THE REPUBLIC OF UGANDA

Ministry of Education and Sports

HOME-STUDY LEARNING

SENIOR 3

MATHEMATICS

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NCDC NATIONAL CURRICULUM DEVELOPMENT CENTRE

UGANDA NATIONAL EXAMINATIONS BOARD

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This material has been developed as a home-study intervention for schools during the lockdown caused by the COVID-19 pandemic to support continuity of learning.

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FOREWORD

Following the outbreak of the COVID-19 pandemic, government of Uganda closed all schools and other educational institutions to minimize the spread of the coronavirus. This has affected more than 36,314 primary schools, 3129 secondary schools, 430,778 teachers and 12,777,390 learners.

The COVID-19 outbreak and subsequent closure of all has had drastically impacted on learning especially curriculum coverage, loss of interest in education and learner readiness in case schools open. This could result in massive rates of learner dropouts due to unwanted pregnancies and lack of school fees among others.

To mitigate the impact of the pandemic on the education system in Uganda, the Ministry of Education and Sports (MoES) constituted a Sector Response Taskforce (SRT) to strengthen the sector's preparedness and response measures. The SRT and National Curriculum Development Centre developed print home-study materials, radio and television scripts for some selected subjects for all learners from Pre-Primary to Advanced Level. The materials will enhance continued learning and learning for progression during this period of the lockdown, and will still be relevant when schools resume.

The materials focused on critical competences in all subjects in the curricula to enable the learners to achieve without the teachers' guidance. Therefore effort should be made for all learners to access and use these materials during the lockdown. Similarly, teachers are advised to get these materials in order to plan appropriately for further learning when schools resume, while parents/guardians need to ensure that their children access copies of these materials and use them appropriately.
I recognise the effort of National Curriculum Development Centre in responding to this emergency through appropriate guidance and the timely development of these home study materials. I recommend them for use by all learners during the lockdown.

Alex Kakooza
Permanent Secretary
Ministry of Education and Sports

ACKNOWLEDGEMENTS

National Curriculum Development Centre (NCDC) would like to express its appreciation to all those who worked tirelessly towards the production of home–study materials for Pre-Primary, Primary and Secondary Levels of Education during the COVID-19 lockdown in Uganda.

The Centre appreciates the contribution from all those who guided the development of these materials to make sure they are of quality; Development partners - SESIL, Save the Children and UNICEF; all the Panel members of the various subjects; sister institutions - UNEB and DES for their valuable contributions.

NCDC takes the responsibility for any shortcomings that might be identified in this publication and welcomes suggestions for improvement. The comments and suggestions may be communicated to NCDC through P.O. Box 7002 Kampala or email admin@ncdc.go.ug or by visiting our website at http://ncdc.go.ug/node/13.

Grace K. Baguma
Director,
National Curriculum Development Centre

ABOUT THIS BOOKLET

Dear learner, you are welcome to this home-study package. This content focuses on critical competences in the syllabus.

The content is organised into lesson units. Each unit has lesson activities, summary notes and assessment activities. Some lessons have projects that you need to carry out at home during this period. You are free to use other reference materials to get more information for specific topics.

Seek guidance from people at home who are knowledgeable to clarify in case of a challenge. The knowledge you can acquire from this content can be supplemented with other learning options that may be offered on radio, television, newspaper learning programmes. More learning materials can also be accessed by visiting our website at www.ncdc.go.ug or ncdc-go-ug.digital/. You can access the website using an internet enabled computer or mobile phone.

We encourage you to present your work to your class teacher when schools resume so that your teacher is able to know what you learned during the time you have been away from school. This will form part of your assessment. Your teacher will also assess the assignments you will have done and do corrections where you might not have done it right.

The content has been developed with full awareness of the home learning environment without direct supervision of the teacher. The methods, examples and activities used in the materials have been carefully selected to facilitate continuity of learning.

You are therefore in charge of your own learning. You need to give yourself favourable time for learning. This material can as well be used beyond the home-study situation. Keep it for reference anytime.

Develop your learning timetable to ca ter for continuity of learning and other responsibilities given to you at home.

Enjoy learning

HEALTH TIPS

Dear Learner,

Welcome to use this study material. As you prepare to start these activities, remember that you are studying from home due to the Covid-19 pandemic. It is therefore important that you keep safe by doing the following:

1. Regularly wash your hands with soap and running water or use a sanitizer to sanitize your hands,
2. always wear a face mask when you are in a crowded place
3. keep a distance of 2 metres from other people.

TERM 1

TOPIC 1: THE EQUATION OF A STRAIGHT LINE

LEARNING OUTCOMES

By end of this topic, you should be able to:

(i) Find the gradient of a line.
(ii) state the gradient and intercept from the equation of a line.
(iii) find the equation of a straight line given its gradient and a point on it.
(iv) determine the equation of a straight line given two points.
(v) determine the equation when a line is given on the graph.
(vi) Apply the relationship of gradient of parallel and perpendicular lines to get the equation of a straight line

MATERIALS REQUIRED

ruler, pencil, graph papers, and pen

LESSON 1: TO FINDTHE GRADIENT OF A LINE.

LEARNING OUTCOME

By the end of this lesson, you should be able to **find the gradient of a line.**

INTRODUCTION

Have you ever climbed a stair case or a slope? Some are very steep while others are not. Others slope from left to right while others slope from right to left. The gradient of a line is a number which shows how steep a line is and the direction of sloping of the line.

Prior knowledge: knowledge about finding the gradient of a line.

Recall:

The gradient, **m,** of the line segment joining two points $A(x_1,y_1)$ and $B(x_2,y_2)$ is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Recap

1. Find the gradient of the lines joining the following pairs of points

(a) (2, 6) and (-1, 0) (b) (3,-5) and (2,2) (c) (5,-6) and (1,-4)

(e)(6,-1) and (3, 2) (f) (p, 0) and (0, q) (g) (4, 0) and (0,-4)

2. State the formula for calculating the gradient of a straight line.

LESSON 2: TO FINDTHE GRADIENT AND Y-INTERCEPT OF A LINE GIVEN THE EQUATION.

LEARNING OUTCOME

By the end of this lesson, you should be able to **find the gradient and y-intercept of a line given the equation.**

INTRODUCTION

Equation of a line.

An equation is any expression containing an "equals" sign.

The equation of the straight line is an equation containing the x and y coordinates of any point on it.

Consider the equation $y = 3x + 6$. We need to represent this on a graph paper by choosing at least two points.

x	0	-2
y	6	0

Insert sketch of the graph

You notice that the equation represents a line, and it cuts the y-axis when $x = 0$ and $y = 6$.

Represent the equation $y = -2x + 4$ on a graph paper.

Choose two points and enter them into the table below.

x	0	2
y	4	0

Insert graph

You notice that the equation$y = -2x + 4$ is also a straight line which cuts the y-axis when $y = 4$and$x = 0$.

The general equation of a straight line is always in the form

$$y = mx + c$$

c is the y-intercept(or value of y when $x = 0$

m is the gradient of the line

This is also called the slope-intercept formula

In order to find the equation of a line one needs to know the gradient and the intercept!!

Example

State the gradient and the y-intercept of the following lines;

(i) $y = 8x + 9$ (ii)$3y - 6x = 2$

Solution

(i) The gradient is 8 and the y-intercept is 9

(ii) $3y - 6x = 2$ *the equation is not in the form y=mx+c !!!*

$3y - 6x = 2 \Rightarrow 3y = 6x + 2$ (*Adding 6x to both sides*)

$\Rightarrow \frac{3y}{3} = \frac{6x}{3} + \frac{2}{3}$ (*dividing through by 3*)

$\therefore y = 2x + \frac{2}{3}$ the gradient is 2 and the y-intercept is $\frac{2}{3}$

Activity

1. Copy and complete the table below;

$Equation$	$gradient$	$y - intercept$
$y = 4x - 6$		
	$\frac{1}{2}$	$\frac{3}{4}$
$y = 7 - 5x$		
$y = x + 3$		
$y = 5x$		
$y - 10x = 5$		
$2y + 4x = 6$		
$y + 3x + 7 = 0$		

2. Find the equation of the straight lines in the form $y = mx + c$

(a) gradient 4, y-intercept 2

(b) gradient 8, y-intercept -3

(c) gradient -3, y-intercept -3

(d) gradient $\frac{3}{4}$, y-intercept -1

ACTIVITY

(a)Draw the lines on separate graphs (i) $y = 2x$ (ii)$y = 5x$ (iii) $y = -2x$

(b) Name the point which is common to all the above lines

(c) State the y-intercept of the above lines.

All the above lines are passing through the origin (0, 0), and the y-intercept is 0.

A straight line which passes through the origin (0,0)
the form $y = mx$

LESSON 3: EQUATION OF A STRAIGHT LINE GIVEN ITS GRADIENT AND A POINT ON IT.

LEARNING OUTCOME

By the end of this lesson, you should be able to find the equation of a straight line given its gradient and a point on it.

INTRODUCTION.

We can find the equation of a straight line if we know the gradient and a point that lies on it.

Example.

A line has a gradient of 5 and passes through the point (3, 3). Determine its equation.

Solution

Equation of a line is in the from $y = mx + c$

Notice that m=5 is given, what about c? 

The required equation of the line is of the form $y = 5x + c$

The line passes through the point (3, 3) i.e. when $x = 3, y = 3$

Hence $y = 5x + c \Rightarrow 3 = 15 + c \Rightarrow c = -12$

The required equation is $y = 5x - 12$.

Check this out! Given that two points (x, y) and (x_1, y_1)

$$m = \frac{y - y_1}{x - x_1}$$

Rearranging this becomes $y - y_1 = m(x - x_1)$ this is called the point-slope formula. This formula can also be used to find the equation of a line given the gradient and a point on the line.

Example

Find the equation of the straight line with gradient 3 and passes through the point (1, 5).

m=3 and the given point is $(1,5)$

using $y - y_1 = m(x - x_1) \Rightarrow y - 5 = 3(x - 1)$

$\Rightarrow y = 3(x - 1) + 5$

$\Rightarrow y = 3x - 3 + 5$

$\Rightarrow y = 3x + 2$ is the required equation.

ACTIVITY

Find the equation of the straight lines of given gradient and passing through the given points.

(a) m=4 and (2, 5)

(b) m=-2 and (4, 0)

(c) m = $\frac{2}{5}$ and (-3,-1)

LESSON 4: EQUATION OF A STRAIGHT LINE PASSING THROUGH TWO POINTS

LEARNING OUTCOME

By the end of this lesson, you should be able to find the equation of a straight line given two points.

INTRODUCTION

The equation of a straight line can also be determined when two points on the line are given.

Step 1. Find the gradient of the line using the two points

$$m = \frac{y - y_1}{x - x_1}$$

Step 2: choose one of the points and the gradient and substitute in the point-slope formula $y - y_1 = m(x - x_1)$

Step 3: rearrange the formula and make y the subject so that it is in the form $y = mx + c$

Find the equation of the line passing through the points $(5,9)$ and $(7,13)$

Step 1. Find the gradient of the line using the two points

$$m = \frac{y - y_1}{x - x_1}$$

$$m = \frac{13 - 9}{7 - 5} = \frac{4}{2} = 2$$

Step 2: choose one of the points say (5, 9) and the gradient m= and substitute in the point-slope formula $y - y_1 = m(x - x_1) \Rightarrow y - 5 = 2(x - 9)$

$$\Rightarrow y = 2(x - 9) + 5$$

$$\Rightarrow y = 2x - 18 + 5$$

Step 3: rearrange the formula and make y the subject so that it is in the form

$$y = mx + c$$

The required equation is $y = 2x - 13$

Repeat the above example while following the steps below:

Step 1. Find the gradient of the line using the two points

$$m = \frac{y - y_1}{x - x_1}$$

Step 2: substitute the value of the gradient in the slope-intercept formula

$$y = mx + c$$

Step 3: choose any one of the given points, substitute it in the formula obtained in step 2 in order to find the value of c.

ACTIVITY.

Find the equations of lines passing through the points

(a) (4, 0) and (0, 8)
(b) (-3, 4) and (8, 1)
(c) (9, 5) and (1, -1)

LESSON 5. TO DETERMINE THE EQUATION OF A LINE GIVEN ON THE GRAPH.

LEARNING OUTCOME

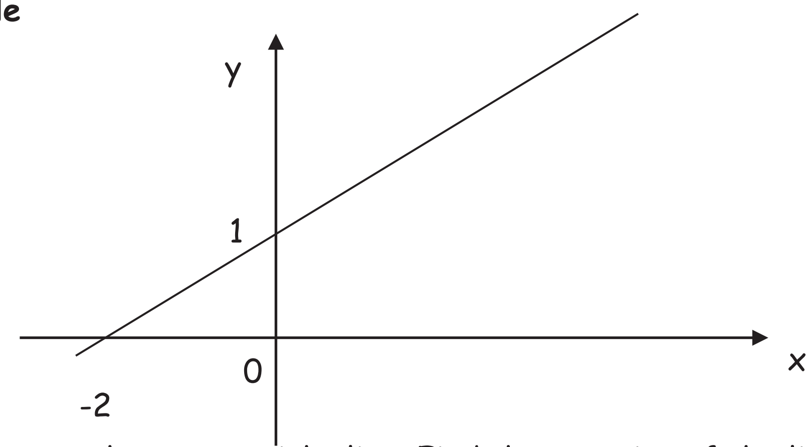
By the end of this lesson, you should be able to find the equation of a straight line given on a graph.

INTRODUCTION

Now that we know how to find the equation of a straight line given the gradient and a point on it, we can find the equation of a line shown on a graph.

The following steps can help us in this work.
Step 1. We **identify two points** (one being the y- intercept) on the line shown on the graph.
Step 2. We **use the two points** to find the gradient.
Step 3. We **find the equation** of the line as in the previous lessons.

Example



The diagram shows a straight line. Find the equation of the line.

Solution.

The visible points on the line are (0, 1) and (-2, 0).

The gradient is $m = \frac{0-1}{-2-0} = \frac{1}{2}$

Choose one of the points say (0 , 1) and substitute in the point-slope formula

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{1}{2}(x - 0)$$

$$\Rightarrow y - 1 = \frac{1}{2}x$$

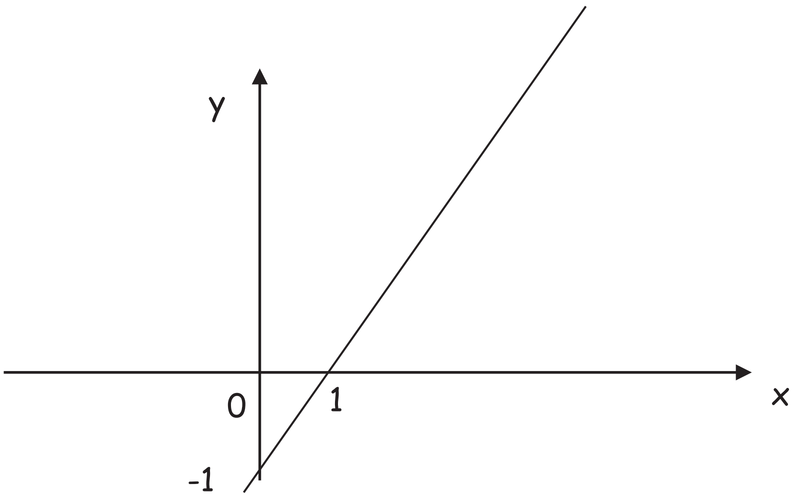
$$\Rightarrow 2y - 2 = x$$

$$\therefore 2y = x + 2$$

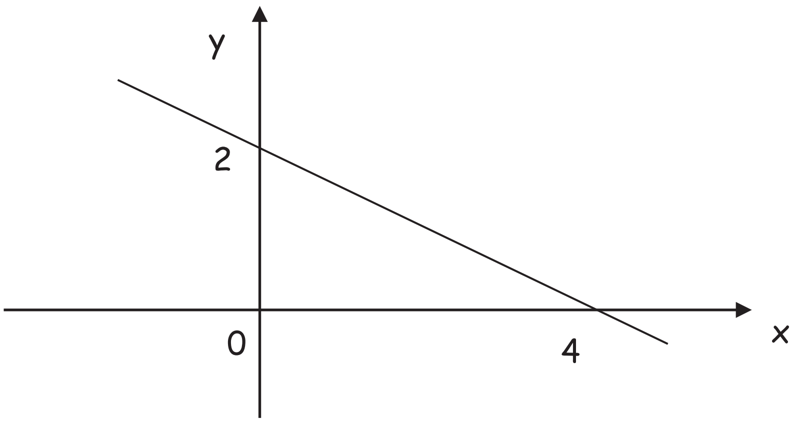
ACTIVITY.

Find the equation of each of the lines shown.

(a)



(b)



LESSON 6. TO APPLY THE RELATIONSHIP OF GRADIENT OF PARALLEL AND PERPENDICULAR LINES TO GET THE EQUATION OF A STRAIGHT LINE

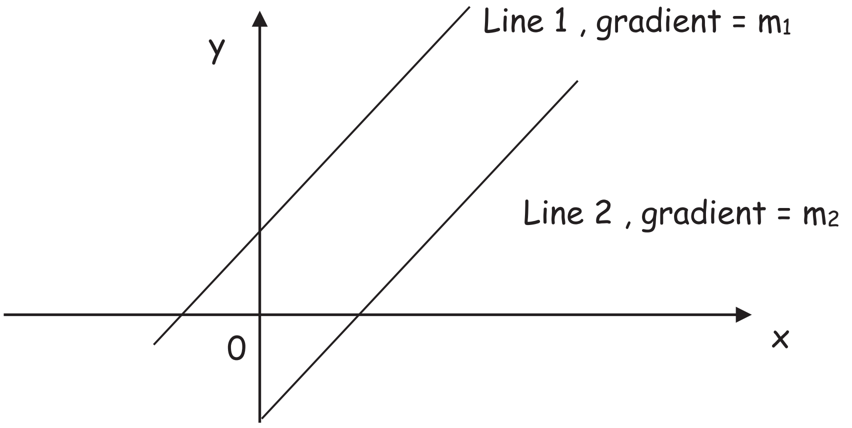
LEARNING OUTCOME

By the end of this lesson, you should be able to find the equation of a straight line given on a graph.

