FOR GOD AND MY COUNTRY
THE REPUBLIC OF UGANDA

Ministry of Education
and Sports

HOME-STUDY LEARNING

SENIOR 2

MATHEMATICS

August 2020

NCDC
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This material has been developed as a home-study intervention for schools during the lockdown caused by the COVID-19 pandemic to support continuity of learning.

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FOREWORD

Following the outbreak of the COVID-19 pandemic, government of Uganda closed all schools and other educational institutions to minimize the spread of the coronavirus. This has affected more than 36,314 primary schools, 3129 secondary schools, 430,778 teachers and 12,777,390 learners.

The COVID-19 outbreak and subsequent closure of all has had drastically impacted on learning especially curriculum coverage, loss of interest in education and learner readiness in case schools open. This could result in massive rates of learner dropouts due to unwanted pregnancies and lack of school fees among others.

To mitigate the impact of the pandemic on the education system in Uganda, the Ministry of Education and Sports (MoES) constituted a Sector Response Taskforce (SRT) to strengthen the sector's preparedness and response measures. The SRT and National Curriculum Development Centre developed print home-study materials, radio and television scripts for some selected subjects for all learners from Pre-Primary to Advanced Level. The materials will enhance continued learning and learning for progression during this period of the lockdown, and will still be relevant when schools resume.

The materials focused on critical competences in all subjects in the curricula to enable the learners to achieve without the teachers' guidance. Therefore effort should be made for all learners to access and use these materials during the lockdown. Similarly, teachers are advised to get these materials in order to plan appropriately for further learning when schools resume, while parents/guardians need to ensure that their children access copies of these materials and use them appropriately.
I recognise the effort of National Curriculum Development Centre in responding to this emergency through appropriate guidance and the timely development of these home study materials. I recommend them for use by all learners during the lockdown.

Alex Kakooza
Permanent Secretary
Ministry of Education and Sports

ACKNOWLEDGEMENTS

National Curriculum Development Centre (NCDC) would like to express its appreciation to all those who worked tirelessly towards the production of home–study materials for Pre-Primary, Primary and Secondary Levels of Education during the COVID-19 lockdown in Uganda.

The Centre appreciates the contribution from all those who guided the development of these materials to make sure they are of quality; Development partners - SESIL, Save the Children and UNICEF; all the Panel members of the various subjects; sister institutions - UNEB and DES for their valuable contributions.

NCDC takes the responsibility for any shortcomings that might be identified in this publication and welcomes suggestions for improvement. The comments and suggestions may be communicated to NCDC through P.O. Box 7002 Kampala or email admin@ncdc.go.ug or by visiting our website at http://ncdc.go.ug/node/13.

Grace K. Baguma
Director,
National Curriculum Development Centre

ABOUT THIS BOOKLET

Dear learner, you are welcome to this home-study package. This content focuses on critical competences in the syllabus.

The content is organised into lesson units. Each unit has lesson activities, summary notes and assessment activities. Some lessons have projects that you need to carry out at home during this period. You are free to use other reference materials to get more information for specific topics.

Seek guidance from people at home who are knowledgeable to clarify in case of a challenge. The knowledge you can acquire from this content can be supplemented with other learning options that may be offered on radio, television, newspaper learning programmes. More learning materials can also be accessed by visiting our website at www.ncdc.go.ug or ncdc-go-ug.digital/. You can access the website using an internet enabled computer or mobile phone.

We encourage you to present your work to your class teacher when schools resume so that your teacher is able to know what you learned during the time you have been away from school. This will form part of your assessment. Your teacher will also assess the assignments you will have done and do corrections where you might not have done it right.

The content has been developed with full awareness of the home learning environment without direct supervision of the teacher. The methods, examples and activities used in the materials have been carefully selected to facilitate continuity of learning.

You are therefore in charge of your own learning. You need to give yourself favourable time for learning. This material can as well be used beyond the home-study situation. Keep it for reference anytime.

Develop your learning timetable to ca ter for continuity of learning and other responsibilities given to you at home.

Enjoy learning

Dear Learner, welcome to this study material. As you prepare to start working on activities in here, remember that you are studying from home due to the Covid-19 pandemic. It is therefore important that you keep safe by doing the following:

1. **Regularly wash your hands with soap and clean water or use a sanitizer to sanitize your hands.**
2. **Always wear a face mask when you are in a crowded place.**
3. **Keep a distance of 2 metres away from other people.**

Topic: Indices

Identifying the base number and index.

We use large and very small numbers in our daily life. For example a counties budget being in trillions, the diameter of a virus, the mass of the earth, the speed of light.

Indices help us to express large numbers in a compact and convenient way.

In this topic you will learn how to work with indices.

Index numbers are written in the form $\boldsymbol{a^b}$

An index number has two parts, the lower part is the **base** and the upper part is the **index**. For example in the number $\boldsymbol{a^b}$, $\boldsymbol{a}$ is the base and $\boldsymbol{b}$ is the index.

The index of a number determines how many times the base is multiplied by itself. It is written as a small number to the right and above the base number.

