontal from a point 0.5m above the level ground. It just clears a wall which is 70 metres on the horizontal plane from the point of projection. Find the:

(a) (i) time taken for the particale to reach the wall. (Ans: 2.5384 s)

(ii) height of the wall. (Ans: 27.664 m)

(b) Maximum height reached by the particle from the point of projection. (Ans: 27.320)

**15.** (a) Show that the iterative formula based on Newton Raphson's method for solving the equation $lnx + x - 2 = 0$ is given by

$$x_{n+1} = \frac{x_n(3-lnx_n)}{1+x_n},\ n = 0, 1, 2,$$

(b) (i) Construct a flow chart that:

- reads the initial approximation as $r$
- computes using the iterative formula in (a), and prints the roots of the equation $lnx + x - 2 = 0$ when the error is less than $1 \times 10^{-4}$.

(Ans: root = 1.557 (3 dp))

(ii) Perform a dry run of the flow chart when $r = 1.6$.

**16.** A research station supplies three varieties of seeds $S_1$, $S_2$ and $S_3$ in the ratio 4:2:1. The probabilities of germination of $S_1$, $S_2$, and $S_3$ are 50%, 60% and 80% respectively.

(a) Find the probability that a seed selected at random will germinate. (Ans: 0.5714)

(b) Given that 150 seeds are selected at random, find the probability that less than 90 of the seeds will germinate. Give your answer to two decimal places. (Ans: 0.73 (2 dp))

# 2014 PAPER I

**1.** Solve the simultaneous equations:

$x - 2y - 2z = 0$

$2z + 3y + z = 1$

$3x - y - 3z = 3$ (Ans: $z = -2, y = 3, z = -4$)

**2.** A focal chord $PQ$ to the parabola $y^2 = 4x$ has gradient $m = 1$. Find the coordinates of the midpoint of $PQ$. (Ans: (3, 2))

**3.** Given that $2A - \cos 2B = -p$ and $\sin 2A - \sin 2B = q$, prove that $\sec(A+B) = \frac{1}{q}\sqrt{p^2+q^2}$.

**4.** Differentiate $\log_5\left(\frac{e^{\tan x}}{\sin^2 x}\right)$ with respect to $x$.

(Ans: $\frac{dy}{dx} = \frac{1}{\ln 5}(\sec^2 x - 2\cot x)$)

**5.** Find the equation of a line through $S(1, 0, 2)$ and $T(3, 2, 1)$ in form $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$. Hence deduce the Cartesian equation of the line.

(Ans: $\mathbf{r} = \begin{pmatrix}1\\0\\2\end{pmatrix} + \lambda\begin{pmatrix}2\\2\\-1\end{pmatrix}$, $\frac{x-1}{2} = \frac{y}{2} = \frac{z-2}{-1}$)

**6.** Solve the equation $\sqrt{2x+3} - \sqrt{x+1} = \sqrt{x-2}$. Verify your answer. (Ans: $x = 3$)

**7.** Find $\int x(1-x^2)^{\frac{1}{2}}\,dx$. (Ans: $\frac{1}{3}(1-x^2)^{\frac{3}{2}} + c$)

**8.** A cylinder has radius $r$ and height $8r$. The radius increases from 4cm to 4.1cm. Find the approximate increase in the volume. (use $\pi = 3.14$).

(Ans: $120.576\ cm^3$)

**9.** (a) Given that the complex number $Z$ and its conjugate $\overline{Z}$ satisfy the equation $Z\overline{Z} + 2iZ = 12 + 6i$, find $Z$.

(Ans: $Z = 3 + 3i$, $Z = 3 - i$)

(b) One root of the equation $Z^3 - 3Z^2 - 9Z + 13 = 0$ is $2 + 3i$. Determine the other roots. (Ans: *No root*)

**10.** A circle is described by the equation $x^2 + y^2 - 4x - 8y + 16 = 0$. A line given by the equation $y = 2(x - 1)$ cuts the circle at points $A$ and $B$. A point $P(x, y)$ moves such that its distance from the midpoint of $AB$ is equal to its distance from the centre of the circle.

(a) Calculate the coordinates of $A$ and $B$.

(Ans: A(2, 2), B(3.6, 5.2))

(b) Determine the centre and radius of the circle.