INTRODUCTION

Now that we know how to find the equation of a straight line given the gradient and a point on it, we can find the equation of a line parallel or perpendicular to a give line.

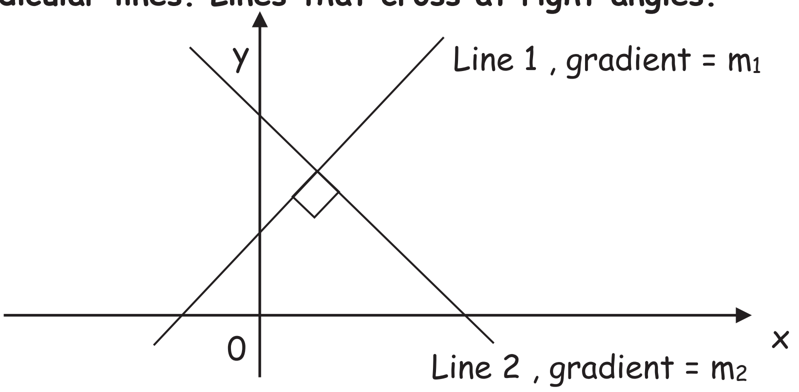
Parallel lines.



Since parallel lines slope in the same way, **parallel lines have equal gradient**.

i. e. $\mathbf{m_1 = m_2}$

Perpendicular lines. Lines that cross at right angles.



In general, if **two lines are perpendicular, the product of their gradient is -1**.

i.e. $\mathbf{m_1 \times m_2 = -1}$

Example.

Given the line L with equation y = 2x + 1, find the gradient of any line :-

(a) parallel to line L.

(b) perpendicular to line L.

Solution.

(a) Since the new line is to be parallel to L, they have equal gradients.

By comparing with the form.

$$y = mx + c$$

c is the y-intercept(or value of y when $x = 0$

m is the gradient of the line

The gradient of line L is 2.
So the gradient of the new line is also 2.

(b) Let the gradient of line L be $m_1 = 2$ and that of the new line be m_2.

For perpendicular lines, $\mathbf{m_1 \times m_2 = -1}$

$$2 \times m_2 = -1$$

Dividing through by 2, $\frac{2 \times m_2}{2} = \frac{-1}{2} \Rightarrow m_2 = -\frac{1}{2}$

Example.

Given the line L with equation y = 4x + 1, find the equation of a line passing through the point:-

(a) (1, 1) and parallel to line L.

(b) (0, -1) and perpendicular to line L.

Solution.

(a) By comparing with the form y = m x + C, the gradient of L = 4

The gradient of the line parallel is also 4. Using the point (1, 1)

$$y - y_1 = m(x - x_1)$$
$$y - 1 = 4(x - 1)$$
$$\Rightarrow y - 1 = 4x - 4$$
$$\Rightarrow y = 4x - 3$$

(b) For perpendicular lines, $\mathbf{m_1 \times m_2 = -1}$

$$4 \times m_2 = -1$$

Dividing through by 4, $\frac{4 \times m_2}{4} = \frac{-1}{4} \Rightarrow m_2 = -\frac{1}{4}$

Using the point (0 , -1)

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = \left(-\frac{1}{4}\right)(x - 0)$$
$$\Rightarrow 4(y + 1) = -x$$
$$\Rightarrow 4y = -x - 4$$

ACTIVITY.

1. Given the line L with equation y = 3x + 1, find the equation of a line passing through the point:-

   (a) (2, 1) and parallel to line L.

   (b) (1, -1) and perpendicular to line L.

2. Given the line L with equation 2y = 4x + 3, find the equation of a line passing through the point:-

   (a) (1, 3) and parallel to line L.

   (b) (3, -2) and perpendicular to line L.

TOPIC 2. SET THEORY

LEARNING OUTCOME

By the end of this topic, you should be able to:

(i) Describe and use the complement set.

(ii) Use Venn diagrams to represent sets and the number of elements in a set

(iii) Apply practical situations using two and three sets

Materials required:

ruler, pencil, coloured, and pencils

LESSON 1: DESCRIBING AND USING THE COMPLEMENT SET.

LEARNING OUTCOME

By the end of this lesson, you should be able to identify and use the complement set.

INTRODUCTION.

In senior one, you considered sets and venn diagrams. Here we are going to look at sets with members that belong to it and members that do not belong to it.

Prior Knowledge: you need to have knowledge about a universal set and subset

Complement of A Set-

Recall: A universal set is denoted by the symbol ε. It's that set that contains all the elements in the given sets.

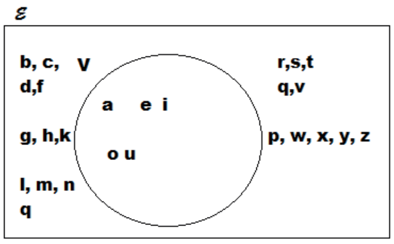
Everything that is NOT in that set!

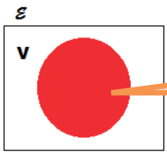
ILLUSTRATION

If our universal set is the districts in the different regions in Uganda. Then a possible subset R is the set of some of the districts in the eastern region coloured maroon.

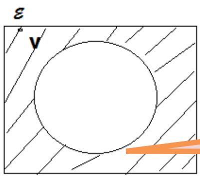
R= $\{Amuria, mbale, Iganga, Jinja, Busia, Pal$

Given that $\varepsilon = \{letters\ of\ the\ alphabet\ \} V = \{a, e, i, o, u\}$. These sets can be represented in a Venn diagram





The shaded region inside is set V



The shaded region inside ε but not including setV

The shaded region mentioned in 2 is called the **complement** of set V.

The complement of a set is the set of all elements in the universal set which do not belong to that set.

The complement of a set is denoted using symbols as A' read as (*A complement*).

Using the set -builder notation A' is written as $A' = \{x: x \in \varepsilon \text{ and } x \notin A\}$

LESSON 2: USING VENN DIAGRAMS TO REPRESENT SETS AND THE NUMBER OF ELEMENTS IN A SET

LEARNING OUTCOME

By the end of this lesson, you should be able to use venn diagrams to represent sets and the number of elements in a set

INTRODUCTION

Do you remember how we represent sets on a venn diagram? What figures and symbols are used?

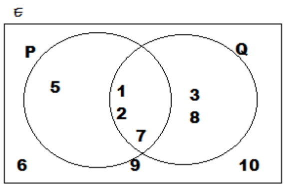
Prior knowledge: Intersection of sets

COMPLEMENT OF A SET and INTERSECTION IN TWO SETS

The complement of a set and intersection is more clearly understood in the Venn diagram

Activity

Use the Venn diagram below:



List the members that (i) do not belong to set P

(ii) do not belong to set Q

(iii) belong to set P but not set Q.

(iv) do not belong to set P but belong to set Q.

(v) neither belong to set P nor to set Q.

(vi) do not belong to both P and Q.

(vii) belong to the universal set

- The set notation P' is read as the elements that do not belong to set P.

$$\therefore P' = \{3,4,6,8,9,10\}$$

- The set notation $Q'$is read as the elements that do not belong to set Q.
  $\therefore\ Q' = \{4,5,6,9,10\}$
- The set notation $P \cap Q'$is read as the elements that belong to set P but not set Q. This can also be read as elements that belong to set P only.
  $\therefore\ P \cap Q' = \{5\}$
- The set notation $P' \cap Q$ is read as the elements that do not belong to set P but belong to set Q. This can also be read as elements that belong to set Q only.
  $\therefore P' \cap Q = \{3,8\}$
- The set notation $P' \cap Q'$ is read as the elements that neither belong to set P nor to set Q. This can also be read as elements that do not belong to set P and do not belong to set Q.

  $\therefore P' \cap Q' = \{6,9,10\}$

- The set notation $(P \cap Q\ )'$is read as the elements that do not belong to both P and Q.

$\therefore (P \cap Q\ )' = \{3,4,5,6,8,9,10\}$

Example

Given that $\varepsilon = \{11,12,13,14,15,16,17,18,19\}\ T = \{12,13,15,17\}, R = \{11,13,15,19\}$

List the elements of (i) T'(ii) R' (iii) $T' \cap R$ (iv) $T \cap R'$ (v) $T' \cap R'$(vi) $(T \cap R)'$

Solution

(i) $T' = \{11,14,16,18,19\}$ (ii) $R' = \{12,14,16,17,18\}$

(iii) $T' \cap R = \{11,14,16,18,19\} \cap \{11,13,15,19\} = \{11,19\}$

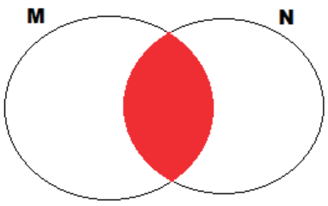
(iv)$T \cap R' = \{12,13,15,17\} \cap \{12,14,16,17,18\} = \{12,17\}$

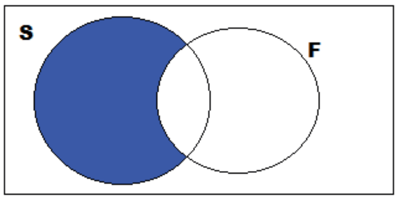
(v) $T' \cap R' = \{11,14,16,18,19\} \cap \{12,14,16,17,18\} = \{14,16,18\}$

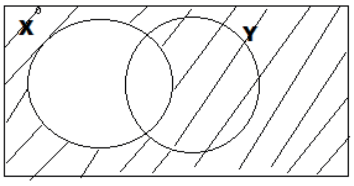
(vi)$(T \cap R)' = \{11,12,14,16,17,18,19\}$

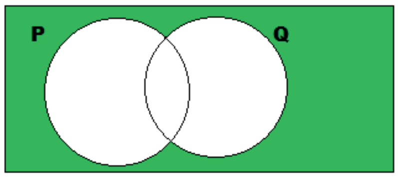
ACTIVITY

1. Describe in words and using set notation representing the shaded regions in each of the following Venn diagrams.

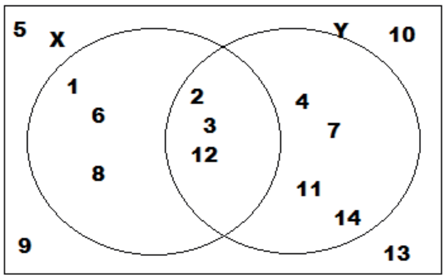








2. Use the Venn diagram to list the elements of:



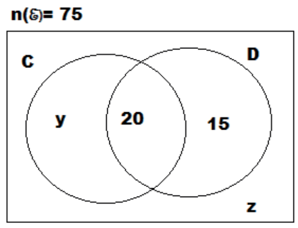
(i) universal ε (ii) X (iii) Y

(iv) X' (v) Y' (vi) $X' \cap Y$ (vii) $X \cap Y'$ (viii) $X' \cap Y'$(ix) $(X \cap Y)'$

Example

Given that $n(\varepsilon) = 75, n(C \cap D) = 20, n(D) = 35$ and , $n(C') = 50$. With the help of a Venn diagram find (i) $n(C)$(ii) $n(C' \cap D')$ (iii) $n(C' \cap D)$ (iv) $n(C \cap D')$

Solution let $(C)only = y$, $n(C' \cap D') = z$



$n(C') = 25 + z = 50 \Rightarrow z = 35$

But $y + 20 + 15 + 35 = 75 \Rightarrow y + 70 = 75 \therefore y = 5$

(i) $n(C) = y + 20 = 5 + 20 = 25$

(ii) $n(C' \cap D') = z = 35$

(iii) $n(C' \cap D) = 15$

(iv) $n(C \cap D') = y = 5$

Exercise

1. Given that $\varepsilon = \{n: -7 < n < 8\}$ and $M = \{-6, -4, -2, 4, 6\}$ find M'

2. Given that $\varepsilon = \{a, b, c, d, e, f, g, h\}, X = \{c, d, e, f, g\}, Y = \{a, c, d, f, g\}$ find the following sets

   (i) X' (ii) Y' (iii) $X' \cap Y$ (iv) $X \cap Y'$ (v) $X' \cap Y'$(vi) $(X \cap Y)'$

3. Given that $n(\varepsilon) = 45, n(F \cap G) = 15, n(F' \cap G') = 12$ and $n(F) = 30$. With the help of a Venn diagram find (i) $n(G)$(ii) $n(F')$ (iii) $n(F' \cap G)$ (iv) $n(F \cap G')$ (v) $(F \cap G)'$

4. Given that $n(W) = 20, n(V) = 27, n(W \cup V) = 32$ and $n(\varepsilon) = 38$. Use a Venn diagram find:

   (i) $n(W \cap V)$(ii) $n(W' \cap V)$ (iii) $n(W \cap V')$ (iv) $n(W' \cap V')$

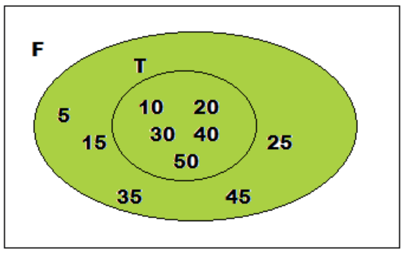
COMPLEMENT OF A SET and UNION OF TWO SETS

Draw a Venn diagram of the following sets shading the union

$T = \{the\ first\ five\ multiples\ of\ ten\}, F = \{multiples\ of\ five\ \ less\ than\ 55\}$

Solution

$T = \{10,20,30,40,50\} F = \{5,10,15,20,25,30,35,40,45,50\}$



Draw separate Venn diagrams to represent each of the following regions:

(i) $A \cup B$ (ii) $A \cup B'$ (iii) $A' \cup B'$ (iv) $(A \cup B)'$

COMPLEMENT OF A SET and INTERSECTION OF THREE SETS

In certain circumstances we need to use three sets and find the relationship among them. In these cases, a Venn diagram is very helpful.

Consider the sets

$\varepsilon = \{months\ of\ the\ year\}$

$X = \{Jan, Feb, March, April\}$

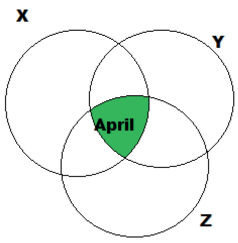
$Y = \{March, April, May, June, July\}$

$Z = \{Jan, April, June, Aug, Sept, Nov\}$

What do the above sets have in common?

All the three sets contain the element April. $\therefore$ April is the intersection of the three sets X, Y and Z.

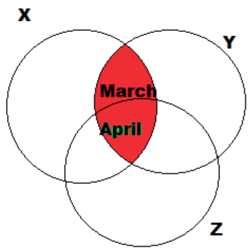
This is shown in the Venn diagram below:



The shaded region is written using the set notation $X \cap Y \cap Z = \{April\}$

We need to compare the different combinations of (i) $X \cap Y$

$X = \{Jan, Feb, March, April\}$ $Y = \{March, April, May, June, July\}$

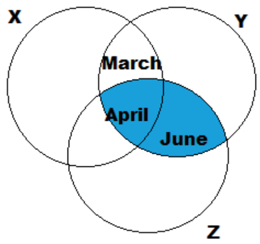


$\therefore X \cap Y = \{March, April\}$ the region $X \cap Y$ is shaded on the Venn diagram . Notice that it includes $X \cap Y \cap Z$

Where the element April is placed, March belongs in the region where only X and Y overlap.

(ii)
$Y = \{March, April, May, June, July\}$

$Z = \{Jan, April, June, Aug, Sept, Nov\}$

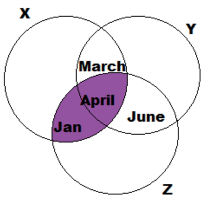


$Y \cap Z = \{April, June\}$

The shaded region represents $Y \cap Z$ on the Venn diagram. Notice that it also includes $X \cap Y \cap Z$

Where the element April is placed, so June belongs in the region where only Y and Z overlap.

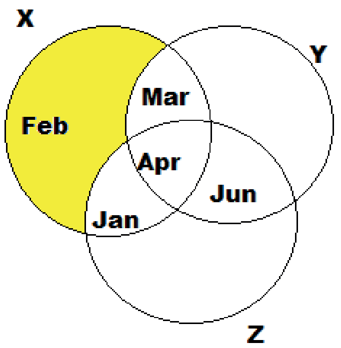
(iii)$X = \{Jan, Feb, March, April\}$ $Z = \{Jan, April, June, Aug, Sept, Nov\}$

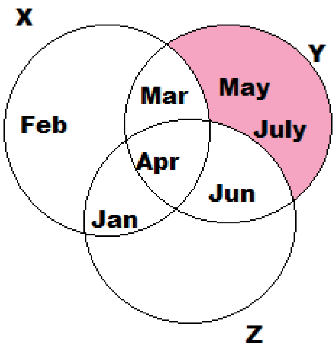


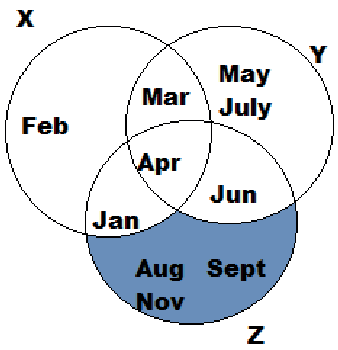
$X \cap Z = \{April, Jan\}$

The shaded region represents $Y \cap Z$ on the Venn diagram. As with the other intersections the April has already been placed in the region $X \cap Y \cap Z$ so Jan belongs in the region where only X and Z overlap.

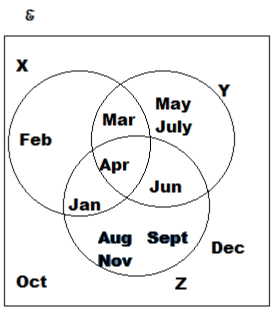
The remaining elements from each set are then placed in the respective regions i.e. elements that belong to $X only, Y only, Z only$ and outside the three sets.