For example, $\boldsymbol{a^2 = a \times a}$ **;** and $\boldsymbol{6^3 = 6 \times 6 \times 6}$

The plural of index is indices. Other names for index are **exponent** or **power**.

Exercise

1. Write these expressions in index form.
   a. $8 \times 8 \times 8 \times 8 \times 8 \times 8 =$

b. $1.2 \times 1.2 \times 1.2 =$

c. $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} =$

d. $\left(-\frac{12}{5}\right) \times \left(-\frac{12}{5}\right) \times \left(-\frac{12}{5}\right) \times \left(-\frac{12}{5}\right) =$

2. Identify the base and index in each of the following.

a. $\left(-\frac{12}{5}\right)^2$ Base: _______ Index: _______

b. 5^{-3} Base: _______ Index: _______

c. x^2 Base: _______ Index: _______

d. $\left(\frac{1}{x}\right)^{-1}$ Base: _______ Index: _______

3. Evaluate the following index numbers.

a. $5^3 =$

b. $2^3 =$

c. $3^2 =$

d. $3^5 =$

4. Is 3^4 bigger than 4^3 ?

LAWS OF INDICES

Index numbers have three main laws. These laws are used to guide mathematicians to evaluate index numbers very fast and get correct results. For example;

If you are to evaluate the expression: $a^3 \times a^2$

Without using the laws of indices you can work it out as follows.

Step 1: Expand the index numbers. $a^3 = a \times a \times a$ While $a^2 = a \times a$

Step 2: Multiply the expressions $\underbrace{a \times a \times a}_{\boxed{a^3}} \times \underbrace{a \times a}_{\boxed{a^2}}$

Step 3: Write the final expression in index form. $a \times a \times a \times a \times a = a^5$

Therefore $a^3 \times a^2 = a^5$

The First law of Indices

Using the first law of indices this same expression can be done using one step. The first law of indices is as follows:

When multiplying index numbers with the same base, add the powers (indices).

Mathematically this is written as $\boldsymbol{a^m} \times \boldsymbol{a^n} = \boldsymbol{a^{(m+n)}}$

Using the first law of indices:$a^3 \times a^2 = a^{3+2} = a^5$

Exercise

Simplify these expressions by writing your final answer in index form.

1. $a^6 \times a^3 =$
2. $3^4 \times 3^2 =$
3. $8^8 \times 8^5 =$
4. $3^4 \times 3^2 =$
5. $m^5 \times m^2 \times m^3 =$

Special indices

1. When the index is 1, it may be written or not.

For example; $a^1$is the same as a ; $m^1 = m$; $x^1 = x$

2. The zero index. Any number to power of zero is equal to one.
   This means $2^0 = 1$; $3^0 = 1$; $12^0 = 1$; $x^0 = 1$; $b^0 = 1$;

The Second law of Indices

When dividing index numbers with the same base, subtract the powers (indices).

Mathematically this is written as $\boldsymbol{a^m \div a^n = a^{(m-n)}}$

The example below shows the application of the second law of indices

$$a^6 \div a^3 = a^{6-3}$$

$$= a^3$$

At times the expression my combine more than one law. The example below requires application of the first law and the second law as shown.

Simplify the expression $\frac{b^4 \times b^3}{b^5}$

$$\frac{b^4 \times b^3}{b^5} = \frac{b^{4+3}}{b^5}$$

$$= \frac{b^7}{b^5}$$

$$= b^{7-5}$$

$$= b^2$$

Exercise

1. Simplify the following expressions
   a. $8^7 \div 8^2 =$
   b. $\frac{q^{16}}{q^7} =$
2. Copy each of the following statements and fill in the missing numbers.
   a. $a^{13} \div a \quad = a$
   b. $p^{16} \div p \quad = p^7$
   c. $\frac{x}{x^5} = x^5$
   d. $4 \quad \div 4^8 = 4^2$

The Third Law of Indices

The third law of indices states that

$$(a^m)^n = a^{m \times n} \text{ Where m and n are integers.}$$

For example,

Using the third law of indices simplify the expression $(\mathbf{2^3})^2$ leaving the answer in index form.

$$(\mathbf{2^3})^2 = 2^{3\times 2}$$

$$= 2^6$$

Simplify the expression $(\mathbf{2^6} \times 2^3)^4$

$$(\mathbf{2^6} \times 2^3)^4 = (2^{6+3})^4$$

$$= (2^8)^4$$

$$= 2^{8\times 4}$$

$$= 2^{32}$$

Fractional indices

When indices are written as fractions, they are still evaluated using the three laws of indices. The following meaning will guide you to work with fractional indices.

i. You have always known that $\sqrt{25} = 5$ in index form, this is the same as

$$25^{\frac{1}{2}} = (5^2)^{\frac{1}{2}} = 5^{2\times\frac{1}{2}} = 5$$

Therefore the square root is the same as the fractional index of a half.

ii. Similarly, $\sqrt[3]{x}$ is the same as $x^{\frac{1}{3}}$ while $\sqrt[4]{y}$ is the same as $y^{\frac{1}{4}}$

Example 1: Without using a calculator, evaluate the expression $\sqrt[3]{\left(\frac{125}{1000}\right)}$