(Ans: centre = (2, 4); radius = 2)

(b) Find the locus of $P$. (Ans: $y = 2x - 1$)

**11.** (a) Differentiate $y = \cot^{-1}(\ln x)$ with respect to $x$

(Ans: $\frac{dy}{dx} = \frac{-1}{x(1+(\ln x)^2)}$)

(b) Evaluate $\int_{\pi/3}^{\pi} x\sin x\,dx$ (Ans: 2.7992)

**12.** (a) Find the Cartesian equation of the plane through the points whose position vectors are $2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $-2\mathbf{j} + 4\mathbf{k}$. (Ans: $5x - y + 67 = 26$)

(b) Determine the angle between the plane in (a) and the line $\frac{x-2}{2} = \frac{y}{-4} = z - 5$. (Ans: 33.66°)

**13.** (a) Find the first three terms of the expansion $(2 - x)^6$ and use it to find $(1.998)^6$ correct to two decimal places. (Ans: $64 - 192x + 240x^2$, 63.62 (2 dp))
(b) Expand $(1 - 3x + 2x^2)^5$ in ascending powers of $x$ as far as the $x^2$ term. (Ans: $1 - 15x + 100x^2$)

**14.** (a) Find the equation of a normal to a curve whose parametric equations are $x = b\sec^2\theta$, $y = b\tan^2\theta$.
(Ans: $y + x = b(\tan^2\theta + \sec^2\theta)$)
(b) The area enclosed by the curve $x^2 + y^2 = a^2$, the $y$-axis and the line $y = \frac{-1}{2}a$ is rotated through 90° about the $y$-axis. Find the volume of the solid generated. (Ans: $V = \frac{5}{96}\pi a^3$ or $V = \frac{9}{32}\pi a^3$)

**15.** Solve:
(a) $4\sin^2\theta - 12\sin 2\theta + 35\cos^2\theta = 0$ for $0° \le \theta \le 90°$
(Ans: $\theta = 68.1986°$, $\theta = 74.0546°$)
(b) $3\cos\theta - 2\sin\theta = 2$, for $0° \le \theta \le 360°$
(Ans: $\theta = 22.6199°$, 270°)

**16.** A substance loses mass at a rate proportional to the amount $M$ present at time $t$.
(a) Form a differential equation connecting $M$, $t$ and the constant of proportionality $k$.
(Ans: $\ln M = -kt + c$.)
(b) If initially the mass of the substance is $M_o$, show that $M = M_O e^{-kt}$.
(c) Given that half of the substance is lost in 1600 years, determine the number of years 15g of the substance would take to reduce to 13.6g
(Ans: 226.1694 years)

# 2014 PAPER II

**1.** The daily number of patients visiting a certain hospital is uniformly distributed between 150 and 210.
(a) Write down the probability distribution function (pdf) of the number of patients.
(Ans $f(x) = \begin{cases} \frac{1}{60}, & 50 \le x \le 210 \\ 0, & \text{elsewhere} \end{cases}$)
(b) Find the probability that between 170 and 194 patients visit the hospital on a particular day.
(Ans: 0.4)

**2.** A particle starts from rest at the origin (0, 0). Its acceleration in $ms^{-2}$ at time $t$ seconds is given by $a = 6t\mathbf{i} - 4\mathbf{j}$. Find its speed when $t = 2$ seconds.
(Ans: 14.4222 $ms^{-1}$)

**3.** Use the trapezium rule with four sub-intervals to estimate $\int_{0.2}^{1.0}\left(\frac{2x+1}{x^2+1}\right)dx$, correct to two decimal places.
(Ans: 2.20 (2 dp))

**4.** Tom's chance of passing an examination is $\frac{2}{3}$. If he sits for four examinations, find the probability that he passes:
(a) Only two examinations (Ans: 0.2963 (Calc))
(b) More than half of the examinations
(Ans: 0.5926 (Calc))

**5.** Forces of $\begin{pmatrix}1\\-4\end{pmatrix}$N, $\begin{pmatrix}3\\6\end{pmatrix}$N, $\begin{pmatrix}-9\\1\end{pmatrix}$N and $\begin{pmatrix}5\\-3\end{pmatrix}$N act at the points having position vectors $(3\mathbf{i} - \mathbf{j})$m, $(2\mathbf{i} + 2\mathbf{j})$m, $(-\mathbf{i} - \mathbf{j})$m and $(-3\mathbf{i} - 4\mathbf{j})$m respectively. Show that the forces reduce to a couple.