The complete Venn diagram then looks like as shown below:



Activity: Use the Venn diagram to list the members that

(i) belong to both X and Y only

(ii) belong to both Y and Z only

(iii) belong to both X and Z only.

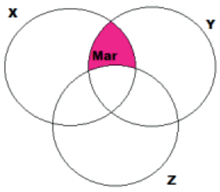
(iv) do not belong to any of the three sets

(vii) belong to the universal set

SET NOTATION IN THREE SETS

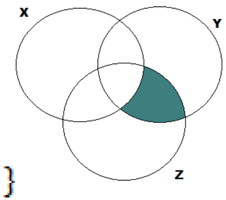
Using the results obtained above, the elements which:

(i) belong to both X and Y only also means the elements which belong to sets X and Y but not Z . The set notation is written as

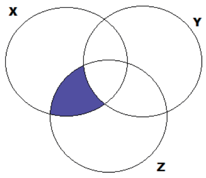


$X \cap Y \cap Z' = \{march\}$

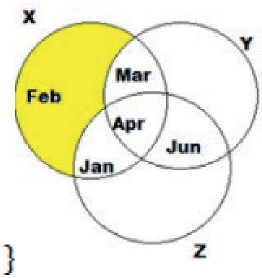
(ii) belong to both Y and Z only also means the elements which belong to sets Y and Z but not X. This is written as $X' \cap Y \cap Z = \{June\}$



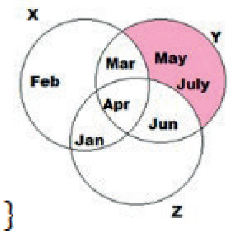
(iii) belong to both X and Z only also means the elements which belong to sets X and Z but not Y. This is written as $X \cap Y' \cap Z = \{Jan\}$



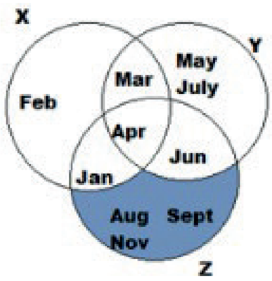
(iv) belong to X only which also means the elements which belong to set X but not Y and not Z is written as $X \cap Y' \cap Z' = \{feb\}$



(v) belong to Y only also means the elements which belong to set Y but not X and not Z. This is written as $X' \cap Y \cap Z' = \{May\ ,June\ \}$

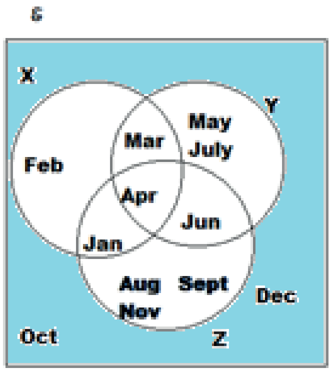


(vi) belong to Z only also means the elements which belong to set Z but not X and not Y.



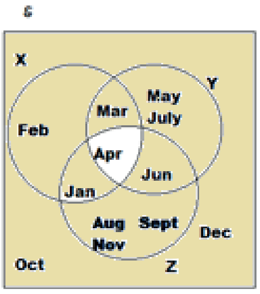
This is written as $X' \cap Y' \cap Z = \{Aug, Sept, Nov\}$

(vii) do not belong to any of the three sets also means the elements which do not belong to set X and not Y and not Z.

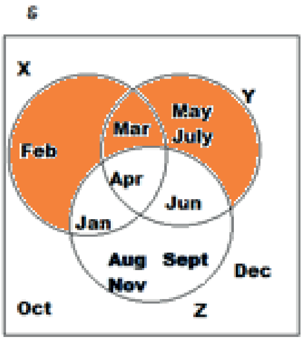


This is written as $X' \cap Y' \cap Z' = \{Oct, Dec\}$

This can also be written as $(X \cup Y \cup Z)'$



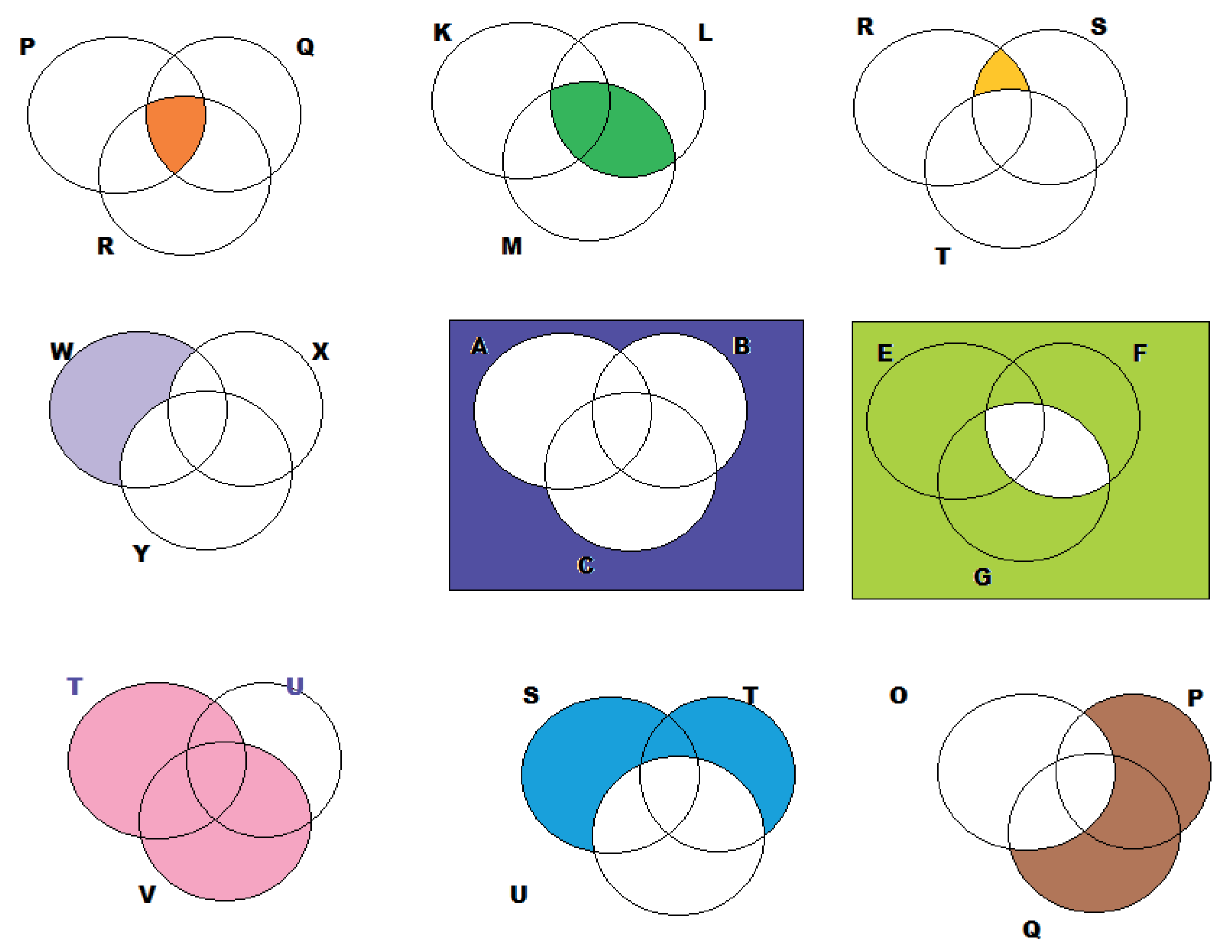
(viii) do not belong to any of the three sets X and Y and Z



(ix) belong to either X or Y but not Z is written as $X \cup Y \cap Z'$

ACTIVITY

1. Write using set notation the meaning of the shaded regions in each of the following Venn diagrams



2. Given that $\varepsilon = \{1,2,3,4,5,6,9,10,11,12,14,15,16,17,18,19\} A = \{1,2,3,4,6,9,10\}$

$B = \{1,2,5,10,14,15,17\}$ $C = \{1,2,3,4,5,11,12,17\}$

(a) Draw a Venn diagram to represent the above sets

(b) Use the Venn diagram to list the elements of the following:

(i) $A \cap B'$ (ii) $A' \cap B'$ (iii) $A \cap B \cap C$ iv) $A \cup B \cup C$ (v) $(A \cup B \cup C)'$(vi)$(A \cap B \cap C)'$ (vii) $A \cup B \cap C$ (viii) $A \cup B \cap C'$ (ix) $A \cap (B \cup C)'$

Lesson 3. Application to Real Life Problems

Learning Outcome

By the end of this lesson, you should be able to apply sets to real life problems.

Introduction

There are word problems that can be solved using set theory.

Of the 95 people who were tested for COVID 19, 53 had a high fever (F), 52 had a dry cough (D) and 57 had Chest pain (C). 25 had both a high fever and chest pain. 27 had both a chest pain and a dry cough and 31 had both a high fever and a dry cough. 2 did not have any of the three symptoms.

(a) Represent this information in a Venn diagram, showing clearly the number of drivers in each region.

(b) use it to find:

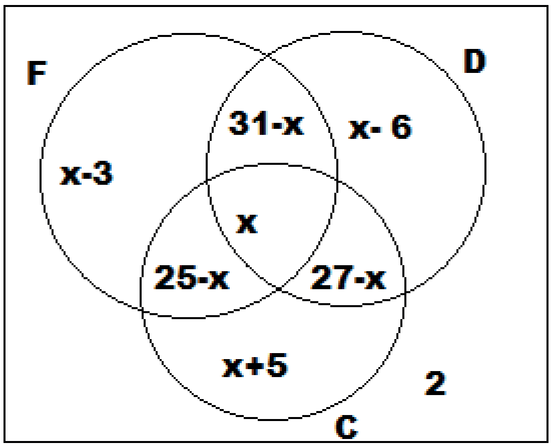
(i) the number of people who had all the three symptoms

(ii) the number of people who had exactly two of the symptoms

(iii) the number of people who had only one of the symptoms

Solution

Before drawing the Venn diagram summarise the given information using set notation as shown below.

$n(\varepsilon) = 95$, $n(F) = 53$ $n(D) = 52$ $n(C) = 57$ $n(F \cap C) = 25$ $n(C \cap D) = 27$ $n(F \cap D) = 31$ $n(F' \cap C' \cap D') = 2$ Let the $n(F \cap C \cap D) = x$ F only $53 - (31 - x + x + 25 - x) = x - 3$ D only $52 - (31 - x + x + 27 - x) = x - 6$ C only $57 - (25 - x + x + 27 - x) = 5 + x$	**n(ε)=95** 

(ii) $n(F) + x - 6 + 27 - x + x + 5 + 2 = 95$

$53 + x - 6 + 27 - x + x + 5 + 2 = 95$

$81 + x = 95$

$\therefore x = 14$

(iii) number of people who had exactly two symptoms =
$31 - x + 25 - x + 27 - x = 83 - 3x$

$= 83 - (3 \times 14)$

$= 41$

(iv) number of people who had only one symptom $= x - 3 + x - 6 + x + 5$

$= 3x - 4$

$= (3 \times 14) - 4$

$= 38.$

2. A school enrolled 76 students in S. 5 offering science subjects. 32 are to study Physics.40 are to study Chemistry. 48 are to study Biology. 14 of them are to study Physics and Chemistry, 16 are to study Physics and Biology while 20 are to study Biology and Chemistry. 6 are to study all three subjects.

a) Draw a labelled Venn diagram to represent the data.

b) Determine the number of students who study at least two subjects.

c) Find the number of students who study physics and chemistry only.

d) State the number of students who study biology only.

TERM 2

TOPIC 1: PROPORTION

Learning outcome
By the end of this topic, you should be able to;
(i) State Joint and partial variations.
(ii) Apply joint and partial variations in solving problems.
(iii) Use Compound proportion to solve real life problems.

Marerials required

Notebook, calculator .

LESSON 1: STATE JOINT VARIATION AND PARTIAL VARIATION.

LEARNING OUTCOME

By the end of this lesson, you should be able to state joint and partial variation.

Introduction
In S. 2 we were introduced to Ratio and Proportion. We saw that when two quantities say x and y are related then that gives us the ratio of x: y. If the ratio x: y is constant for all of values of x and y, we say that x varies directly as y. If one quantity increases (or decreases) by a certain value, the other quantity also increases (or decreases) by the **same** value. In that case x varies directly as y. ($x \alpha y$). The reverse is inverse proportion or variation. That is direct and inverse variations.

Joint Variation: This is when one quantity depends on two or more other quantities. For example the volume (V) of a cylinder depends on the radius (r) and height (h). We say that V varies jointly as the square of radius r and height h. $V \alpha r^2h$.

Partial Variation: Consider the cost (C) of printing books. First of all before a book is printed there are fixed expenses (K). Then there is the cost of papers and printer's charges of doing the job. Thus the total cost of printing books is made up of two parts, one of which is constant and the other which varies as the number of books (N).$C = K + aN$.

Activity

1. The volume of a cone (V) varies as jointly with the height (h) and the square of its radius (r).
   (i) Identify the type of variation
   (ii) Write an equation connecting V, h and r.

2. Given that Y varies directly as M and inversely as the square root of N.
   (i) Identify the type of variation
   (ii) Find Y in terms of M and N.

3. The daily cost (C) of feeding a family of (N) children is partly constant and partly varies as N.
   (i) Identify the type of variation
   (ii) Write the formula connecting C and N.

4. The time (T) taken to roast meat is partly constant and partly varies as the weight (W) of the piece of meat to be roasted.
   (i) Identify the type of variation
   (ii) Write a formula connecting T and W.

LESSON 2: APPLY JOINT AND PARTIAL VARIATIONS IN SOLVING PROBLEMS.

LEARNING OUTCOME

By the end of this lesson, you should be able to apply joint and partial variations in solving problems.

INTRODUCTION

In the previous lesson, we were able to state the formula connecting quantities which vary either jointly or partially from any given information. In this lesson we are going to apply the formulae formed to solve problems.
After forming the equation connecting two or more quantities given, some quantities may be given which we substitute in the equation given to solve for the ones missing.

Example 1:
Given that x varies jointly as y and the square of z. If x = 48, when y = 3 and z = 2, find z when x = 180 and y = 5.

Solution
$x \propto yz^2$.
(Since x varies jointly as y and the square of z)

$$x = kyz^2$$

(We remove the proportionality symbol $\propto$ and introduce a constant k)
We then substitute the values of x, y and z given to get the value of k

$$48 = k \times 3 \times 2^2$$

$$\therefore k = \frac{48}{3 \times 4} = \frac{48}{12} = 4$$

The equation of variation is $x = 4yz^2$.

When x = 180 and y = 5 we get;
$180 = 4 \times 5z^2$

$$z^2 = \frac{180}{4 \times 5} = \frac{180}{20} = 9$$

$z = \sqrt{9}$

$z = \pm 3.$

Example 2

M is partly constant and partly varies as N. Given that M = 15 when N = 3 and M = 16 when N = 6. Find;

(i) The equation connecting M and N

(ii) the value of M when N = 21.

Solution

(i) Equation connecting M and N

M is partly constant and partly varies as N

$M = C + \alpha N$

$M = C + KN$

(we substitute the values of M and N)

$15 = C + 3K.$ --------------------(i)

$16 = C + 6K.$ -------------------- (ii)

(we solve the two equations to get the values of C and K)

Eqn (ii) – Eqn (i). $3K - 1,$

$K = \frac{1}{3}.$

(substitute K in (i) to get C)

$15 = C + 3 \times \frac{1}{3}$

$15 = C + 1$

$14 = C.$

$M = 14 + \frac{N}{3}.$

(ii) Value of M when N = 21

(we substitute for N in the equation to get M)

$M = 14 + \frac{N}{3}$

$M = 14 + \frac{21}{3}$

$M = 14 + 7$

$M = 21.$

Activity

1. P varies directly as M and as the square root of N. Given that P = 420 when M = 6 and N = 25,
   Find the: (i) equation connecting P, M and N
   (ii) the value of P when M = N = 4

2. The daily cost (T) of feeding a family of (N) children is partly constant and partly varies as N. Given that the daily cost of feeding a family of 2 children is Shs. 9,000, and the cost of feeding a family Of 1 child is Shs. 7,000. Find;
   (i) The formula connecting T and N
   (ii) The cost of feeding a family of 5 children.

3. Given that x varies directly as the square of y and inversely as z. If x = 2 when y = 12 and z = 6, find x when y = 18 and z = 1.5.

4. The cost (C) Shs. Of a roll of cloth is partly constant and partly varies as the square of length l meters of the cloth. The cost of a roll of 50 m is Shs 50,000. The cost of a roll of length 80 m is Shs. 96,800.
   (a) Form an equation relating the cost, C and the length l
   (b) Calculate the;
      (i) Cost of a roll of length 20 m.
      (ii) Length of a roll which costs Shs. 34,700.

LESSON 3: COMPOUND PROPORTIONS

LEARNING OUTCOME

By the end of this lesson, you should be able to identify and use compound proportions.

INTRODUCTION

In S.2 we learnt direct and inverse proportion. We saw that if 'a' is directly proportional to 'b' and 'a' increases in the ratio 4 : 3 then 'b' will also increase in the ratio 4 : 3. But if 'a' is inversely proportional to 'b' and 'a' increases in the ratio 4: 3 then 'b' will decrease in the ratio 3 : 4. In compound proportion it's convenient to define a special compound unit e.g. men – hours and determine whether it is directly proportional or inversely proportional such that if one has increased or decreased in a particular ratio you also increase or decrease.

Example 1

If 5 men can dig a piece of land in 12 days, how long will 15 men take to dig the same piece of land. The compound unit here is men – days. If men increase, the days used will reduce

Men have increased in the ratio 15 : 5, so the days must reduce in the ratio 5 : 15.
Hence number of days will be; $12 \times \frac{5}{15} = 4$ days.

Example 2

Four people can do a job in nine days. How long will three people take to do the same job working under similar conditions? The compound unit is people and number of days. If the people reduce, the days will increase. The people have reduced in the ratio 3 : 4, then the days will increase in the ratio 4 : 3

$\therefore$the number of days will be $9 \times \frac{4}{3} = 12$ days.