We have just seen that $\sqrt[3]{\left(\frac{125}{1000}\right)}$ is the same as $\left(\frac{125}{1000}\right)^{\frac{1}{3}}$

First write 125 and 1000 in index form

$125 = 5 \times 5 \times 5 = 5^3$ while$1000 = 10 \times 10 \times 10 = 10^3$

$$\therefore \left(\frac{125}{1000}\right)^{\frac{1}{3}} = \left(\frac{5^3}{10^3}\right)^{\frac{1}{3}}$$

$$= \frac{5^{3\times\frac{1}{3}}}{10^{3\times\frac{1}{3}}}$$

$$= \frac{5}{10}$$

Example 2: simplify fully $\sqrt{\left(\frac{50\pi a^5}{2\pi a^3}\right)}$

Step 1: simplify the expression in the brackets using the first and second laws of indices.

$$\sqrt{\left(\frac{50\pi a^5}{2\pi a^3}\right)} = \sqrt{\left(\frac{50}{2} \times \pi^{1-1} \times a^{5-3}\right)}$$

$$= \sqrt{(25 \times 1 \times a^2)}$$

Write 25 in index form and the square root sigh to a fractional index.

$$= (5^2 \times a^2)^{\frac{1}{2}}$$

Use the third law of indices to simplify the expression further.

$$= 5^{2\times\frac{1}{2}} \times a^{2\times\frac{1}{2}}$$

$$= 5a$$

Negative indices

There are times when the expression has negative indices.

For example and expression such as a^{-2}

Using the second law of indices $a^{-2} = a^{0-2} = \frac{a^0}{a^2}$

The zero index $a^0 = 1$

Therefore $a^{-2} = \frac{1}{a^2}$

PRACTICE EXERCISE

1. Simplify these expressions

   a. $m^3 \times m^5$

   b. $\frac{m^8}{m^2}$

   c. $(m^3)^4$

   d. $(2m^2)^3$

   e. x^0

   f. $(16y^6)^{\frac{1}{2}}$

   g. $5x^4y^4 \times x^2y$

h. $\frac{45e^6f^6}{5ef^2}$

i. $15y^6 \div 3y^2$

j. $3s^2t^3 \times 4s^4t^2$

2. Find the value of the following

a. 4^0

b. 4^{-2}

c. $16^{\frac{3}{2}}$

d. 64^0

e. $64^{-\frac{2}{3}}$

Topic: Statistics (Mean, Mode and Median)

We use numbers in our daily life. For example when you want to know how old you are you state it using numbers. An example is I am 15 years old. 15 is a number.

Having numbers like 2, 3, 8, 5, 2 does not give you what they are all about. They do not have meaning until you do some more work about them.

In this topic you will learn how to add meaning to numbers using the Mean Mode and Median.

The Mean

Mean has the same meaning as the word average. Five students were given a chance to pick sweets from a shop. The table below shows the number of sweets each student picked.

Student	**Number of sweets picked.**
Student 1	2
Student 2	3
Student 3	5
Student 4	8
Student 5	2

Some students have more while others have less. Your task is to re-distribute the sweets so that all students get the same number of sweets.

If you are to do this very first, it is better you first collect all the sweets from the students, give one sweet out to one student at a time until all sweets are finished. In mathematics, you can do this by following these steps.

Step one: Add all the sweets together.

2+3+5+8+2 = 20 sweets.

Step 2: Divide the total number of sweets by the total number of students.

$20 \div 5 = 4$

What you have obtained is the mean or average.

$$Mean/Average = \frac{Sum\ of\ all\ the\ items}{Total\ number\ of\ items}$$

The frequency table.

A frequency table is used to organise data. For example, that data below is of the ages of 15 students in senior 2.

12, 13, 15, 12, 14, 13, 15, 12, 13, 14, 14, 15, 13, 13, 15.

The frequency table helps you know the number of students for each of the ages represented in the data shown. All these numbers are appearing more than once. The number of times a number appears is called **frequency**. The tallies are to help you count the frequency without errors.

Age	Tally	Frequency
12	///	3
13	~~////~~	5
14	///	3
15	////	4

	Total = 15

The same formula used earlier still works for this kind of data.

$$Mean/Average = \frac{Sum\ of\ all\ the\ items}{Total\ number\ of\ items}$$

$$Mean = \frac{12 + 12 + 12 + 13 + 13 + 13 + 13 + 13 + 14 + 14 + 14 + 15 + 15 + 15 + 15}{15}$$

$$Mean/Average = \frac{203}{15}$$

$$Mean/Average = 13.53$$

This is a very long method and you are likely to make a mistake in the process of adding many numbers. Here is a shorter alternative method.

Step 1: Improve on your frequency table

Got by multiplying x value with f value in each row.

Age (x)	Tally	Frequency (f)	fx
12	///	3	12 x 3 = 36
13	~~////~~	5	13 x 5 = 65
14	///	3	14 x 3 = 42
15	////	4	15 x4 = 60
		$\sum f = 15$	$\sum fx = 203$

Where $\sum f$ is the total of all frequencies and $\sum fx$ is the sum of all products of frequency and the age value x.

$$Mean = \frac{\sum fx}{\sum f}$$

$$Mean = \frac{203}{15} = 13.53$$

The Median

The median is the middle number for a given set of data. To find the middle number you need to

Step 1: arrange the numbers in order from smallest to biggest or biggest to smallest.