**6.** Given the table below:

| $x$ | 0 | 10 | 20 | 30 |
|---|---|---|---|---|
| $y$ | 6.6 | 2.9 | -0.1 | -2.9 |

use linear interpolation to find:
(d) $y$ when $x = 16$ (Ans: $y = 1.1$)
(e) $x$ when $y = -1$ (Ans: $x = 23.2$)

**7.** The table below shows the scores of students in Mathematics and English tests.

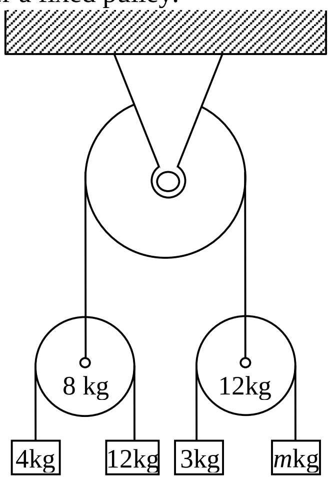
| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Maths | 72 | 65 | 82 | 54 | 32 | 74 | 40 | 60 |
| English | 58 | 50 | 86 | 35 | 76 | 43 | 40 | 60 |

Calculate the rank correlation coefficient for the students' performance in the two subjects.
(Ans: 0.143)

**8.** A bullet of mass 50 grammes travelling horizontally at 80 $ms^{-1}$ hits a block of wood of mass 10 kg resting on a smooth horizontal plane. If the bullet emerges with a speed of 50 $ms^{-1}$, find the speed with which the block moves. (Ans: 0.15 $ms^{-1}$)

**9.** (a) A bag contains 30 white ($W$), 20 blue ($B$) and 20 red ($R$) balls. 3 balls are drawn at random one after the other without replacement. Determine the probability that the first ball is white and the third ball is also white. (Ans: 0.18012)
(b) Events $A$ and $B$ are such that $P(A)=\frac{4}{7}$,
$P(A\cap B') = \frac{1}{3}$, and $P(A/B) = \frac{5}{14}$. Find
(i) P(B) (Ans: 2/3)
(ii) $P(A'\cap B')$ (Ans: 0)

**10.** The diagram below shows two pulleys of mass 8kg and 12kg connected by a light inextensible string hanging over a fixed pulley.




The accelerations of 4kg and 12kg masses are $\frac{g}{2}$ and $\frac{g}{2}$ downwards respectively. The accelerations of the 3kg and $m$ kg masses are $\frac{g}{3}$ upwards and $\frac{g}{3}$ downwards respectively. The hanging portions of the strings are vertical. Given that the string of the fixed pulley stationary, find the:

(a) tensions in the strings,

(Ans: 98N, $T_1$ = 58.8 N, $T_2$ = 39.2N)

(b) value of $m$. (Ans: $m$ = 6 kg)

**11.** The numbers $x$ and $y$ are approximated with possible errors $\Delta x$ and $\Delta y$ respectively.

(a) Show that the maximum absolute error in the quotient $\frac{x}{y}$ is given by $\frac{y\Delta x + x\Delta y}{y^2}$

(b) Given $x$ = 2.68 and $y$ = 0.9 are rounded to the given number of decimal places, find the interval within which the exact value of $\frac{x}{y}$ is expected to lie. (Ans: (2.8158, 3.1588)

**12.** A lorry starts from point $A$ and moves along a straight horizontal road with constant acceleration of 2 $ms^{-2}$. At the same time a car moving with a speed of 20 $ms^{-1}$ and constant acceleration of 3 $ms^{-2}$ is 400m behind the point $A$ and moving in the same direction as the lorry. Find:

(a) how far from $A$ the car overtakes the lorry.

(Ans: 214.3589 m)

(b) the speed of the lorry when it is being overtaken

(Ans: 29.282 $ms^{-1}$)

**13.** The cumulative frequency table below shows the ages in years of employees of a certain company.

| Age(years | Cumulative frequency |
|---|---|
| < 15 | 0 |
| < 20 | 17 |
| < 30 | 39 |
| < 40 | 69 |
| < 50 | 87 |
| < 60 | 92 |
| < 65 | 98 |

(a) (i) Use the data in the table to draw a cumulative frequency curve (Ogive)

(ii) Use the curve to estimate the semi-interquartile range. (Ans: 9.5)

(b) Calculate the mean age of the employees.