Example 3

10 women can make 24 dresses in three days, how many women are needed to make 32 dresses in two days working on similar terms and conditions.

The compound units are women – dresses – days. If the dresses to be made increase, the women must increase. If the days reduce, the women must increase.

The dresses have increased in the ratio 32 : 24, the days have reduced in the ratio 2 : 3

Therefore the number of women will be; $10 \times \frac{32}{24} \times \frac{3}{2} = 20$ women.

Activity:

1. If 9 men working 8 hours a day can dig a trench in 12 days, how long would 6 men working 9 hours a day take to dig the same trench?

2. Ten men clear a field of area 10 hectares in four days. How many hectares would 15 men clear in six days?

3. 20 pupils can plant 1000 eucalyptus trees in four hours. How many trees can 30 pupils plant in six hours?

4. 120 Teachers can mark 4,800 papers of a national examination in 5hours. How many Teachers are required to mark 7,200 papers in 6 hours?

5. 400kg of food will feed 80 prisoners for 6 days. How many kgs of food is required to feed 40 prisoners for 30 days?

TOPIC 2 BUSINESS MATHEMATICS

LEARNING OUTCOME

By the end of this topic, you should be able to:

(i) apply the compound interest formula for calculating interest.
(ii) describe and calculate hire purchase.
(iii) calculate income tax given income tax bands.

Materials required

Notebook, calculator

LESSON 1: APPLY COMPOUND INTEREST FORMULA WHEN CALCULATING INTEREST

LEARNING OUTCOME

By the end of this lesson, you should be able to apply compound interest formula when calculating interest.

INTRODUCTION

In S. 2 we were introduced to simple interest as interest calculated on the original principal only. We also saw that interest can be added to the money borrowed or lent (Principal) after a certain period of time. And that the interest is calculated on this total amount for the next period. Adding the interest is known as **compounding the interest.**

This can be done using step by step (i.e.) adding the interest at the end of every year to get the new principal until all the years are finished. Or we can also use the compound interest formula which is given as;

The compound interest formula is given as;

$$A = P\left(1 + \frac{R}{100}\right)^n.$$

Where;

- A = Amount after the whole period for which the money was borrowed or lent.
- P = Principal or the money we borrowed or lent
- R = The rate % for which the money was borrowed or lent
- n = Period or number of years or months for which the money is going to be borrowed.

Note: The units for R and n must be consistent, i.e.

If R is per annum, n must be in years

If R is per month, n must be in months. And vice versa.

Example 1

Find the compound interest on Shs 50,000 at 12% per annum for 3 years.

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$A = 50{,}000\left(1 + \frac{12}{100}\right)^3$$

$$A = 50{,}000(1.12)^3$$

$$A = 50{,}000 \times 1.4049$$

$$A = 70{,}246.4$$

Compound interest = Amount – Principal

= 70,246.4 - 50,000

= **Shs. 20,246.4**

Activity:

Using the compound interest formula, find the compound interest of;

1. Shs. 45,000 at 8% per annum for 2 years.

2. Shs. 120,000 at 3% per month for half a year.

3. Shs. 355,000 at 15% per annum for 3 years.

LESSON 2: DESCRIBE AND CALCULATE HIRE PURCHASE

LEARNING OUTCOME

By the end of this lesson, you should be able to describe and calculate hire purchase.

INTRODUCTION

Quite often you may wish to buy an article e.g. a bicycle, a mobile phone which is costly and you do not have enough money to pay for it. To help you in such a situation, the shopkeeper may allow you to buy the article by paying part of the price in cash and then making a fixed payment each month for a number of months.

The first payment is called **deposit**; the monthly fixed payment is called **monthly instalment**. This mode of payment is called **hire purchase.** Under hire purchase, the hire purchase price is always more than the cash price because you are allowed to take the article before paying for it fully. You therefore pay for hiring and for buying.

Hire purchase price = Deposit + Total instalments

Example

The cash price of a Television set is Shs. 800,000. You can get it on hire purchase by paying a down payment of Shs. 350,000 and 4 monthly instalments of Shs.120, 000 each.

Find; (i) The hire purchase price

(ii) The extra money one pays in hire purchase than paying cash.

(i) Hire purchase price = Deposit + Total instalments

= 350,000 + 120,000 x 4

= 350,000 + 480,000

= **Shs. 830,000.**

(ii) Extra money = Hire purchase price - Cash price

= 830,000 - 800,000

= **Shs. 30,000**.

Activity:

1. A calculator is marked at Shs. 45,000. Under hire purchase it is available for a deposit of Shs. 20,000 and three monthly instalments of Shs. 9,000 each. Calculate the;
   (i) Hire purchase price
   (ii) Extra money paid over cash price.

2. The cash price of a bicycle is Shs. 280,000. If the bicycle is bought under hire purchase terms, then there is a deposit of 25% and 5 monthly instalments of Shs. 45,000 each. Find the difference between the two prices.

3. The cash price of a refrigerator is Shs. 920,000 and its hire purchase prices 15% higher under 12 monthly instalments of Shs. 55,000 each. Determine the amount of deposit.

4. A cash discount of 10% is allowed on a radio whose marked price is 200,000. Hire purchase terms are: a deposit of 20% and 7 monthly instalments of Shs. 25,000 each. Find the difference between the cash price and the hire purchase price.

LESSON 3: CALCULATE INCOME TAX

LEARNING OUTCOME

By the end of this lesson, you should be able to calculate income tax.

INTRODUCTION

Every person who works and earns a salary is required to pay tax to the government on his/her income. This tax on personal income is called **Income tax.**

Income tax is calculated from the **Gross Income** which is the total income that an individual receives as salary including allowances such as Transport, Housing, Medical, etc.

Taxable income is the income on which tax is levied. It is got by deducting from the gross all the allowances. I.e. **Taxable income = Gross income – Total allowances.**

Tax structure: This show the tax bands which are: This shows the tax bands which are followed when calculating income tax where different ranges of the taxable income have different rates.

Example:

1. Mr. Kayemba earns a gross monthly income of Shs. 545,250 which included the following allowances; Transport Shs. 50,000 and Medical Shs. 35,000. His taxable income is subjected to tax as follows; first 100,000 at 8%, the next 150,000 at 12% the rest 15%. Calculate Mr. Kayemba's income tax.

   His gross income is Shs. 545,250

   Allowances;

ITEM	**AMOUNT**
Transport	50,000

Medical	35,000
Total allowances	**85,000**

Taxable income = Gross income – Total allowances

= 545,250 - 85,000

= 460,250

To determine the income tax.

Taxable income	**Amount**	**Rate**	**Income tax**
1^{st} 100,000	100,000	$\frac{8}{100} \times 100{,}000$	8,000
next 150,000	150,000	$\frac{12}{100} \times 150{,}000$	18,000
rest (460,250 – 250,000)	210,250	$\frac{15}{100} \times 210{,}250$	31,537.5
Total	**460,250**		**57,537.5**

Mr. Kayemba pays income tax of **Shs. 57,537.5**

Activity:

1. Juliet earns a gross income of Shs. 961,500 per month. She gets a housing monthly allowance of Shs. 158,000 and a Marriage allowance of Shs. 73,400. Income tax is charged on 18% of the first 265,000 and 25% of the rest. Calculate Juliet's;
   (i) Total allowances
   (ii) Taxable income

(iii) Income tax.

2. Muwada is a married man with two children; he earns a gross income of Shs. 8,570,000 per month. He gets transport allowance of Shs. 478,000, medical allowance of 8% of his gross income and child allowance of 258,000 per child. Income tax is charged on 25% of the first Shs. 2,650,000 then on 35% of the next 3,550,000 and on 45% of the remaining income. Calculate Muwada's;
   (i) Total allowances
   (ii) Taxable income
   (iii) Income tax

TOPIC 3 SIMULTANEOUS EQUATIONS

Learning Outcome

By the end of this topic, you should be able to:

(i) solve simultaneous equations using substitution and elimination.
(ii) draw graphs of simultaneous equations and find the solution.
(iii) State the difference between linear equation and quadratic equation.
(iv)draw the graph of the line and the curve and solve the two equations from the graph.

Materials Required

notebook, calculator, graph book and Geometry set.

Lesson 1: Solve Simultaneous Equations using Substitution method.

Learning Outcome

By the end of this lesson, you should be able to solve simultaneous equations using substitution method.

Introduction

Consider the following equations $x + 5 = 11$, $e - 2 = 7$, and $3m + 1 = 2m - 4$. Such equations with one unknown are called linear equations. In each of these unknowns, the value of the unknown is only one it cannot be changed.

i.e.; in $x + 5 = 11$, x can only be 6.

But if we have a single equation with two unknowns, we can find so many values of the unknown which satisfy it e.g. $x + y = 8$, then x could be any number depending on the value of y. If $y = 7$, $x = 1$. If $y = 2$, $x = 6$. If $y = 5$, $x = 3$ etc.

Look at the following pairs of equations:

$x + y = 8$ $\qquad$ $a - b = 3$ $\qquad$ $2m - n = 5$

$x - y = 6$ $a + b = 7$ $m + n = 4$.

Each pair of these equations has two unknowns. Such pairs of equations are called **Simultaneous Equations.** And they have only one pair of values which satisfy both of them.

Substitution Method

In this method of solving simultaneous equations we choose any equation and express the unknown in terms of the other and then express it in the other equation and solve it.

Example

1. Solve the equations

$x - y = 2$ ....(i)

$2x - y = 7$..................(ii)

From (i), $x = 2 + y$. We substitute $2 + y$ for x in (ii)

$2(2 + y) - y = 7$

$4 + 2y - y = 7$

$y = 3$.

Substitute 3 in $x = 2 + y$, we get

$x = 2 + 3$

$x = 5$.

The solution is: $\mathbf{x = 5}$ and $\mathbf{y = 3}$.

2. Solve the equations

$5a - b = 8$ .... (i) $a + b = 10$ ....(ii)

From (ii) $a = 10 - b$. We substitute $10 - b$ for a in (i)

$5(10 - b) - b = 8$

$50 - 5b - b = 8$

$-5b - b = 8 - 50$

$-6b = -42$ (dividing both sides by -6)

$b = 7$

but; $a = 10 - b$

$a = 10 - 7$

$a = 3$.

The solution is $\mathbf{a = 3}$ and $\mathbf{b = 7}$.

NOTE: When choosing the unknown to substitute, take the one whose coefficient is 1 if there isn't take one with the lowest coefficient.

Example 3: Solve the simultaneous equation using substitution method

$3x + 4y = -1$ ………………(i)
$2x + 5y = 4$ ……………….(ii)

Solution: from (ii) $2x = 4 - 5y$, $x = \frac{4-5y}{2}$

Substitute for x in (i)

$3\left(\frac{4-5y}{2}\right) + 4y = -1$

Multiply 2 with every term

$12 - 15y + 8y = -2$

$-7y = -14$ (dividing both sides by -7)

$y = 2$

Substitute for x in $x = \frac{4-5y}{2}$

$x = \frac{4-10}{2}$

$x = -3.$

The solution is $\mathbf{x = -3}$ and $\mathbf{y = 2}$

Activity:

Solve the following simultaneous equations using substitution method.

1. $2x + 3y = 14$
   $2x - y = 6$
2. $3a + 2b = 4$
   $a + b = 1$
3. $11p + q = 2$
   $9p + q = -6$
4. $3x - y = 10$
   $x - y = 2$
5. $5y = 3x - 10$
   $4y = 2x - 8.$

Lesson 2: Elimination Method

LEARNING OUTCOME

By the end of this lesson, you should be able to solve simultaneous equation by elimination.

INTRODUCTION

In this method we eliminate any of the two unknowns given by adding or subtracting the two equations using the rule of **SSS** and **DSA**.

Where; SSS means **S**ame **S**ign **S**ubtract and **DSA** means **D**ifferent **S**ign **A**dd

Example 1. Solve the equations:

$3x - 2y = 7$ ……….. (i)

$x + 2y = 5$ ………… (ii)

We eliminate y because y has the same coefficient (2) in both equations.

Since they are of different signs, we add (DSA)

I.e. (i) + (ii)

$$3x - 2y = 7$$

$$\underline{x + 2y = 5}$$

$$4x \qquad = 12$$

$\therefore\ x = 3$

Substituting $x = 3$ in (ii), we have

$$3 + 2y = 5$$

$$2y = 2$$

$$y = 1.$$

The solution is $\mathbf{x = 3}$ and $\mathbf{y = 1}$.

Note:

When the coefficients of the unknowns in both equations are not the same, we need to multiply either one equation or both equations with a convenient value before we add or subtract.

Example 2:

Solve the equations

$3x + 2y = 13$ ………… (i)
$2x + y = 8$ …………….. (ii)
No unknown can be eliminated by addition and subtraction. But if we multiply (ii) by 2, then y can be eliminated.
i.e. (i) - 2(ii)

$$\begin{array}{r} 3x + 2y = 13 \\ \underline{4x + 2y = 16} \\ -x \quad = -3 \end{array}$$

$\therefore\ x = 3$

Substitute $x = 3$ in (ii) to get y

$$6 + y = 8$$
$$y = 2$$

The solution is $\mathbf{x = 3}$ and $\mathbf{y = 2}$.

Activity .Solve the simultaneous equations below using elimination method;

1. $3x + 2y = 16$
   $3x - y = 1$
2. $3a - b = 8$
   $2a - 3b = 10$
3. $2m + n = 10$
   $2m + 7n = 40$
4. $2x - 3y = 1$
   $3x - 4y = 3$
5. $2p - 5q = 11$
   $3p + 2q = -12.$

LESSON 3: DRAW GRAPHS OF SIMULTANEOUS EQUATIONS AND FIND THE SOLUTION

LEARNING OUTCOME

By the end of this lesson, you should be able to draw graphs of simultaneous equations and use the graphs to solve the equations.

INTRODUCTION

In S. 1 we looked at Coordinate geometry and Graphs. We were able to generate a table of points on a linear equation and plot that equation on the $x - y$ axes.

In solving Simultaneous equations using graphical method we generate points that lie on a linear equation and plot them on a Cartesian plane. We do the same for the other equation given on the same coordinate axes.

The coordinates of the point of intersection of these lines is the solution of the simultaneous equation.

Example: Solve the simultaneous equations below graphically.

$x + y = 8$; $x - y = 6$.

The line $x + y = 8$ has points shown in the table below

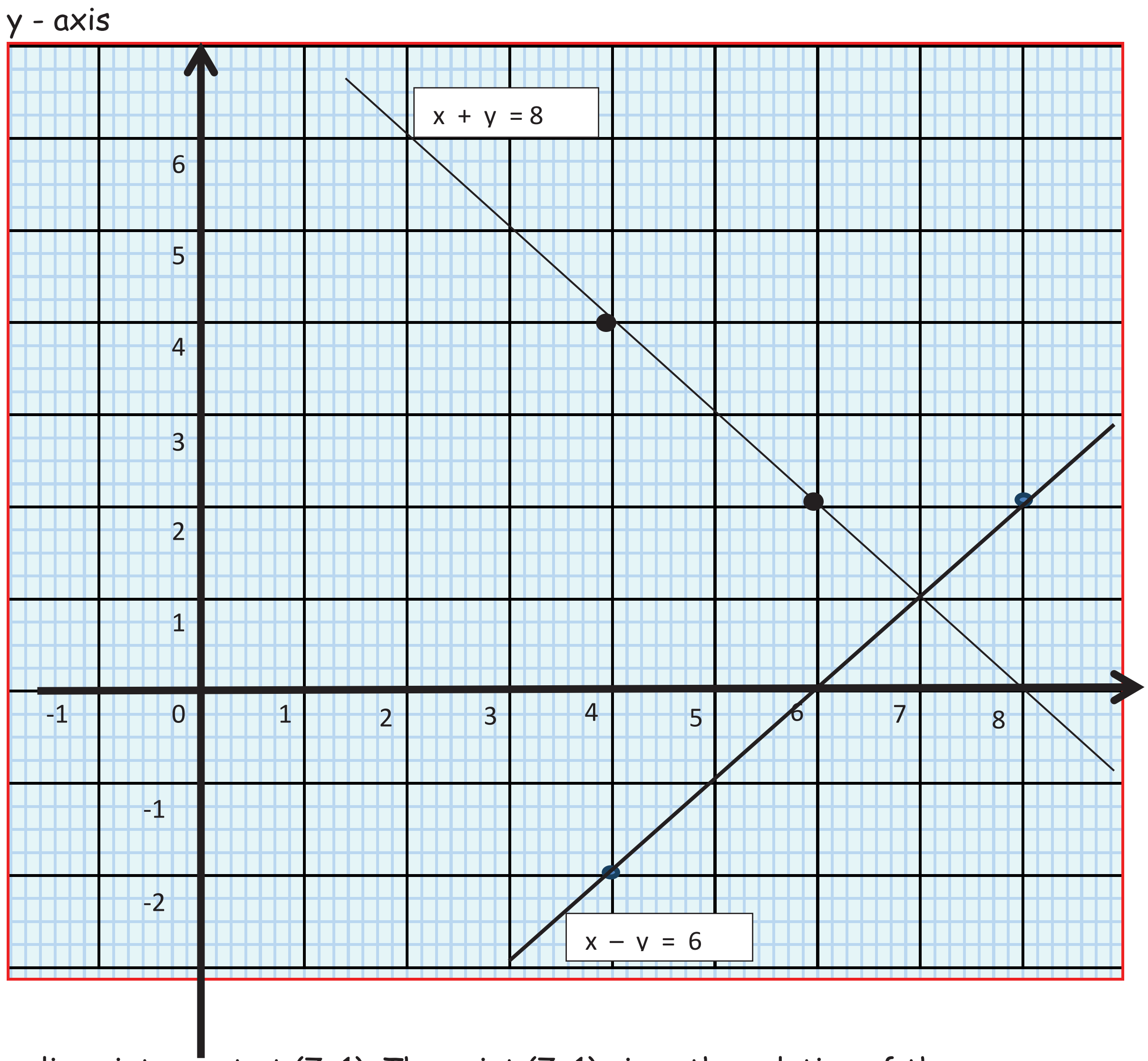
x	2	5	4	0
y	6	3	4	8

We plot any two of these points on the x-y axes and draw a straight line through them.