Step 2: identify the number is the middle as the median.

For example

The numbers 2, 3, 8, 5, 2 show the number of sweets picked by five students from a shop. Find the median.

Step 1: Arrange the numbers in order starting with the smallest.

2, 2, 3, 5, 8

Step 2: Identify the number in the middle of this data.

The median is 3.

The Median is easy to find when the data set has an odd number of data. When the data set has an even number of data, there is no middle number. When that happens, you find the average of the two numbers in the middle and use it as the median.

For example: Find the Median of the numbers 2, 4, 3, 8, 5, 2

Step 1: Arrange the numbers from biggest to smallest.

8, 5, 4, 3, 2, 2.

Step 2: There are two numbers 4 and 3 in the middle of this data. What is left is to find the average of 4 and 3 and record it as median.

$$\text{Median} = \frac{4+3}{2} = \frac{7}{2}$$

Median = 3.5

The Mode

The mode is the most commonly occurring number in a dataset. It is the number that is repeated the highest number of time. For example in the data set 12, 13, 15, 12, 14, 13, 15, 12, 13, 14, 14, 15, 13, 13, 15.

12 appears three times

13 appears five times

14 appears three times, and

15 appears four times

The mode for this data is 13 because it appears the highest number of times.

Topic: Inequalities and Regions

Mathematics is not always about "equals", sometimes we only know that something is greater or less than.

For example James is 12 years old. Jordan is 14 years old. You can write this as in a number of ways. Which symbol do I use?

Common Inequality symbols

Symbol	Words	Example Use
$>$	greater than	$5 > 2$
$<$	less than	$7 < 9$

For each of the two symbols, the small end always points to the smaller number as shown below.

1) Use any of the inequality symbols of “less than”< or “greater than” > to compare these numbers.
    a) 90 91
    b) 176 836
    c) 5 3
    d) 14.8 14.5
    e) -271 -172

2) Write a mathematical for each of the statements below;
    1) Two is less than nine
    2) Fifteen is greater than eight
    3) 100 is more than 88
    4) 5 litres is little compared to 8 litres.

Other inequality symbols

There are other inequalities that include "equals" like:

Symbol	Words	Example Use
$\geq$	greater than **or equal to**	$x \geq 1$
$\leq$	less than **or equal to**	$y \leq 3$

In inequality such as $x > 1$ is true where x has values of 2, 3, 4, 5, … this is a solution set for the expression $x > 1$.

An inequality such as $x \geq 1$is true for when all values where x is greater than 1 but also true when $x = 1$. The possible values of x to make this statement true are x= {1, 2, 3, 4, …} This is the solution set for the inequality $x \geq 1$.

In the same way for an inequality such as $y \leq 3$ y can take any of the following values y = {3, 2, 1, 0, -1, ...}

Example1: Write down the solution set for the inequality

a) $x \geq 6$
b) $x > 6$
c) $x \ \leq 6$

Solutions:

a) This example requires values of x for which the inequality$x \geq 6$ will be true. They are: x = {6, 7, 8, 9, …}

b) The solution set for the inequality $x > 6$ is should have values of x greater than 6. These are x = {7, 8, 9, ...}

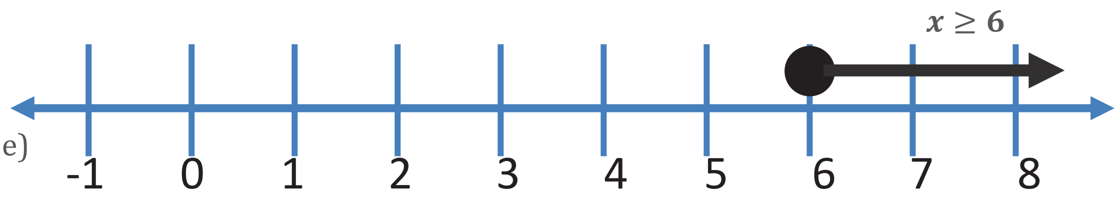
d) The solution set for the inequality $x \leq 6$ is x = {6, 5, 4, 3, ...}

Representing inequalities on a number line.

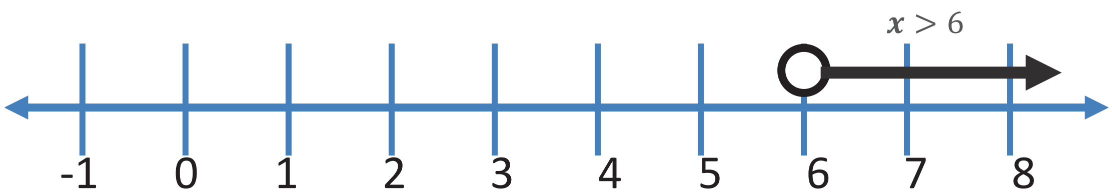
We have learnt that inequalities such as $x \geq 6$ *and* $x > 6$ have a simple difference. The solution set for $x \geq 6$ has 6 part of the solution set while $x > 6$ does not.