**14.** (a) Show that the Newton-Raphson formula for finding the root of the equation $x = N^{1/5}$ is given by

$$x_{n+1} = \frac{4x_n^{\,5} + N}{5x_n^{\,4}},\ n = 0, 1, 2, \ldots$$

(b) Construct a flow chart that:

(i) reads $N$ and the first approximation $x_0$,

(ii) computes the root to three decimal places,

(iii) prints the root ($x_n$) and number of iterations ($n$)

(c) Taking $N$ = 50, $x_0$ = 2.2, perform a dry run for the flow chart. Give your root correct to three decimal places. (Ans: 2.187 (3 dp))

**15.** (a) A non-uniform plan $AB$ of length 4 metres rests in horizontal position on vertical supports A and B. The centre of gravity is at 1.5m from $A$. The reaction at $B$ is 37.5N. Determine the:

(i) mass of the plank (Ans: 10.2 kg)

(ii) reaction at $A$. (Ans: 62.5 N)

(b) Find the coordinates of the centre of gravity of a uniform lamina bound by the curve $y^2 = 2x$ and the line $x$ = 4. (Ans: (2.4, 0)

**16.** The marks in an examination were normally distributed with mean μ and standard deviation σ. 20% of the students scored less than 40 marks and 10% scored more than 75 marks. Find the:

(a) values of μ and σ (Ans: μ = 53.875, σ = 16.478)

(b) percentage of candidates who scored more than 50 marks. (Ans: 59.29%)

# 2015 PAPER ONE

**1.** The first term of an Arithmetic Progression (A.P) is equal to the first term of the Geometric Progression (G.P) whose common ratio is $\frac{1}{3}$ and sum to infinity is 9. If the common difference of the A.P is 2, find the sum of the first ten terms of the A.P. (Ans: $S_{10}$ = 150)

**2.** Find the equation of the line through the point (5, 3) and perpendicular to the line $2x - y + 4 = 0$ (Ans: $2y + x = 11$)

**3.** Solve for $x$ in $\log_a(x+3) + \frac{1}{\log_x a} = 2\log_a 2$

(Ans: $x$ =1)

**4.** Given that D(7, 1, 2), E(3, -1, 4) and F(4, -2, 5) are points on a plane, show that ED is perpendicular to EF.

**5.** In a triangle ABC all angles are acute. Angle ABC = 50°, $a$ = 10cm and $b$ = 9 cm. Solve the triangle.

(Ans: $A$ = 58.34°, B – 71.66°, $c$ = 11.15 cm)

**6.** Differentiate $e^{-x^3}x^3 \sin x$ with respect to $x$.

(Ans: $x^2e^{-x^2}(-2x^2\sin x + 3\sin x + x\cos x)$)

**7.** The region enclosed by the curve $y = x^2$, the $x$-axis and the line $x$ = 2 is rotated through one revolution about the $x$-axis. Find the volume of the solid generated. (Ans: $\frac{32\pi}{5} \approx 20.1062$)

**8.** Solve $\frac{dy}{dx} = e^{x+y}$ given that $y$ = 2 when $x$ = 0.

(Ans: $e^x$ – 1.135)

**9.** (a) Given that $f(x) = (x - \alpha)^2 g(x)$, show that $f'(x)$ is divisible by $\alpha$.

(b) A polynomial $P(x) = x^3 + 4ax^2 + bx + 3$ is divisible by $(x - 1)^2$. Use the result in (a) above to find the values of $a$ and $b$. Hence solve the equation $P(x) = 0$ (Ans: $a = \frac{1}{4}$, $b$ = -5)

**10.** Sketch on the same coordinate axes the graphs of the curve $y = 2 + x - x^2$ and the line $y = x + 1$. Hence determine the area of the region enclosed between the curve and the line. (Ans: 1.333 sq units)

**11.** (a) Solve $Z\bar{Z} - 5iZ = 5(9 - 7i)$ where $\bar{Z}$ is the complex conjugate of $Z$.

(Ans: $Z$ = 7 – 4i OR $Z$ = 7 – $i$)

(b)(i) Find the Cartesian equation of the curve given as $|Z + 2 - 3i| = 2|Z - 2 + i|$

(Ans: $3x^2 + 3y^2 - 20x + 14y + 7 = 0$)

(ii) Show that it represents a circle. Find the centre and radius of the circle.