The line $x - y = 6$ has points shown in the table below;

x	2	5	7	9
y	-4	-1	1	3

Still we plot any two points on the same x-y axes and draw a line through those points.



The two lines intersect at (7, 1). The point (7, 1) gives the solution of the simultaneous equation.

∴**x =7** and **y = 1.**

Activity:

1. (a) Copy and complete the tables for the following equations

(i) $2x + y = 8$

x	0	1	2	3	4
y					0

(ii) $x + 2y = 7$

x	0	1	2	3	4
y	3.5	3			

(b) Using the tables above plot graphs of the lines $2x + y = 8$ and $x + 2y = 7$ on the same x-y axes.

(c) Give the solution of the simultaneous equations from the graph.

2. Use the graph to solve the simultaneous equations below;

(a) $y - x = 3$
$3y + x = 3$

(b) $2x - y = 5$
$x + y = 7$

(b) $3x - y = 2$
$2x + y = 5$

(d) $2y - 3x = 1$
$y + 3x = 5$

LESSON 4: DIFFERENCE BETWEEN LINEAR EQUATION AND QUADRATIC EQUATION

LEARNING OUTCOME

By the end of this lesson, you should be able to distinguish between linear and quadratic equations.

INTRODUCTION

In S. 1 we plotted linear graphs after generating a set of points on the line.

Graph of a function.

y is a function of x, that means for every value assigned to x, there is always a corresponding value of y. Then these pairs of values of x and y can be plotted and a distinctive graph (a curve or a straight line) will be obtained. That is the graph of the function.

If the graph is of the first degree, of which the general form is $y = mx + c$, then this is called a **linear equation** and the graph is a straight line.

The expression $ax^2 + bx + c$ where a, b and c are constants is called a **quadratic function** of x or a function of the second degree (where the highest power of x is 2). If such an expression is plotted against x, the graph will be in form of a curve.

Activity:

Which of the following equations are quadratic and which ones are linear?

(a) $y = 2x + 3$

(b) $y = x^2 + 3$

(c) $2x^2 - 3x + 4 = 0$

(d) $0 = 2x + 3$

(e) $3x^2 = 2x + 3$

LESSON 5: USING GRAPHICAL METHOD TO SOLVE QUADRATIC EQUATIONS

LEARNING OUTCOME

By the end of this lesson, you should be able to quadratic equation by graphical method.

INTRODUCTION

In S. 1 we plotted linear graphs by after generating a set of points which lie on the line. In the same way, we are going to form a table for values of x and y which lie on the function of the curve. These set of points are plotted on the x- y axis and joined with a smooth curve.

Where the curve closes the x- axis are the values of x for which y = 0. Those are the solutions of the function.

Example

1. (a) Make a table of values of x and y for the graph of $y = x^2 - 3x + 2$, for values of x between -2 and 5.

   (b) Use the table to draw the graph of $y = x^2 - 3x + 2$.
   Hence find the solution of the equation $x^2 - 3x + 2 = 0$.

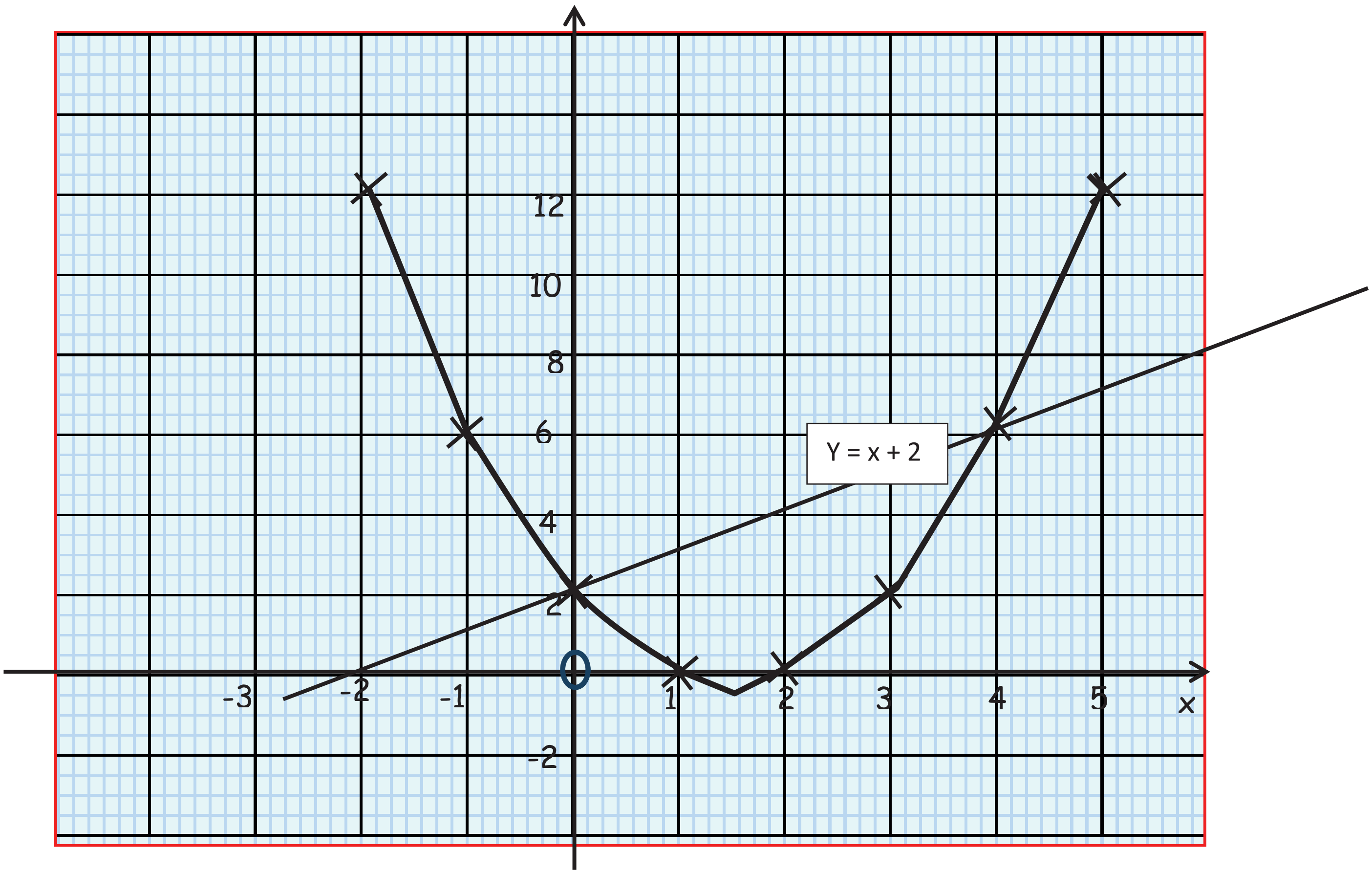
   (c) On the same axes draw the graph of the line $y = x + 2$.
   Hence solve the equation $x^2 - 3x + 2 = x + 2$.

Solution

(a)Table of values of x and y for the graph $y = x^2 - 3x + 2$.

x	-2	-1	0	1	2	3	4	5
x^2	4	1	0	1	4	9	16	25
-3x	6	3	0	-3	-6	-9	-12	-15
2	2	2	2	2	2	2	2	2
y	12	6	2	0	0	2	6	12

(b)



The curve of the function crosses the x- axes where x = 1 and x = 2. These are the points on the curve for which y = 0.

Therefore, **x = 1** and **x= 2** are the solutions of the equation $x^2 - 3x + 2 = 0$.

(c)Draw the line y = x + 2

x	0	2
y	2	4

To get the solution of $x^2 - 3x + 2 = x + 2$, we need the points where the curve and the line meet.
The values of x at the points of intersection of the line and the curve are the solution of the equation.
From the graph, values of x are; **2 and 4.**

Activity

1. (a) Draw the graph of $y = 2x^2 - x - 4$ for values of x from -3 to 3.

   (b) On the same graph draw the line $y = 2x$.

   (c) Use the graph to solve the equations

   (i) $2x^2 - x - 4 = 0$ (ii) $2x^2 - 3x - 4 = 0$.

TOPIC 4 MATRICES

LEARNING OUTCOME.

By the end of this lesson, you should be able to:

(i) Describe a matrix.

(ii) State the order of a matrix.

(iii) State the types of matrix.

(iv)Determine compatibility in addition and multiplication of matrices.

(v) Find determinant of 2 x 2 matrix.

(vi)Find inverse of 2x2 matrix.

(vii)Use matrices to solve simultaneous equations.

Materials Required

Notebook and pen.

LESSON 1: DESCRIBE A MATRIX

LEARNING OUTCOME

By the end of this lesson, you should be able to describe a matrix.

INTRODUCTION

The Family Bakery produces Loaves, Cakes and Bans. The Bakery has two different vehicles which transport their products to the market. Those are; - Fuso, Tata. The Fuso lorry takes 6 boxes of loaves, 4 boxes of cakes and 8 boxes of bans. Tata takes 9 boxes of loaves, 3 boxes of cakes and 7 boxes of bans.

This information can be expressed in a table as;

	Loaves	Cakes	Bans
Fuso	6	4	8
Tata	9	3	7

From the table we can extract out numbers to get;

$\begin{pmatrix} 6 & 4 & 8 \\ 9 & 3 & 7 \end{pmatrix}$.

This arrangement of numbers in a rectangular pattern is called a Matrix.

Activity:

1. Grace went to the market and bought 3 clusters of tomatoes, 4 bunches of matooke, and 1 kg of groundnuts. Simon bought 2 clusters of tomatoes, 6 bunches of matooke and 2 kgs of ground nuts. Silver bought 1 cluster of tomatoes, 2 bunches of matooke and half a kilogram of ground nuts. Represent this information in a matrix form.

2. Mathematical instruments were distributed in each of the four streams of S.3 as follows; 3A was given 4 text books, 5 graph books and 7 geometry sets. 3B was given 3 graph books, 5 geometry sets and 2 text books, 3C received 11 graph books and 6 text books only while 4D got only two items and they were28 in total of which 13 were geometry sets and the rest text books.
   Represent this information in a matrix form.

3. After the first round of the champions league the "big four" English clubs had performed as follows: Arsenal had won 3 games, drawn 2 and lost 1, Liverpool had won 2 and the rest were draws, Manchester United had won 2 drawn 2 and lost 2, while Chelsea had won 2 drawn 1 and lost the rest. Represent this information in a matrix form.

LESSON 2: ORDER AND TYPES OF MATRICES

LEARNING OUTCOME.

By the end of this lesson, you should be able to describe the order and types of matrices.

INTRODUCTION

In our first lesson we saw a matrix as arrangement of elements in a definite shape.

The shape of a matrix is described by giving the **number of rows** and the **number of columns** and this is called the **order of a matrix.** A matrix consisting of m rows and n columns is a matrix of order **m X n** read as (m by n).

Rows are the lines running horizontally and columns are those lines running vertically. Bold letters are used to represent matrices.

Given the matrix A = $\begin{pmatrix} 4 & 2 \\ 1 & 8 \\ 15 & 7 \end{pmatrix}$. Matrix A has 3 rows and 2 columns. Therefore, its order is 3 X 2.

Types of matrices:

(i) **A row matrix.** This is a matrix of 1 row but having several columns

Eg, $\begin{pmatrix} 3 & -7 & 6 \end{pmatrix}$.

(ii) **A column matrix**. This is a matrix of 1 column but having several rows

E.g., $\begin{pmatrix} 5 \\ 8 \\ 17 \end{pmatrix}$.

(iii) **A Square matrix**. This is a matrix which has the same number of rows and columns.

Eg, $\begin{pmatrix} 5 & 6 \\ 3 & 9 \end{pmatrix}$ or, $\begin{pmatrix} -2 & 0 & 4 \\ 8 & 11 & 6 \\ 7 & -3 & 9 \end{pmatrix}$.

(iv)**An identity matrix.** This is a square matrix with all elements in the major diagonal equal to 1 and the rest zeros.

Eg, $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ or $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

An identity matrix is denoted by a letter I.

(v) **A zero / Null matrix.** This is a matrix of any order with all elements equal to zeros

E.g., $\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$.

A zero matrix is also called a Null matrix and it's denoted by a letter O.

Activity

1. State the order of the following matrices:

(a) M = $\begin{pmatrix} r & d & g \\ e & n & q \\ w & t & y \end{pmatrix}$, (b) K = $\begin{pmatrix} 4 \\ 15 \\ 7 \end{pmatrix}$ (c) B = $\begin{pmatrix} -5 & 35 & 0 \\ 16 & -8 & 3 \end{pmatrix}$.

2. Give a matrix of your choice of order:

(a) 1 X 4 (b) 3 X 2 (d) 2 X 5.

3. Write the matrices below;

(a) O = A zero matrix of order 4 X 2

(b) I = An identity matrix of order 4 X 4

(c) A = A row matrix with 5 columns

(d) B = A column matrix with 4 rows.

LESSON 3: ADDITION, SUBTRACTION OF MATRICES AND SCALAR MULTIPLICATION

LEARNING OUTCOME.

By the end of this lesson, you should be able to add subtract and carry out scalar multiplication of matrices.

INTRODUCTION

In the previous lesson we saw the order of a matrix as number of rows by number of columns.

In addition and subtraction of matrices, we add or subtract the corresponding elements together.

Matrices can only be added or subtracted when they are of the same order. The result matrix will also have the same order.

Example:

1. Given that $A = \begin{pmatrix} 2 & 3 \\ 5 & -4 \end{pmatrix}$, $B = \begin{pmatrix} 6 & 2 \\ -3 & 7 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 4 \\ 9 & -8 \end{pmatrix}$. Find;

   (i) A + B   (ii) B – C

   (i) A + B

   $$\begin{pmatrix} 2 & 3 \\ 5 & -4 \end{pmatrix} + \begin{pmatrix} 6 & 2 \\ -3 & 7 \end{pmatrix} = \begin{pmatrix} 2+3 & 3+2 \\ 5+-3 & -4+7 \end{pmatrix}$$

   $$= \begin{pmatrix} 5 & 5 \\ 2 & 3 \end{pmatrix}.$$

   (ii) B - C

   $$\begin{pmatrix} 6 & 2 \\ -3 & 7 \end{pmatrix} - \begin{pmatrix} 1 & 4 \\ 9 & -8 \end{pmatrix} = \begin{pmatrix} 6-1 & 2-4 \\ -3-9 & 7--8 \end{pmatrix}$$

   $$= \begin{pmatrix} 5 & -2 \\ -11 & 1 \end{pmatrix}.$$

Multiplication of a matrix with a scalar

A scalar is any quantity that may be multiplied by a scalar. If a scalar 2 is multiplied by M then that scalar must be multiplied by every element of matrix M

Example 2

Given that P = $\begin{pmatrix} 4 & 2 \\ -1 & 3 \\ 0 & 1 \end{pmatrix}$ and R = $\begin{pmatrix} 2 & 3 \\ 5 & 4 \\ -2 & 1 \end{pmatrix}$. Find

(i) 2P (ii) 3R.

(i) $2P = 2\begin{pmatrix} 4 & 2 \\ -1 & 3 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 8 & 4 \\ -2 & 6 \\ 0 & 2 \end{pmatrix}$

(ii) $3R = 3\begin{pmatrix} 2 & 3 \\ 5 & 4 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 6 & 9 \\ 15 & 12 \\ -6 & 3 \end{pmatrix}$.

Activity

1. Work out the following matrices

   (a) $\begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + \begin{pmatrix} 4 \\ 5 \\ -2 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & -3 \\ 5 & 9 \\ 2 & -3 \end{pmatrix} - 2\begin{pmatrix} 4 & -2 \\ 1 & 3 \\ 0 & 2 \end{pmatrix}$.

2. Given the matrices below;

   $A = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}$, $B = \begin{pmatrix} 5 & -1 \\ 3 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 6 & 2 \\ 0 & 7 \end{pmatrix}$. Find;

   (i) 3A + 2B
   (ii) 2C - 3B
   (iii) 2A + 4(B – C).

LESSON 4: MULTIPLICATION OF MATRICES

LEARNING OUTCOME

By the end of this lesson, you should be able to multiply matrices.

INTRODUCTION

We were introduced to matrices and we saw the different types of matrices. Among the types of matrices, we saw a row and a column matrix. When multiplying matrices, we **multiply row by column**. i.e. each element in the row of the first matrix is multiplied by corresponding element in the column of the second matrix, then the products are added together. Matrices can only be multiplied when the **number of columns of the first matrix** is equal to the **number of rows of the second matrix**. Then the order of the product matrix will be number of rows of the first matrix by number of columns of the second matrix. For Example

1. Given that $A = \begin{pmatrix} 3 & 2 & 1 \\ 2 & -4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 3 \\ 1 & 2 \\ 5 & 1 \end{pmatrix}$. Find AB

A x B = AB

(2 x 3) by (3 x 2) = (2 x 2) (order of product AB)

$$\begin{pmatrix} 3 & 2 & 1 \\ 2 & -4 & 1 \end{pmatrix}\begin{pmatrix} -2 & 3 \\ 1 & 2 \\ 5 & 1 \end{pmatrix} = \begin{pmatrix} -6+2+5 & 9+4+1 \\ -4+\ -4+5 & 6+\ -8+1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 14 \\ -3 & -1 \end{pmatrix}.$$

2. Given that $P = \begin{pmatrix} 2 & 1 & -3 \end{pmatrix}$ and $Q = \begin{pmatrix} 3 & -1 \\ 2 & 4 \\ 1 & 0 \end{pmatrix}$. Find PQ

P x Q = PQ

(1 x 3) by (3 x 2) = (1 x 2)

$$\begin{pmatrix} 2 & 1 & -3 \end{pmatrix}\begin{pmatrix} 3 & -1 \\ 2 & 4 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 6+2+\ -3 & -2+4+0 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 2 \end{pmatrix}.$$

Activity

1. (a) A matrix of K order 3 x 4 is multiplied by matrix D of order 4 x 2. What is the order of the product matrix KD?