This is how the inequality $x \geq 6$ is represented on a number line.

e)



And this is how the inequality $x > 6$ is represented on a number line.



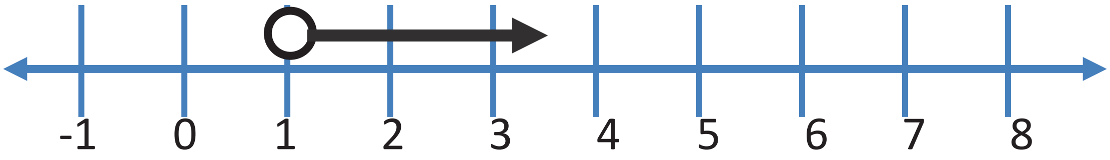
The two number lines are different in that, the starting point has **a solid circle** for **greater or equal to** and **an empty** circle for **greater than.** The solid circle is for both the inequalities of greater or equal to and less or equal to. Less than and greater than are represented by the empty circle.

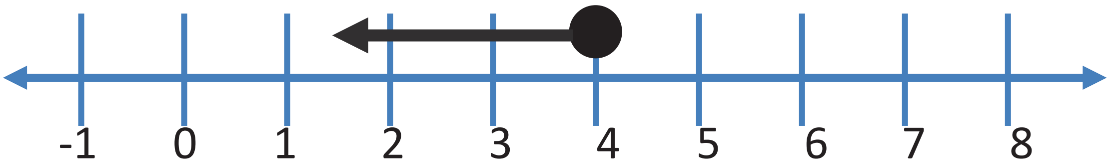
Exercise

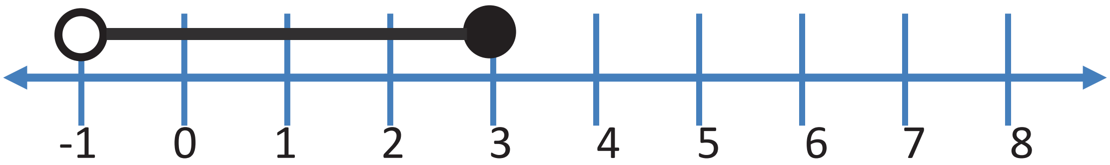
1. Represent the following inequalities on the number line.

a) $n > 2$
b) $y \geq -1$
c) $n < 3$
d) $x \leq 2$
e) $-2 < n \leq 2$

Write the inequality for each of the following diagrams. Use the letter **y** in your inequality.







Graphing linear inequalities.

In addition to representing inequalities on a number line, they can be represented on a graph paper. As you graph inequalities these are the simple rules to follow.

a) The greater than and less than inequalities are represented by a dotted line.
b) The less or equal to and greater or equal to inequalities are represented by a solid line.
c) Shade the region that does not fulfil the inequality (Shade the unwanted region).

Representing inequalities on a graph.

Inequalities represent regions. To show this region on a graph, you need to

Step 1: Identify and draw the boundary line for the region.

> **Learning Tip:** If the inequality has the ≤ or ≥ sign, its boundary line is a solid line. The boundary line for an inequality with the < or > signs is drawn with a dotted line.

Step 2: plot and draw the boundary line on a Cartesian plane.

Step 3: Identify the unwanted region and shade it (shade the region that does not satisfy the inequality)

Example 1:

Show the region represented by the following inequality $y < 2x$

Learning Tip: A table of values for each of the boundary lines is helpful when determining the points on the boundary line.

Three points are good enough for you to draw a line. It is faster to use x-intercept and y-intercept and one other point. These will form your coordinates when plotting the boundary lines.