(Ans: Centre $(\frac{10}{3}, \frac{-7}{3})$ ; radius = 3.771)

**12.** (a) Simplify $\frac{\cos 3\theta + \cos 5\theta}{\sin 5\theta - \sin 3\theta}$ (Ans: cot $\theta$)

(b) Show that $\cot 2\theta = \frac{1-\tan^2\theta}{2\tan\theta}$

**13.** Express $\frac{1}{x^2(x-1)}$ as partial fractions. Hence evaluate $\int_2^3 \frac{dx}{x^2(x-1)}$ correct to 3 decimal places.

(Ans: $\frac{-1}{x} - \frac{1}{x^2} + \frac{1}{x-1}$, = 0.12102)

**14.** (a) Show that the lines $\boldsymbol{a} = \begin{pmatrix}3\\-4\\2\end{pmatrix} + \alpha\begin{pmatrix}-1\\1\\2\end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix}5\\0\\-2\end{pmatrix} + \beta\begin{pmatrix}-1\\-1\\2\end{pmatrix}$ intersect

(b) Find the:

(i) point of intersection, *P* of the lines in (a)

(Ans: (2, -3, 4)

(ii) Cartesian equation of the plane which contains ***a*** and ***b***. (Ans: $2x + z = 8$)

**15.** The tangent at the point $P(cp, c/p)$ and $Q(cq, c/q)$ on the rectangular hyperbola $xy = c^2$ intersect at *R*. Given that *R* lies on the curve $xy = \frac{c^2}{2}$, show that the locus of the midpoint of the line *PQ* is given by $xy = 2c^2$.

**16.** The rate of increase of a population of certain birds is proportional to the number in the population present at that time. Initially, the number in the population was 32,000. After 70 years, the population was48,000. Find the:

(a) number of birds in the population after 82 years.

(Ans: 51455)

(b)time when the population doubles the initial number. (Ans: 119.67 years)

# 2015 PAPER TWO

**1.** Find the magnitude and direction of the resultant of the forces $\begin{pmatrix}-3\\1\end{pmatrix}$N, $\begin{pmatrix}4\\2\end{pmatrix}$N and $\begin{pmatrix}1\\2\end{pmatrix}$N

(Ans: 5.83N, 30.96°)

**2.** Use the trapezium rule with five subintervals to estimate $\int_2^4 \frac{5}{(x+1)}dx$. Give your answer correct to three decimal places. (Ans: 2.558 3 dp)

**3.** The table below shows the mass of boys in in a certain school.

| Mass (kg) | 15 | 20 | 25 | 30 | 35 |
|---|---|---|---|---|---|
| Number of boys | 5 | 6 | 10 | 20 | 9 |

Calculate the mean. (Ans: 27.2 kg)

**4.** Two cyclists *A* and *B* are 36m apart on a straight road. Cyclist *B* starts from rest with an acceleration of $6 ms^{-2}$ while *A* is in pursuit of *B* with velocity of 20 m/s and acceleration of 4 $ms^{-2}$. Find the time when *A* overtakes *B*.(Ans: 2 s)

**5.** Events *A* and *B* are independent. $P(A) = x$, $P(B) = x + 0.2$ and $P(A \cup B) = 0.65$. find the value of *x*.

(Ans: $x = 0.3$)

**6.** The table below shows the values of a function *f*(*x*) for a given value of *x*.

| *x* | *f*(*x*) |
|---|---|
| 9 | 2.66 |
| 10 | 2.42 |
| 11 | 2.18 |
| 12 | 1.92 |

Use linear interpolation or extrapolation to find:

(a) *f*(10.4) (Ans: 2.324)

(b) the value of *x* corresponding to *f*(*x*) = 1.46

(Ans: 13.769 N)

**7.** The marks in an examination were found to be normally distributed with mean 53.9 and standard deviation 16.5. 20% of the candidates who sat this examination failed. Find the pass mark for the examination. (Ans: 40%)

**8.** A fixed hollow hemisphere has centre O and is fixed so that the plane of the rim is horizontal. A particle *A* of weight $30\sqrt{2}$ N can move on the inside surface of the hemisphere. The particle is acted upon by a horizontal *P*, whose line of action is in a vertical plane through *O* and *A*. *OA* makes an angle of 45° with the vertical. If the coefficient of friction between the particle and the hemisphere is 0.5 and the particle is just about to clip downwards, find the:

(a) normal reaction. (Ans: 40N)

(b) value of *P*. (Ans:*P* = 14.1421 N)