(b) $\begin{pmatrix} 3 & 5 & -7 \end{pmatrix}R = \begin{pmatrix} 0 & 1 & 2 \\ -9 & 4 & 6 \\ 4 & 7 & 8 \end{pmatrix}$. Find the order of matrix R.

2. $\begin{pmatrix} 2 & 5 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$. Find the value of a and b.

3. Given the matrices $A = \begin{pmatrix} 2 & -2 \\ 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 5 \\ 2 & -1 \end{pmatrix}$ and $C = \begin{pmatrix} 3 & 4 \\ -2 & 3 \end{pmatrix}$.

Find; (i) AB (ii) BC (iii) CA (iv) A (B + C).

LESSON 5: DETERMINANT AND INVERSE OF A MATRIX

LEARNING OUTCOME

By the end of this lesson, you should be able to find determinant and inverse of a matrix.

INTRODUCTION

In our previous lesson we saw the types of matrixes and we saw a square matrix as a matrix with the same number of rows and columns. We are going to concentrate on a square matrix of order 2.

Determinant of a matrix.

Let A be a square matrix of order 2 x 2. $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

The elements in a square matrix from the left hand top corner to the right hand bottom corner are the major or leading diagonal; the other elements are the minor diagonal. Determinant of a matrix = product of elements in the leading diagonal minus the product of elements in the minor diagonal.

For the matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, determinant (det) A = ad - bc .

Example

1. If M = $\begin{pmatrix} 2 & -4 \\ 5 & -6 \end{pmatrix}$ determinant (det) M = 2x-6 - 5x-4

   = -12 - -20

   = 8.

2. If the determinant of matrix $\begin{pmatrix} 5 & 1 \\ 3 & e \end{pmatrix}$ is 7. Find the value of e

   Determinant = 7

   5xe - 3x1 = 7

   5e - 3 = 7

   5e = 10

   e = 2.

Inverse of a Matrix

If A is a square matrix, then its inverse written as A^{-1} is the matrix that if multiplied by A gives an identity matrix. That is, if A^{-1} is the inverse of A, then $\mathbf{AA^{-1} = I}$.

How to find the inverse of a matrix;

- Interchange the elements of the leading diagonal, and change the signs of the elements of the minor diagonal.
- Multiply the resultant matrix by the reciprocal for the determinant of the original matrix.
- Write the inverse in its simplest form.

i.e. if $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then its inverse is given by;

$A^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$. Where; determinant A is the determinant of matrix A

Note: If the determinant of a matrix is zero, that matrix has no inverse. Such a matrix is called a singular matrix.

Example

1. Find the inverse of matrix R = $\begin{pmatrix} 4 & 1 \\ 6 & 2 \end{pmatrix}$.

Solution

$R^{-1} = \frac{1}{\det R}\begin{pmatrix} 2 & -1 \\ -6 & 4 \end{pmatrix}$

det R = (4x2 - 6x1) = 8 - 6 = 2

$R^{-1} = \frac{1}{2}\begin{pmatrix} 2 & -1 \\ -6 & 4 \end{pmatrix}$

$R^{-1} = \begin{pmatrix} 1 & -0.5 \\ -3 & 2 \end{pmatrix}$.

Activity:

1. Find the inverse of the following matrices.

   (i) $\begin{pmatrix} 3 & 7 \\ 2 & 5 \end{pmatrix}$ (ii) $\begin{pmatrix} 5 & 2 \\ 3 & 2 \end{pmatrix}$ (iii) $\begin{pmatrix} 3 & -4 \\ 1 & -2 \end{pmatrix}$

2. Which of the following matrices is singular;

   (i) $\begin{pmatrix} 6 & 3 \\ 8 & 4 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & -2 \\ 2 & 3 \end{pmatrix}$ (iv) $\begin{pmatrix} -1 & \frac{1}{2} \\ 2 & -1 \end{pmatrix}$

3. Given that *A* = $\begin{pmatrix} -3 & 9 \\ 2 & x \end{pmatrix}$, find the value of x if *A* is a singular matrix.

LESSON 6: USING MATRICES TO SOLVE SIMULTANEOUS EQUATIONS

LEARNING OUTCOME

By the end of this lesson, you should be able to solve simultaneous equations using matrices.

INTRODUCTION

In the previous lesson, we saw how to solve simultaneous equations using substitution, elimination and graphical methods. We are going to use matrix method following the procedure below;

- Re – arrange the equations given such that the unknowns are in line with each other.
- Express the equations in matrix form.
- Find the inverse of the left hand 2x2 matrix.
- Pre – multiply the inverse on both sides of the equation of matrices
  Recall $MM^{-1} = \mathbf{I}$.
- Equate the unknowns to the values got on the right.

Example:

Use matrix method to solve the simultaneous equations

$x + 3y = 5$
$2x + 4y = 6$

The equations are already arranged so we express them into matrices

$\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$

Find the inverse of $\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$. Inv. $= \frac{1}{det}\begin{pmatrix} 4 & -3 \\ -2 & 1 \end{pmatrix}$

det = 4x1 - 3x2

= 4 - 6

= -2

Inv. $= \frac{1}{-2}\begin{pmatrix} 4 & -3 \\ -2 & 1 \end{pmatrix}$

$$\text{Inv.} = \begin{pmatrix} -2 & \frac{3}{2} \\ 1 & \frac{-1}{2} \end{pmatrix}$$

Pre- multiply this inverse on both sides of the equation of matrices

$$\begin{pmatrix} -2 & \frac{3}{2} \\ 1 & \frac{-1}{2} \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & \frac{3}{2} \\ 1 & \frac{-1}{2} \end{pmatrix}\begin{pmatrix} 5 \\ 6 \end{pmatrix}.$$

We get an identity matrix on the left **since** $\mathbf{MM^{-1} = I}$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -10+9 \\ 5+\ -3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$\therefore \mathbf{x = -1}$ and $\mathbf{y = 2}$.

Activity:

Use the matrix method to solve the following pairs of simultaneous equations:

1. $4x + 2y = 5$
   $x + 6y = 4.$

2. $3x - y = 4$
   $6x - 3y = 6.$

3. $4m + 6n = 15$
   $3m - 4n = 7$

4. $2a + 3b = 7$
   $2b + a = 3.$

TOPIC 5 ALGEBRAIC EXPRESSIONS

Materials required

Notebook and pen.

LESSON 1: BUILD A FORMULA FROM WORD PROBLEMS

LEARNING OUTCOME

By the end of this lesson, you should be able to build a formula from word problems.

INTRODUCTION

We have met many formulae in Mathematics. For example; the area of a triangle is a half times base and height. $A = \frac{1}{2}bh$. The letters are used to represent quantities and the formula describes the relationship between those quantities. A formula is a mathematical statement in which one quantity is expressed in terms of other quantities but when all these quantities are represented by letters. In building a formula, we choose appropriate letters to represent quantities involved in the statement given and then connect them to come up with an algebraic expression of only letters without words and that is called the formula.

For Example

1. A farmer has chickens and goats. Build a formula for the total number of legs that his animals have.

We need to choose appropriate letters to represent the quantities involved;

Let c represent chickens
" g represent goats
" L represent legs.

Each chicken has 2 legs, the total legs for chicken = 2c
Each goat has 4 legs, the total legs for goats = 4g

$\therefore\ L = 2c + 4g$
$L = 2(c + 2g)$.

2. To measure the depth of a soak pit, you drop a stone down it and record the time until you hear it hit the bottom. Then the depth will be nine times the square of the time taken to hit the bottom. Build a formula from that statement.

   Choose appropriate letters to represent quantities
   d to represent depth
   t to represent time taken to hit the bottom.

   $d = 9 \times t^2$
   $d = 9t^2$.

Activity

1. In a shop, pencils cost Shs. 250 each, books cost Shs. 1500 each and rulers cost Shs. 500 each. Tom bought pencils, books and rulers. Build a formula for the total cost spent by Tom.

2. Dick packs eggs in boxes of twelve. How many boxes will he need for N eggs?

3. A trader buys tomatoes and yams. Write an expression for the number of vegetables he can sell. If he sells tomatoes at Shs. 250 each and yams at Shs. 3500 each, how much money would he receive if he sold all the tomatoes and yams that he bought?

4. Last week I dug up potato crop. On Tuesday I dug up twice as much as what I dug on Monday. On Wednesday I dug up three times as much as on Monday. On Thursday I dug up only half as much as I dug on Wednesday. On Friday I dug the same amount as on Tuesday. If I dug up P kg on Monday, how much did I dig up in the week?

5. A book has 120 pages. Each leaf is p mm thick, and the covers are each c mm thick. If the total thickness is T mm, complete the formula T = ……….

LESSON 2: CHANGING THE SUBJECT OF A FORMULA

LEARNING OUTCOME

By the end of this lesson, you should be able to change the subject of a formula.

INTRODUCTION

In the previous lesson we saw a formula as a mathematical statement with quantities represented by letters. For example the area of a rectangle is length times breadth given as; **A = lb,** **y = mx + c,**

In any formula, the single letter on the left e.g. A or y in the formulae above is called the **subject of the formula**.

The subject of any formula must occur only once and isolated on the left hand side of the equation. In $V = \frac{1}{3}\pi r^2 h$, V is the subject of the formula. V is expressed in terms of other letters. V should not appear again on the right hand side of the equation.

But in $x = \frac{b + cx}{A}$, x is not the subject of the formula since x appears on both sides, i.e. x is not isolated.

A formula can be rearranged so as to make another letter the subject. That process is called changing the subject of a formula.

When changing the subject of a formula i.e. express one letter in terms of the other letters we take note of the following;

- Do the reverse of the operations in the formula, if a letter is added we subtract it from both sides, if multiplied we divide it and vice versa.
- Square both sides if the letter is under square root sign
- Remove fractions by multiplying all the terms by the LCM of the denominators
- Arrange the terms containing the subject (required letter) on one side of the equation
- Factorise and divide by the coefficient of the subject (required letter)

Example 1

Make x the subject of the formula in (a) and (b)

(a) $y = mx + c$

re arrange the equation to keep the terms containing x on the LHS

$mx + c = y$

transfer c to the RHS of the equation

$mx = y - c$

divide both sides by m the coefficient of x

$\therefore x = \frac{y - c}{m}$.

(b) $\frac{m}{x} + p = q$

multiply all terms by x

$m + px = qx$

correct terms with x on one side LHS

$qx - px = m$

factorise x

$x(q - p) = m$

divide both sides by $(q - p)$ the coefficient of x

$$x = \frac{m}{q - p}.$$

Example 2

Make p the subject of the formula.

$$b = \frac{\sqrt{(5p - a)}}{h}$$

multiply h on both sides

$bh = \sqrt{(5p - a)}$

square both sides

$(bh)^2 = 5p - a$

$5p - a = (bh)^2$

$5p = a + (bh)^2$

$p = \dfrac{a + (bh)^2}{5}$.

Activity

Make the letters given in bracket the subject in the following formulae.

1. $v = u + at$ (t)
2. $a = \dfrac{b + 5c}{d}$ (c)
3. $R = \dfrac{P}{Q + 5}$ (Q)
4. $A = \pi r^2 h$ (r)
5. $G = b\sqrt{e} + m$ (e)
6. $y = \sqrt{\dfrac{x - a}{x - b}}$ (x).

TERM 3
TOPIC 1. QUADRATIC EQUATIONS

By the end of this topic, you should be able to:

i) solve quadratic equations using factorization, completing square and formula.
ii) make tables of values from given quadratic equations.
iii) solve quadratic equations using graphs.
iv) form quadratic equations from given roots.
v) form and solve quadratic equations from given situations.

Materials Required
You will need a pen/pencil and exercise book to take important notes and try out the activities.
You will need a simple calculator, pen/pencil and exercise book to take important notes and try out the activities.

LESSON 1: SOLVE QUADRATIC EQUATIONS USING FACTORISATION

LEARNING OUTCOME

By the end of this lesson, you should be able to solve quadratic equations by factorisation.

INTRODUCTION
You may recall expansion and factorization of algebraic expressions done in term two of senior two. We start by expanding the product of two brackets $(x+2)$ and $(x+3)$

$$(x+2)(x+3) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6$$

x is an unknown letter. The kind of expression like $x^2 + 5x + 6$ where the highest power of x is two is called a **quadratic expression**.
If we equate such an expression to zero, we get a **quadratic equation** like $x^2 + 5x + 6 = 0$.
We are going to **solve** by finding the possible values of the unknown letter (or **roots**) that satisfy such equations.
In the method of factorisation we use a property of zero.

If we have two numbers P and Q, and PQ=0,
Then, either P =0 or Q = 0 or both are zero.
We start by solving quadratic equations with **two terms**. You will realise that these are **easy to factorise**.

EXAMPLE 1
Solve the quadratic equation$x^2 + 7x = 0$.
$x^2 + 7x = 0$
$x(x + 7) = 0$
Using the property of zero stated above, either $x = 0$
Or $(x + 7) = 0$ so, $x = -7$
For quadratic equations with **three terms**, you have to apply the skills of factorizing three term quadratic expressions.

EXAMPLE 2
Solve the quadratic equation $x^2 + 5x + 6 = 0$.

$x^2 + 5x + 6 = 0$
Step 1. We identify the **coefficient** of the term with x^2(quadratic term) and the number term.
Confirm that they are 1 and +6 in that order.
Step2 We multiply them to get a product.
Confirm the product as 1 X 6 = 6.
Step 3. We identify two numbers with a sum equal to the coefficient of the term in x (confirm that it
is +5) and a product of 6. (You should confirm that in this example, the numbers are 2 and 3).
Step 4. We then split the x term using the numbers identified in step 3. (here, $5x=2x + 3x$).
Step 5. We then pair up terms so that we can factorize and get the roots by equating each factor to zero in turns.
Let us now put all these steps in an orderly flow of work.

$x^2 + 5x + 6 = 0$
$x^2 + (2x + 3x) + 6 = 0$
$(x^2 + 2x) + (3x + 6) = 0$
$x(x + 2) + 3(x + 2) = 0$
$(x + 3)(x + 2) = 0$

Either $(x+3)=0$ or $(x+2)=0$
Therefore $x=-3$ or $x=-2$

EXAMPLE 3

Solve the quadratic equation $2x^2-3x+1=0$.

Solution:

$2x^2-3x+1=0$

$2x^2+(-2x-x)+1=0$
$(2x^2-2x)-(x-1)=0$
$2x(x-1)-(x-1)=0$
$(2x-1)(x-1)=0$

Step	SIDEWORK		
1	Coeff. of x^2 term	is 2	
1	Number term	is 1	
2	Their product	Is 2X1	= 2
3&4	Split coeff. of x term	-3	= -2 + -1
3&4		-2 X -1	= 2

Either $(2x-1)=0$

Or $(x-1)=0$
Therefore $x=\frac{1}{2}$ or $x=1$

ACTIVITY:

1. Factorize the expression x^2+4x+3. Therefore, solve the quadratic equation $x^2+4x+3=0$
2. Solve the quadratic equations
   (a) $x^2-4x=0$
   (b) $x^2-3x-10=0$
   (c) $3x^2+5x+2=0$

LESSON 2: SOLVING QUADRATIC EQUATIONS USING COMPLETING SQUARES

LEARNING OUTCOME

By the end of this lesson, you should be able to:

INTRODUCTION

You may recall senior two works on perfect squares of two-term algebraic expressions. Here, we will form a perfect square on the left hand side (LHS) by **completing squares**. We can be guided by the following steps against an example.

EXAMPLE 1

(a) Complete squares in the expression $2x^2 + 4x + 1$.

Step		Example	comment
1	We collect terms having$x$ into one bracket	$(2x^2 + 4x) + 1$	
2	We factorise to leave x^2 in the bracket	$2(x^2 + 2x) + 1$	
3	We identify the coefficient of x and divide it by 2 to get a number.	2÷2 = 1	This number will be in the squared bracket.
4	We square the number obtained in step3 then add and subtract the result inside the bracket.	$2(x^2 + 2x + 1 - 1) + 1$	
5	We take the difference out of the bracket and combine it with the original number term.	$2(x^2 + 2x + 1) - 2 + 1$	
6	We form the square and simplify.	$2(x + 1)^2 - 1$	The completed square form.

Therefore $2x^2 + 4x + 1 = 2(x + 1)^2 - 1$

(b) *Use the form in (a) to solve the quadratic equation* $2x^2 + 4x + 1 = 0$.

We can now proceed to use the completed square to solve the quadratic equation.
$2x^2 + 4x + 1 = 0$
We write the new form $2(x + 1)^2 - 1 = 0$
We start putting (x + 1) on the LHS $2(x + 1)^2 = 1$
$(x + 1)^2 = \frac{1}{2}$
Taking the square root on both sides $(x + 1) = \pm\sqrt{\frac{1}{2}} = \pm 0.7071$
Taking the positive square root $(x + 1) = 0.7071$. $x = 0.7071 - 1 = -0.2929$
Taking the negative square root $(x + 1) = -0.7071$. $x = -0.7071 - 1 = -1.7071$

EXAMPLE 2: *Solve the quadratic equation* $x^2 + 4x - 5 = 0$.
$x^2 + 4x - 5 = 0$
$(x^2 + 4x) - 5 = 0$
$(x^2 + 4x + 4 - 4) - 5 = 0$
$(x^2 + 4x + 4) - 4 - 5 = 0$
$(x + 2)^2 - 9 = 0$
$(x + 2)^2 = 9$
$(x + 2) = \pm 3$
Take the positive square root $(x + 2) = 3$. $x = 3 - 2 = 1$
Take the negative square root $(x + 2) = -3$. $x = -3 - 2 = -5$

ACTIVITY:

1. (a) Express the quadratic expression $x^2 + 2x - 3$ in the form $A(x + B)^2 + C$.
   (b) Use the form in part (a) to solve the quadratic equation $x^2 + 2x - 3 = 0$.
2. Solve the quadratic equations;
   (a) $x^2 + 2x = 0$
   (b) $x^2 - 4x + 3 = 0$

LESSON 3: SOLVING QUADRATIC EQUATIONS USING THE QUADRATIC FORMULA

LEARNING OUTCOME

By the end of this lesson, you should be able to solve quadratic equations using the quadratic formula.