Table of values for y = 2x

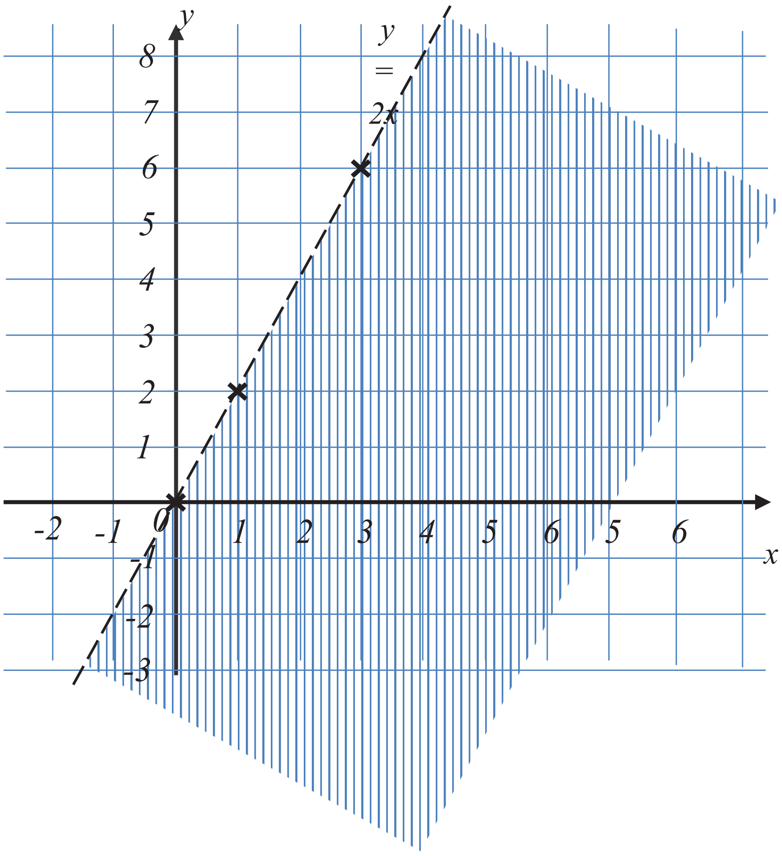
x	0	1	3
y	0	2	6

Inequality	**Boundary line**	**Points on the boundary line**
$y < 2x$	$y = 2x$ (dotted line)	$(0, 0)$; $(1, 2)$; $(3, 6)$

Using the points from the table of values, plot the draw the boundary line.

To identify the unwanted region, choose a point from either side of the boundary line and substitute its values in the inequality. If the inequality remains true, then the point is in the wanted region. Shade the opposite side. In this example let us choose a point (0,4) This means that when x = 0, y = 4

Using the inequality $y < 2x$, substituting for x and y the inequality remains true. So the point (0,4) is in the wanted region. Shade the opposite side of the boundary line.



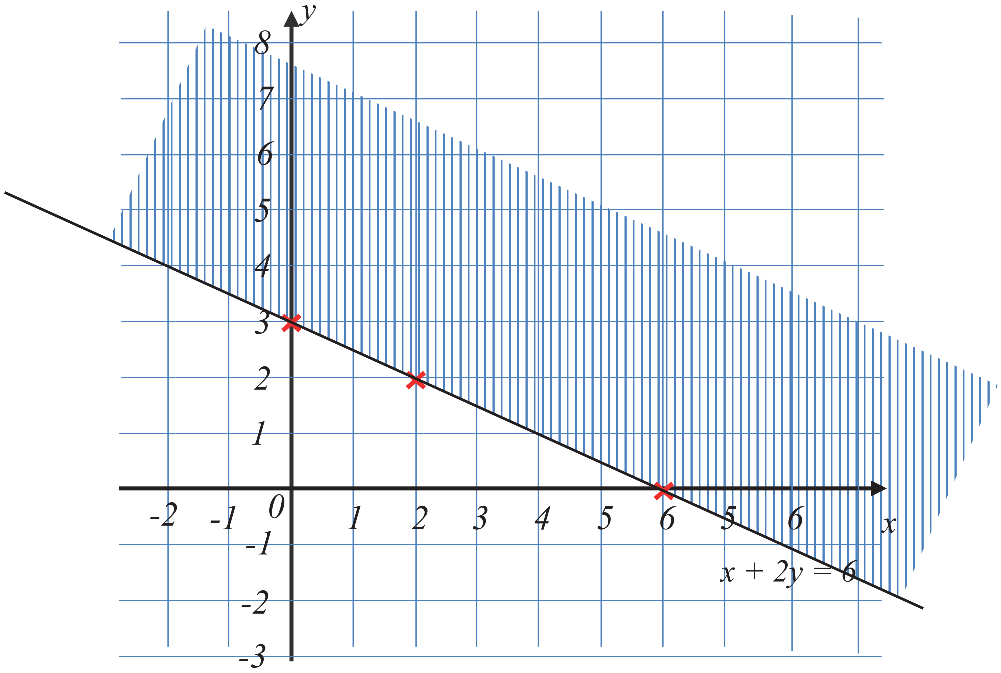
Example 2:

Show the region represented by the inequality $x + 2y \leq 6$

Table of values for x + 2y = 6

x	0	6	2
y	3	0	2

Inequality	**Boundary line**	**Points on the boundary line**
$x + 2y \leq 6$	$x + 2y = 6$	(0, 3); (6, 0); (2, 2)



Example 3:

Show the region $x + y \geq 2$ on a graph paper.

Table of values for x + y = 2

x	0	2	4
y	2	0	- 2

Inequality	Boundary line	Points on the boundary line
$x + y \geq 2$	$x + y = 2$	(0, 2); (2, 0); (4, -2)

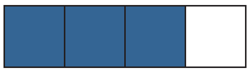
Topic: Ratios and Proportion.

Importance of ratios

Ratios are used in in many areas. For our day to day lives, ratios are used in the catering industry to maintain a consistent taste of food and drinks. A restaurant may have many chefs, but if they all use the same ratios, the taste of food prepared remains the same irrespective of the chef who prepared it.

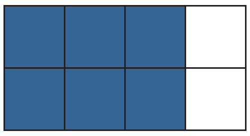
What are ratios?

Ratios are used to compare values. A ratio shows how much of one value there is compared to another value. The diagrams below are made up of blue and white squares of equal sizes. There are three blue boxes for every one white box.