**9.** The probability density function (p.d.f) of a random variable *Y* is given by

$$f(x) = \begin{cases} \frac{(y+1)}{4}, & 0 \le y \le k \\ 0 \quad , & \text{elsewhere} \end{cases}$$

Find:

(a) the value of *k* (Ans: $k = 2$)

(b) the expectation of *Y* (Ans: E(*Y*) = 7/6)

(c) $P(1 \le Y \le 1.5)$ (Ans: 0.28125)

**10.** The numbers *A* = 6.341 and *B* = 2.6 have been rounded to the given number of decimal places.

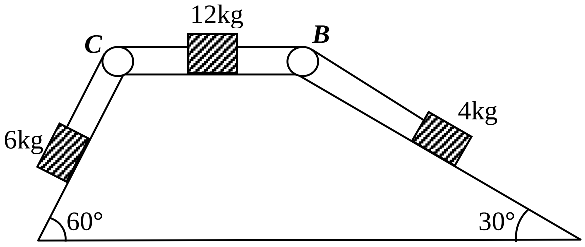
(a) Find the maximum possible error in *AB*.

(Ans: 0.31835)

(b) Determine the interval within which $\frac{A^2}{B}$ can be expected to lie.

(Ans: $15.171 \le$ exact $< 15.770$)

**11.** The diagram below shows a 12kg mass on a horizontal rough plane. The 6kg and 4kg masses are on rough planes inclined at angles 60° and 30° respectively. The masses are connected to each other by light inextensible strings passing over light smooth fixed pulleys *A* and *B*.




The planes are equally rough with coefficient of friction $\frac{1}{2}$. If the system is released from rest, the:

(a) acceleration of the system. (Ans: 0.73833 $ms^{-2}$)
(b) extension in the string. (Ans: 25.3823 N)

**12.** The table below gives the points awarded to eight schools by three judges $J_1$, $J_2$ and $J_3$ during a music competition. $J_1$ was the chief judge.

| $J_1$ | $J_2$ | $J_3$ |
|---|---|---|
| 72 | 60 | 50 |
| 50 | 55 | 40 |
| 50 | 70 | 62 |
| 55 | 50 | 70 |
| 35 | 50 | 40 |
| 38 | 50 | 48 |
| 82 | 73 | 67 |
| 72 | 70 | 67 |

(a) Determine the rank correlation coefficient between the judges of
  (i) $J_1$ and $J_2$. (Ans: 0.7440)
  (ii) $J_1$ and $J_3$. (Ans: 0.7023)

(b) Who of the two other judges had a better correlation with the chief judge? Give a reason
(Ans: $J_2$ had a better correlation because it had a slightly higher value with the chief judge.)

**13.** A ball is projected from a point *A* and falls at a point *B* which is in level with *A* and at a distance of 160m from *A*. The greatest height of the ball attained is 40m. Find the:

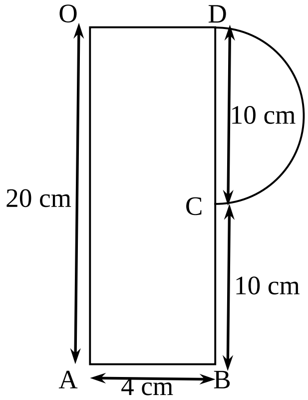
(a) angle and velocity at which the ball is projected.
(Ans: 45°, 39.6 $ms^{-1}$)
(b) time taken for the ball to attain its greatest height.
(Ans: 2.857 s)

**14.** (a) Show that the iterative formula based on Newton Raphson's method for approximating the root of the equation 2 ln $x$ – 1 + 1 = 0 is given by

$$x_{n+1} = x_n\left(\frac{2\ln x_n - 1}{x_n - 1}\right),\ n = 0, 1, 2, \ldots$$

(b) Draw a flow chart that:
  (i) reads the initial approximation $x_o$ of the root.
  (ii) computes and prints the root correct to 2 decimal places, using the formula in (a).
(Ans: root = 3.51 (2 dp))

**15.** The diagram below represents a lamina formed by welding together a rectangular metal. Find the position of the centre of gravity of the lamina from the side *OA*.




(Ans: 3.3576 cm from OA)

**16.** A box *A* contains 4 white and 2 red balls. Another box contains 3 white and 3 red balls. A box is selected at random at two balls are picked one after the other without replacement.

(a) Find the probability that the two balls picked are red. (Ans: 0.1333)
(b) Given that two white balls are picked, what is the probability that they are from box *B*?(Ans: 0.333)