INTRODUCTION

You may have realised that quadratic equations are written in the form $ax^2 + bx + c = 0$. Where the unknown letter is *x* while *a*,*b* and c are numbers.
We can use the **quadratic formula** $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ to solve.

EXAMPLE 1: *Solve the quadratic equation* $2x^2 - 5x + 3 = 0$.

In this equation, we identify that *a* = *2*, *b* = *-5* and c = 3 .
Substituting these figures into the formula $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$.

Gives $x = \frac{-(-5)\pm\sqrt{(-5)^2-4\times2\times3}}{2\times2}$

$= \frac{5 \pm \sqrt{25-24}}{4}$

$= \frac{5 \pm 1}{4}$

Therefore, $x = \frac{5+1}{4} = \frac{6}{4} = \frac{3}{2}$ or $x = \frac{5-1}{4} = \frac{4}{4} = 1$

ACTIVITY:

Solve the quadratic equations:

(a) $x^2 + x - 2 = 0$.

(b) $3x^2 - 5x - 2 = 0$.

LESSON 4: MAKING TABLES OF VALUES FROM GIVEN QUADRATIC EQUATIONS

LEARNING OUTCOME.

By the end of this lesson, you should be able to make tables of values from given quadratic equations.

INTRODUCTION

Given a quadratic equation, we can draw a table by following the steps:
Step1. Fill the first row of the table you have drawn with equally spaced values of x in the range given.
Step2. Fill the first column of the table with the terms in the quadratic expression.
Step3. Compute the values of the terms using the corresponding value of x.
Step4. Find the sum of values in each column as y.
Step5. Pair up the corresponding values of x and y to give the coordinate.

EXAMPLE 1

Draw a table for the relation $x^2 + 2x - 3 = y$ *for values of x ranging from -4 to 2.*

Step 1	x	-5	-4	-3	-2	-1	0	1	2	3
Step 2	x^2	25	16	9	4	1	0	1	2	9 Step 3
	$2x$	-10	-8	-6	-4	-2	0	2	4	6
Step 4	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3
Step 5	y	12	5	0	-3	-4	-3	0	3	12
	Point	(-5,12)	(-4,5)	(-3,0)	(-2,-3)	(-1,-4)	(0,-3)	(1,0)	(2,3)	(3,12)

ACTIVITY:

Draw a table for the relation:
(a) $y = x^2 - x - 2$ for $-3 \leq x \leq 5$
(b) $2x^2 - 5x + 3 = y$. for $-3 \leq x \leq 4$

LESSON 5: SOLVING QUADRATIC EQUATIONS USING GRAPHS

LEARNING OUTCOME

By the end of this lesson, you should be able to solve quadratic equations using graphs.

INTRODUCTION

In lesson 4 of this topic, you learnt how to make tables of quadratic relations. We are going to use such tables to plot points and draw graphs. These graphs will then help us to solve the given quadratic equations.

EXAMPLE 1

(a) Draw a table for the relation $y = x^2 - 3x + 2$ *for* $-1 \leq x \leq 4$

(b) Use the table to draw a graph of the relation $y = x^2 - 3x + 2$.

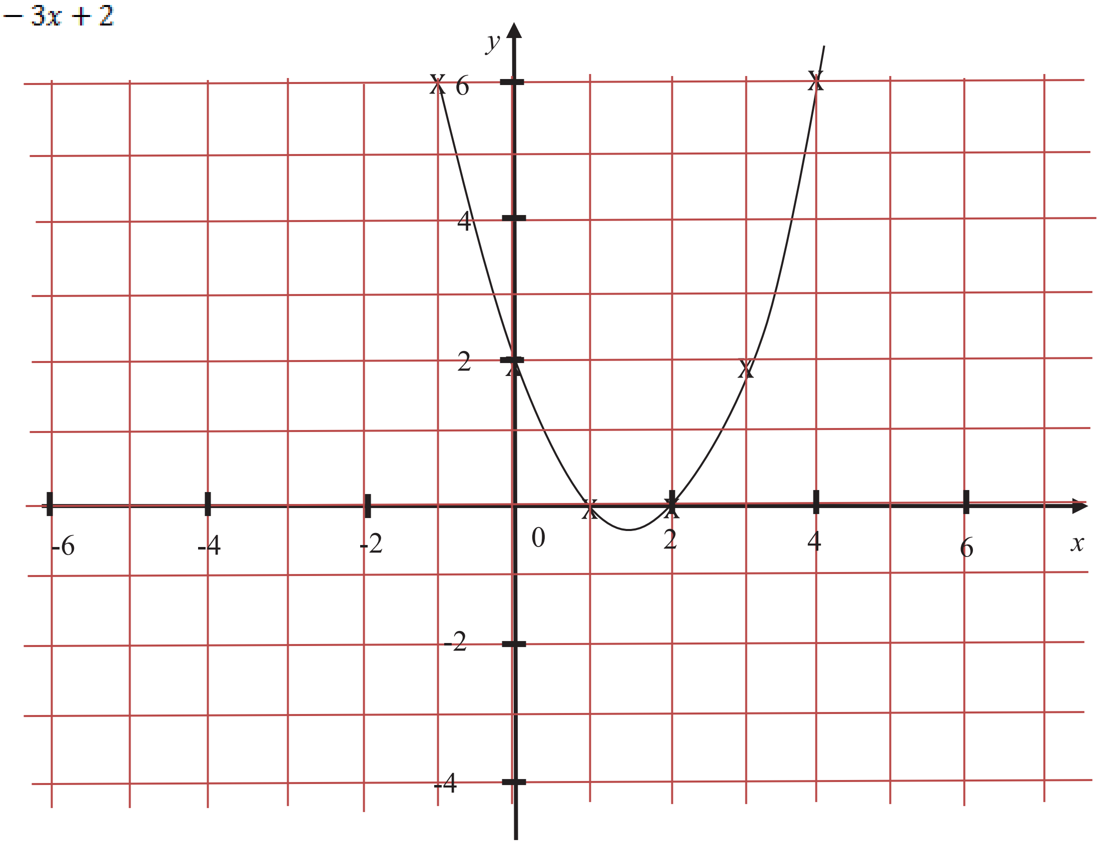
(c) Use the graph in part (b) to solve the quadratic equation $x^2 - 3x + 2 = 0$.

(a)

x	-1	0	1	2	3	4
x^2	1	0	1	4	9	16
$-3x$	3	0	-3	-6	-9	-12
2	2	2	2	2	2	2
y	6	2	0	0	2	6
Point	(-1,6)	(0,2)	(1,0)	(2,0)	(3,2)	(4,6)

(b) We can now plot the points got in the last row of the table in part (a) on the Cartesian axes. (please use sharp pencil to plot the points with X or ʘ signs then join them with a smooth curve).

$y = x^2 - 3x + 2$



(c) To solve the equation $x^2 - 3x + 2 = 0$, you read off the values of x where the graph of the quadratic relation $y = x^2 - 3x + 2$ cuts the x-axis (or where y = 0).

Confirm that for this case they are <u>x = 1 and x = 2</u>. You can check these answers by using factorisation or the quadratic formula.

EXAMPLE 1

(a) Plot a graph of $x^2 - 2x - 3 = y$ *for* $-2 \le x \le 4$

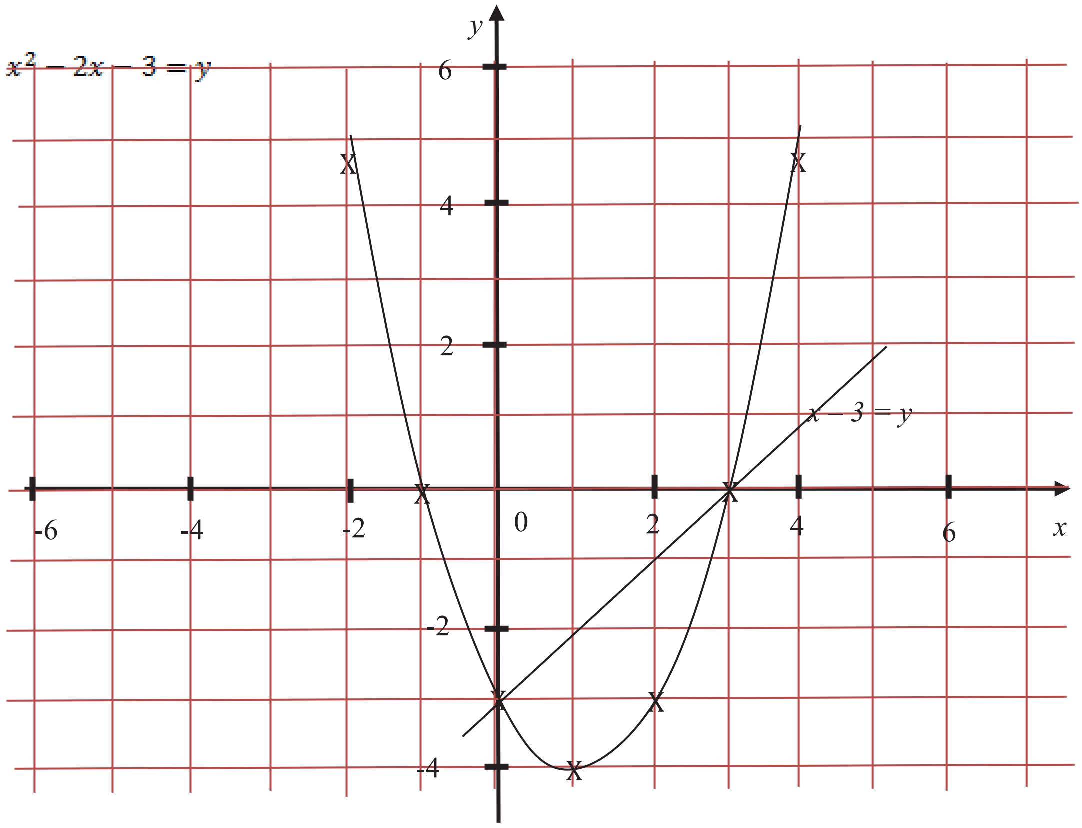
(b) Use the graph to solve the quadratic equation

(i) $x^2 - 2x - 3 = 0$.

(ii)

$x^2 - 3x = 0$

x	-2	-1	0	1	2	3	4
x^2	4	1	0	1	4	9	16
-2x	4	2	0	-2	-4	-6	-8
-3	-3	-3	-3	-3	-3	-3	-3
y	5	0	-3	-4	-3	0	5
Point	(-2,5)	(-1,0)	(0,-3)	(1,-4)	(2,,-3)	(3,0)	(4,5)



The curve cuts the x-axis at x = -1 and x = 3.
Therefore, the roots are <u>x = -1 and x = 3.</u>

(b) In this part, you should note that the quadratic part of the equation is different from that of the relation of the graph.

We form the equation of a line which crosses the curve. The values of x where they cross give the roots of the equation.

$$\begin{array}{l} (x^2-2x-3=y) \\ \underline{-(x^2-3x=o)} \\ \quad x-3=y \end{array}.$$

So the line is $-3 = y$.

x	0	2
y	-3	-1
point	(0,-3)	(2,-1)

We plot the two points and draw the line to pass through them.
Confirm that this line and the curve cross at x = 0 and x = 3.

Therefore, the roots are $x = 0$ and $x = 3$.

ACTIVITIES

1. (a) Draw a table for the relation $y = x^2 - 4x + 3$ for $-1 \le x \le 5$
   (b) Use the table to draw a graph of the relation $y = x^2 - 4x + 3$.
   (c) Use the graph in part (b) to solve the quadratic equation $x^2 - 4x + 3 = 0$.

2. The table below is for the relation $y = 2x^2 - 3x - 5$.

x	-3	-2	-1	0	1	2	3	4
$2x^2$								
$-3x$								
-5								
y								
Points								

(a) Use the results in the table to draw a graph of the relation $y = 2x^2 - 3x - 5$.
(Use a scale of 1cm: 1unit)

(b) Use the graph to: -

(i) find the minimum value of y.
(ii) find the line of symmetry of the graph.
(iii) solve the quadratic equation $2x^2 - 3x - 5 = 0$.
(iv) solve the quadratic equation $2x^2 - 5x + 2 = 0$.

LESSON 6: FORMING QUADRATIC EQUATIONS FROM GIVEN ROOTS

LEARNING OUTCOME

By the end of this lesson, you should be able to form quadratic equations from given roots.

INTRODUCTION

You may recall the property of zero that we used to solve quadratic equations by factorisation.
We are going to use this property to find quadratic equations given the roots.

EXAMPLE 1

Find the quadratic equation whose roots are 1 and 2.

We let $x = 1$ therefore, $x - 1 = 1 - 1$ giving $x - 1 = 0$.
or $x = 2$ therefore, $x - 2 = 2 - 2$ giving $x - 2 = 0$.

Multiplying them, $(x - 1)(x - 2) = 0$.

$$x^2 - 2x - x + 2 = 0$$
$$x^2 - 3x + 2 = 0$$

ACTIVITY

Find the quadratic equation whose roots are:

(a) 3 and 4
(b) -2 and 1
(c) $\frac{3}{2}$ and 5

LESSON 7: USING QUADRATIC EQUATIONS TO SOLVE SOME REAL LIFE SITUATIONS

LEARNING OUTCOME

By the end of this lesson, you should be able to use quadratic equations to solve real life situations.

INTRODUCTION

Do you know of some situations whose solutions require forming and solving quadratic equations? We have to build a formula by choosing a letter to represent an unknown quantity and forming the situation.

For Example

A boy is older than his sister by 2 years. If the product of their ages is 15, find the age of the boy.

Let the age of the boy be x.

The age of the sister will be $x - 2$.

$x(x - 2) = 15.$

$x^2 - 2x = 15$

$x^2 - 2x - 15 = 0$

$x^2 + 3x - 5x - 15 = 0$

$(x^2 + 3x) - (5x + 15) = 0$

$x(x + 3) - 5(x + 3) = 0$

$(x - 5)(x + 3) = 0$

Either $(x - 5) = 0$ giving $x = 5$

or $(x + 3) = 0$ giving $x = -3$ (there is no -3years) so the boy is 5years old.

ACTIVITY

1. A rectangle has an area of 48cm^2. If the length is 2cm longer than the width, find the dimensions.
2. Two positive numbers have a difference of 2 and the sum of their squares is 10. Find the numbers.
3. Gundi is 10 years old and Jal is 7 years old. How many years back was the product of their ages equal to 35?

TOPIC 2. CIRCLE PROPERTIES

LEARNING OUTCOMES.

By the end of this topic, you should be able to:

(i) identify arc, chord, sector and segment.
(ii)relate angle made by an arc at the centre and at the circumference.
(iii) relate angles in the same segment.
(iv)state the angle in a semi-circle.
(v)state the properties of a cyclic quadrilateral.
(vi)find the length of a tangent from a point to a circle.
(vii) find the length of a common chord.
(viii)calculate the area of a sector and a segment.

MATERIALS:
You will need a pen, mathematical set, simple calculator and exercise book to take important notes and try out the activities.

LESSON1: IDENTIFYING ARC, CHORD, SECTOR AND SEGMENT

LEARNING OUTCOME

By the end of this lesson, you should be able to arc, sector and segment of a circle

INTRODUCTION.

We have at one time or another, ever drawn circles. In this topic we will discover more about circles by drawing .

ACTIVITY 1
Draw a circle with a radius of 3cm.
Mark the centre as O.
Mark two points A and B on the circumference of the circle.
Draw a line joining the points A and B.
Draw the radii OA and OB.

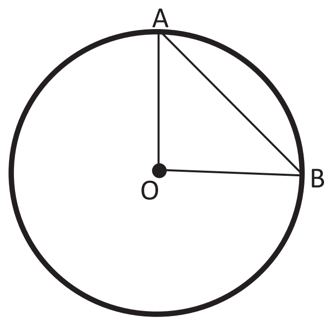


Figure 1

OBSERVATIONS
Your circle looks like figure 1.
We can now label the parts.

Straight line AB is a **chord**. Any chord passing through the centre is a **diameter**. Part of the circumference, AB is an **arc**. From B to A anticlockwise is a **minor (shorter) arc** and clockwise is a **major (longer) arc**.

AOB with the curved side is a **sector**. The larger one is a **major sector**. The smaller is a **minor sector**.
The part bound by an arc and a chord is called a **segment**. A chord divides a circle into a **minor segment** and a **major segment**.

Angle AOB is the angle made (or subtended) by the chord AB at the centre.

LESSON 2: IDENTIFYING ANGLES MADE BY AN ARC AT THE CIRCUMFERENCE

LEARNING OUTCOME

By the end of this lesson, you should be able to identify angles made by an arc at the circumference of a circle.

ACTIVITY 2
Draw a circle with a radius of 3cm.
Mark the centre as O.
Mark four points A, B, C and D, in that order, on the circumference of the circle.
Draw lines CA, CB, DA and DB.

Measure and record angles ACB and ADB. (These are **angles at the circumference**).
Your draw resembles figure 2
Compare the angles and make a conclusion.

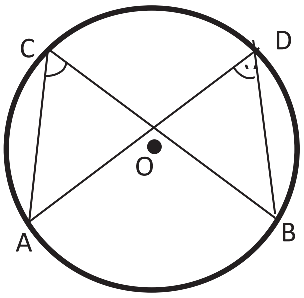


Figure 2

OBSERVATION
Angles made by an arc at the circumference are equal.

ACTIVITY 3

Figure 3 shows a circle with centre at O. Find angle x.