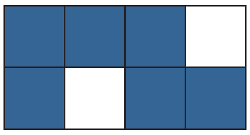


Ratio of Blue to white = 3:1

Image 1 has 3 blue boxes and one white box.
The ratio of blue to white is written as **Blue: White = 3:1**
The ratio of white to blue is written as **White: Blue = 1:3**



Ratio of White to Blue = 2:6



Ratio of Blue to white = 6:2

Images 2 and 3 have 6 blue boxes and 2 white boxes. The arrangement of boxes does not matter in this case.
The ratio of blue boxes to white boxes is written as **Blue: White = 6:2** or White: Blue = 2:6
This can be simplified further to Blue: White = 3:1 since both 6 and 2 are multiples of 2. It comes back to the same ratio showing that for every 3 blue boxes, there is one white box. In this case the ratio 3:1 is simplified compared to 6:2 but both have the same meaning.

Learning Tip:

You are advised to always state the ratio in its simplest form. If the ratio of black cups to blue cups is 6:3, it is advisable to simplify it further and state the ratio in its simplest form. For this case, the ratio becomes as 2:1

Example 1:

There are two boys and one girl in a given family. What is the ratio of boys to girls?

Solution: The ratio of boys to girls is 2:1. This simply means that there are two boys for every one girl in that family.

Example 2:

In juice factory, the perfect mix for pineapple juice is got when two pineapples are mixed in eight jugs of water.

a) *What is the ratio of pineapples to jugs of water in its simplest form?*

Solution:

The ratio of Pineapple to water is 2 pineapples for every 8 jugs of water

= 2:8

To get the simplest form, divide each by two to get the simplest ratio as 1:4

Therefore the simplest ratio of Pineapple to water is 1:4

b) *How many jugs of water are needed to mix 5 pineapples?*

Solution

The ratio Pineapple to water =1:4 simply means that for every one pineapple you can mix it in 4 jugs of water.
For five jugs we shall multiply both the two parts of the ratio with five
= 5x1:5x4
= 5:20
Therefore for five pineapples, are mixed in 20 jugs of water.

Alternatively, if one pineapple can mix 4 jugs
Then five pineapples can mix 5x4 jugs = 20jugs of water.

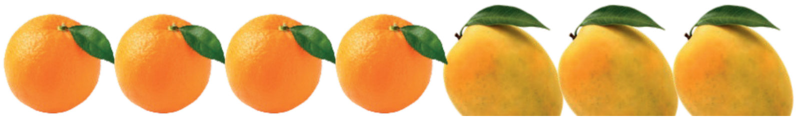
c) *How many pineapples are needed to make juice using 18 jugs of water?*

Solution

Using our ratio of pineapples: jugs of water = 1:4
This means that 4 jugs can be mixed by one pineapple.
1 jug can be mixed by $\frac{1}{4}$of a pineapple.
18 jugs of water will need $\frac{1}{4} \times 18$ pineapples.

$= 4\frac{1}{2}$ pineapples.

Exercise



The image above shows a collection of fruits of oranges and mangoes.

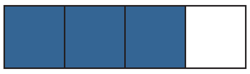
State the ratio of

i. Mangoes to oranges _______________

ii. Oranges to mangoes _______________

Ways of Showing Ratios

There are different ways of showing a ratio. We are going to use the image below to show these different ways.



The ratio of Blue boxes to white boxes can be written as follows;

	Description	Blue boxes to white boxes	White boxes to blue boxes
1.	as values separated by a colon	3:1	1:3
2.	using the word “to”	3 to 1	1 to 3
3.	as a fraction	$\frac{3}{1}$	$\frac{1}{3}$

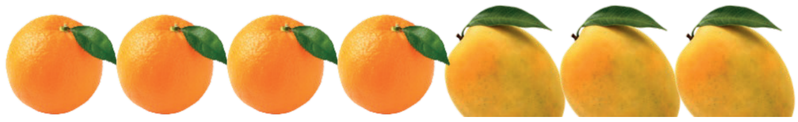
Part to whole ratios.

The examples of ratios we have been using part to part ratios. For example, the ratio of blue boxes to white boxes is an example to part to part ratio. They were comparing one part to another part.

Part to whole ratios compare a given part to the whole. For example the ratio of blue boxes to the total number of boxes. In this case it will be 3 to 4. 4 is the total number of boxes in the whole diagram.

The image above shows a collection of fruits of oranges and mangoes.

State the ratio of



i. Oranges to total fruit?

Solution:

Number of oranges = 4

Total number of fruits = 7

Ratio of oranges to total fruits = 4:7

ii. Mangoes to total fruit?

Solution:

Total number of mangoes = 3

Total number of fruits = 7

Ratio of mangoes to total fruits = 3:7.

Simplifying ratios

Ali and his mother had a discussion on the sizes of their feet. Physically, Ali's foot looked smaller but the mother insisted that they take measurements to explain the relationship between the foot size and height. The table below shows the measurements for foot length and height for the two.

	Ali	Ali's Mum
Length of Foot:	21cm	24cm
Height:	133cm	152cm

The "foot-to-height" ratio in fraction style is:

$$\text{Ali's ratio is } \frac{21}{133} \qquad \text{Mum's ratio is } \frac{24}{152}$$

Let us simplify Ali's ratio by dividing both values by 7

$$\frac{21 \div 7}{133 \div 7} = \frac{3}{9}$$

In the same way, let us simplify Mum's ratio by dividing both values by 8

$$\frac{24 \div 8}{152 \div 8} = \frac{3}{9}$$

The simplified "foot-to-height" ratios for Ali and his mum are the same at $\frac{3}{9}$

For each of the two feet they are as big as they should be for the height of the person.

Using a ratio to share quantities

Example 1:
Share 60 passion fruits between two people in the ratio 2 : 3.

To share using ratios, you need to have the part to whole ratios and use them to share the quantities.
Step 1: form the two part to whole ratios
Total ratio of 2+3 = 5
First part to whole ratio is $\frac{2}{5} \times total\ number\ of\ passion\ fruits$

$$= \frac{2}{5} \times 60$$
$$= 24 \text{ Passion fruits}$$

The second part to whole ratio is $\frac{3}{5} \times total\ number\ of\ passion\ fruits$

$$= \frac{3}{5} \times 60$$
$$= 36 \text{ Passion fruits}$$

Learning Tip:

When a quantity is shared using the ratios, the different parts formed should always add up to the original value. For the example above, 24 +36 = 60 passion fruits.

Example 2:

Share 43,200 shilling between three sister Grace, Gwenn and Gloria in the ratio of 1:2:3

The total ratio for all the siblings = 1+2+3 = 6

Grace's share $= \frac{1}{6} \times \textit{total amount of money to be shared}.$

$= \frac{1}{6} \times 43{,}200$

$= \frac{43200}{6} = 7{,}200\ \textit{shillings}$

Gwenn's share $= \frac{2}{6} \times \textit{total amount of money to be shared}.$

$= \frac{2}{6} \times 43{,}200$

$= \frac{86400}{6} = 14{,}400\ \textit{shillings}$

Gloria's share $= \frac{3}{6} \times \textit{total amount of money to be shared}.$

$= \frac{3}{6} \times 43{,}200 = 21{,}600\ \textit{shillings}$

Exercise

1. Two numbers are in the ratio 2 : 3, and the sum of the numbers is 225. Find the numbers.
2. Divide 275 into two parts which are in the ratio 4 : 7.
3. Decrease the number 225 in the ratio 5 : 3.
4. A number is divided into two parts in the ratio 4 : 9. If the larger part is 108, find the number.
5. Divide 1148 into two parts which are in the ratio 11 : 3.

6. Divide 432 into three parts which are in the ratio 1 : 2 : 3.
7. The angles of a triangle are in the ratio 3 : 5 : 10. Find the angles.
8. Three workers worked for 5 hours, 7 hours and 12 hours. The total wage of 132,000shillings was divided among the three workers according to the number of hours of work they did. How much did each get?

PROPORTIONS

In the topic of fractions you learnt about equivalent fractions.

For example; The following are equivalent fractions

$$\frac{1}{2} = \frac{3}{6} = \frac{50}{100}$$

These fractions are equivalent because when you simplify them they all come to the same fraction.

A proportion is a pair of equivalent ratios.

Suppose a farmer can dig a garden of 2 square meters in 3 days. How many days are needed for the same farmer to dig 8 square meters?

Step 1: State this information in form of two equivalent ratios expressed as fractions.

$$\frac{2\ square\ metres}{3\ days} = \frac{8\ square\ metres}{??\ days}$$

Learning Tip:

The two equivalent ratios have to be stated exactly the same way as shown below.

$\frac{Square\ Metres}{days} = \frac{Square\ Metres}{days}$ or $\frac{days}{Square\ metres} = \frac{days}{Square\ metres}$

Any other arrangement will not give you two equivalent ratios.

Let the number of days required to dig 8 square metres be n

$$\frac{2\ square\ metres}{3\ days} = \frac{8\ square\ metres}{n\ days}$$

Step 2: Cross multiply the two ratios as shown.

$$\frac{2}{3} = \frac{8}{n}$$

$$2 \times n = 8 \times 3$$

Step 3: divide both sides of the equation by 2 to solve for n

$$\frac{2 \times n}{2} = \frac{8 \times 3}{2}$$

$$n = 12 \; days$$

Therefore it takes the farmer 12 days to dig 8 square metres of the same garden.

Try it out
Re do this example using the proportion arranged with days in the numerator.

$$\frac{days}{Square\ metres} = \frac{days}{Square\ metres}$$

Exercise

1. If 5 tickets for a play cost 40,000 shillings, calculate the cost of;
   (a) 6 tickets (b) 9 tickets (c) 20 tickets
2. To make 3 glasses of orange juice, you need 600ml of water. How much water do you need to make
   (a) 5 glasses of orange juice
   (b) 7 glasses of orange juice
3. Barker uses 1800g of flour to make 3 loaves of bread. How much flour will he need to make
   (a) 2 loaves (b) loaves (c) 24 loaves

END

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