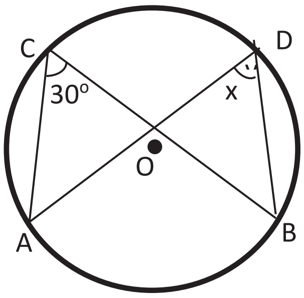


Figure 3

LESSON 4: RELATING ANGLE MADE BY AN ARC AT THE CENTRE AND AT THE CIRCUMFERENCE

LEARNING OUTCOME.

By the end of this lesson, you should be able to relate angles made by an arc at the centre and at the circumference of a circle.

ACTIVITY 4

Draw a circle with a radius of 3cm.
Mark the centre as O.
Mark three points A, B and C, in that order, on the circumference of the circle.
Draw lines CA, CB, OA and OB.
Your circles resembles Figure 4.

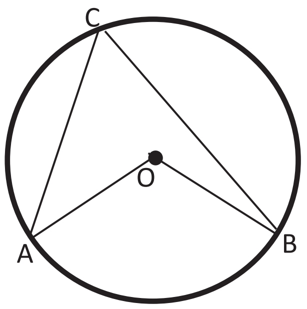


Figure 4

Measure and record angles ACB and AOB.
Compare these angles and make a conclusion.

OBSERVATION

Angle subtended by an arc or chord at the centre is twice the angle subtended by the same arc or chord at the circumference.

ACTIVITY 5

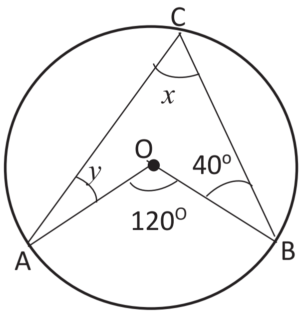


Figure 5

Figure 5 shows a circle with centre at O. find the angles x and y.

LESSON 5: TO STATE THE ANGLES IN SEMI-CIRCLES.

LEARNING OUTCOME:

By the end of this lesson, you should be able to state the angles in semi-circles.

INTRODUCTION;

Lets discover the angle in a semicircle.

ACTIVITY 6

Draw a circle with a radius of 3cm.
Mark the centre as O.
Draw a diameter and mark its ends A and B.
Mark a point C on the circle.
Draw lines AC and BC.
Your circle should resemble figure 6.

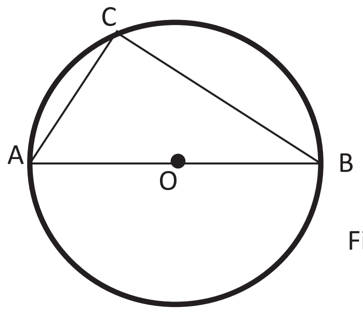


Figure 6

Measure and record angle ACB and make a conclusion.

OBSERVATION

Angle in a semi-circle is a right angle. (Or angle made by the diameter at the circumference is 90^o)

ACTIVITY 7

Figure 7 show a circle with centre at O.

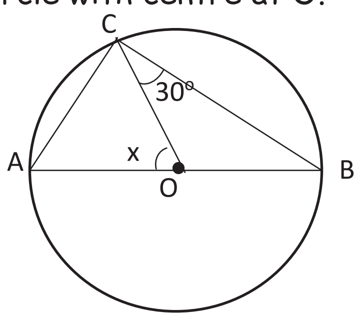


Figure 7

Find the angle x.

LESSON 6: STATE THE PROPERTIES OF A CYCLIC QUADRILATERAL

LEARNING OUTCOME

By the end of this lesson, you should be able to state the properties of a cyclic quadrilateral.

ACTIVITY 8

Draw a circle with a radius of 3cm.
Mark the centre as O.
Mark four points A, B, C and D, in that order, on the circumference of the circle.
Draw lines AB, BC, CD and DA.
(The quadrilateral formed is called a **cyclic quadrilateral**).

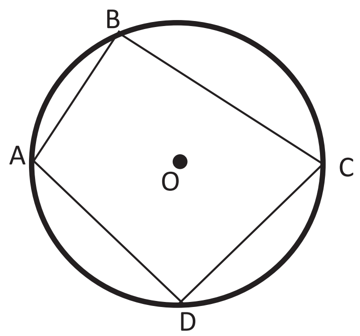


Figure 8

Measure and record angles ABC and ADC. Find their sum.
Measure and record angles BCD and BAD. Find their sum.
Make a conclusion.

OBSERVATION

Opposite angles of a cyclic quadrilateral add up to 180°. (Or they are supplementary).

ACTIVITY 9

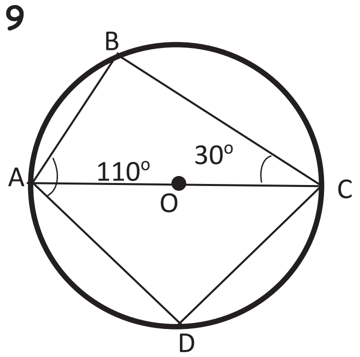


Figure 9

Figure 9 shows a circle with the centre at O. Find angle ACD.

LESSON 7: FINDING THE LENGTH OF A TANGENT FROM A POINT TO A CIRCLE

LEARNING OUTCOME

By the end of this lesson, you should be able to find the length of a tangent from a point to a circle.

INTRODUCTION

Any line touching a circle is called a tangent to the circle.

ACTIVITY 10

Draw a circle with a radius of 3cm.
Mark the centre as O.
Draw a tangent to touch the circle at a point A.
Mark a point B along the tangent such that AB=4cm.
Draw radius OA and measure angle OAB.
A tangent to a circle is always perpendicular to the radius from the point of contact.

Measure and record the distance of point B from the centre of the circle.

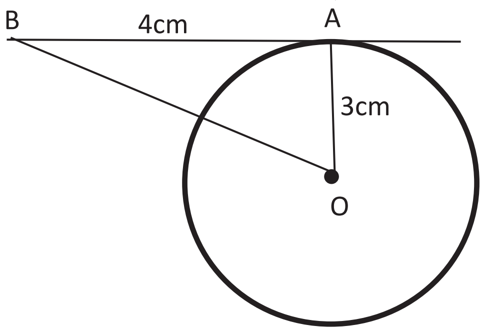


Figure 10

Use figure 10 to find length OB and compare the values.

ACTIVITY 11

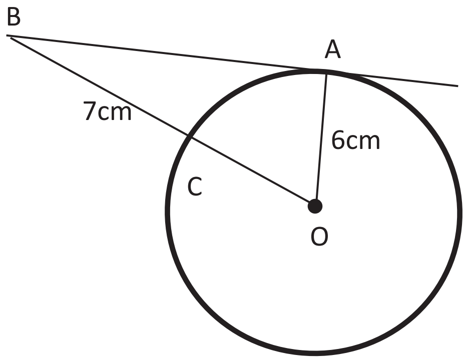


Figure 11

Figure 11 shows a circle of radius 6cm with centre at O. If BA is a tangent to the circle and BC = 7cm, find the length BA.

LESSON 6: FINDING THE LENGTH OF A COMMON CHORD

LEARNING OUTCOME

By the end of this lesson, you should be able to find the length of a common chord.

INTRODUCTION.

When two circles meet at two points. The line joining these two points is a shared(or common) chord.

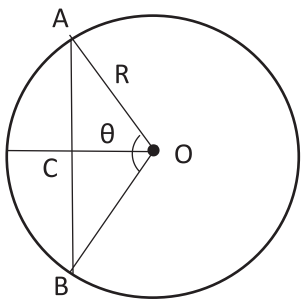


Figure 12

Let us consider a circle of radius, R with a chord AB making an angle θ at the centre. The length of the chord AB = 2 x AC. (Figure 12)
AC can be got by using trigonometry as $= R\sin\frac{\theta}{2}$.
Or by using Pythagoras theorem as $AC = \sqrt{R^2 - (OC)^2}$

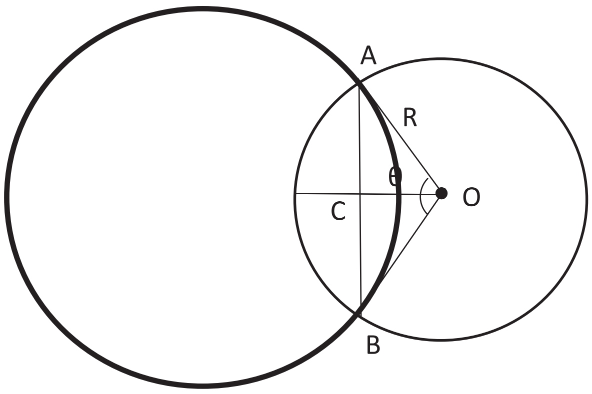


Figure 13

If two circles intersect at points A and B the chord AB is called a common chord. Its length can be obtained in the same way as with the chord of a circle.

ACTIVITY 12

1.

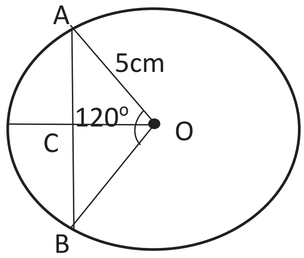


Figure 14

Figure 14 shows a circle of radius 5cm. Chord AB makes an angle of 120° at the centre. Find the:

(a) length of chord AB.

(b) distance of the chord from the centre of the circle.

2.

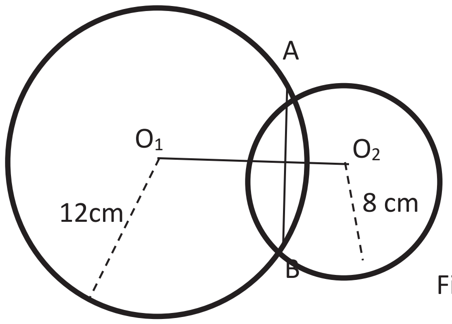


Figure 15

Figure 15 Shows two circles with centres at O_1 and O_2 and radii 12cm and 8cm respectively. If the angles $AO_1B = 60^O$ and $AO_2B = 98^O$, find the:

(a) length of the common chord AB.

(b) distance between the centres of the circles.

LESSON 7: CALCULATE THE AREA OF A SECTOR AND A SEGMENT

LEARNING OUTCOME

By the end of this lesson, you should be able to calculate the area of a sector and a segment.

INTRODUCTION

(a) Area of a sector.

You may recall that a sector is a fraction of a circle.

If a sector subtends an angle θ at the centre of a circle of radius R,

Then the **area of the sector is given by** $\frac{\theta}{360} \times \pi R^2$

(b) Area of a segment.

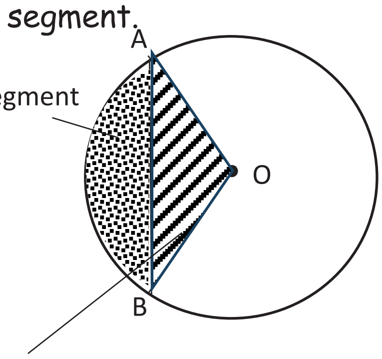


Figure 16

Area of triangle AOB$= \frac{1}{2} R^2 \sin\theta$

Area of a segment on chord AB = Area of its sector AOB– Area of the triangle AOB.

EXAMPLE

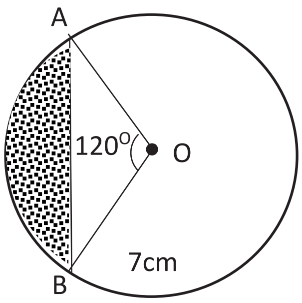


Figure 17

Figure 17 shows a circle of radius 7cm. If O is the centre, find the area of the shaded region.

Area of sector AOB= $\frac{120^o}{360^o} \times \pi \times 7^2 = 51.3cm^2$
Area of triangle AOB $= \frac{1}{2} \times 7^2 \times sin120^o = 21.2cm^2$
Area of segment = 51.3 – 21.2 = 30.1cm^2

EXAMPLE

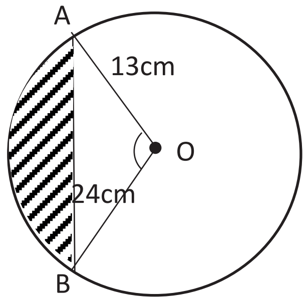


Figure 18

Figure 18 shows a circle of radius 13cm and centre at O. Find the area of the shaded region.

Height of the triangle = $\sqrt{13^2 - 12^2} = \sqrt{169 - 122} = 5cm$
Area of triangle$\frac{1}{2} \times 24 \times 5 = 60cm^2$
Let the angle at the centre be β. $cos\theta = \frac{5}{13} \Rightarrow \theta = cos^{-1}\left(\frac{5}{13}\right) = 67.4^o$. Therefore, β = 2 X 67.4°=134.8°
Area of sector AOB= $\frac{134.8^0}{360^0} \times \pi \times 13^2 = 198.7cm^2$
Shaded area = 198.7 – 60= 138.7cm^2

ACTIVITY 13

1. Find the shaded area in Figure 19.

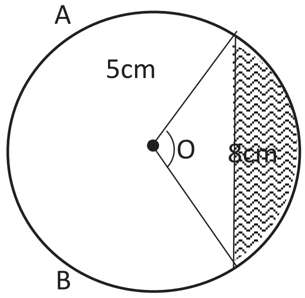


Figure 19

2.

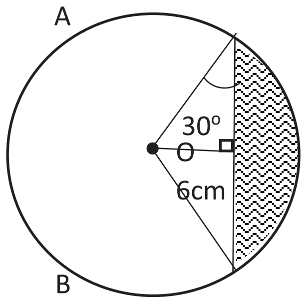


Figure 20

Find the shaded area in Figure 20.

TOPIC 3. TRIGONOMETRY

LEARNING OUTCOMES

By the end of this topic you will be able to:

(i) State the difference between angle of depression and angle of elevation.
(ii)Apply the knowledge of trigonometric ratios to find angles of elevation and depression.
(iii) Apply the knowledge of trigonometrical ratios to real life situations.

Materials required
You will need a pen, mathematical set, simple calculators or log book and exercise book to take important notes and try out the activities.

LESSON 1: STATING THE DIFFERENCE BETWEEN ANGLE OF DEPRESSION AND ANGLE OF ELEVATION

LEARNING OUTCOME

By the end of this lesson, you should be able to state the difference between angle of depression and angle of elevation.

INTRODUCTION

(a) Angle of elevation.

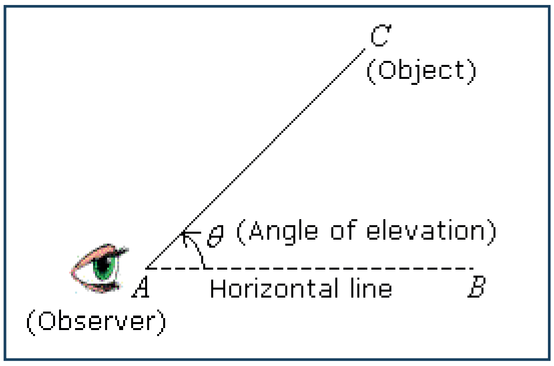


Figure 1

When you look straight ahead, your line of vision is along a **horizontal line**. To view an object high up, you need to change your line of vision **upwards** through an angle. This angle is called the **angle of elevation**. This is illustrated in figure 1.

(b) Angle of depression.

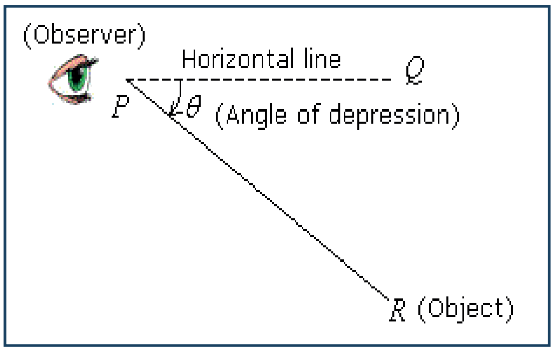


Figure 2

When you look straight ahead, your line of vision is along a **horizontal line**. To view an object low down, you need to change your line of vision **downwards** through an angle. This angle is called the **angle of depression**. This is illustrated in figure 2.

ACTIVITY

You can now go outside and identify objects high up or below and try to identify the angles of elevation or depression.

LESSON 2: APPLYING THE KNOWLEDGE OF TRIGONOMETRICAL RATIOS TO FIND ANGLES OF ELEVATION AND DEPRESSION

LEARNING OUTCOME

By the end of this lesson, you should be able to apply the knowledge of trigonometrical ratios to find agles of elevation and depression.

INTRODUCTION

To apply what we have just learnt, clear sketches will help us to come up with a solution. You may also have to recall and use your knowledge of trigonometrical ratios.

EXAMPLE

A bird rests on a horizontal ground 10m from a vertical tree. It then flies straight to the top of the tree along an angle of elevation of 60°. Find the height of the tree.

<u>Sketch</u>

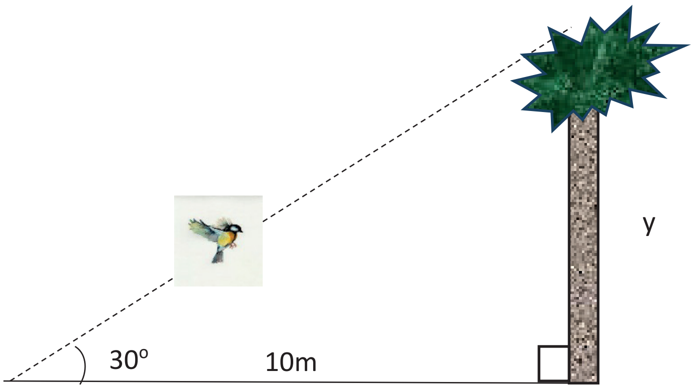


Figure 3

We let the height of the tree to be y.

From the sketch, $tan30^o = \frac{y}{10}$

$10 \times tan30^o = 10 \times \frac{y}{10}$

$y = 10 \times tan30^o = 10 \times 0.5774 = 5.774m$

EXAMPLE

From the top of a vertical building 20m tall, the angle of depression of a mark on a horizontal ground is 18°. Find the distance of the mark from the foot of the building.

<u>Sketch</u> (You will notice that angles of elevation and depression are equal)

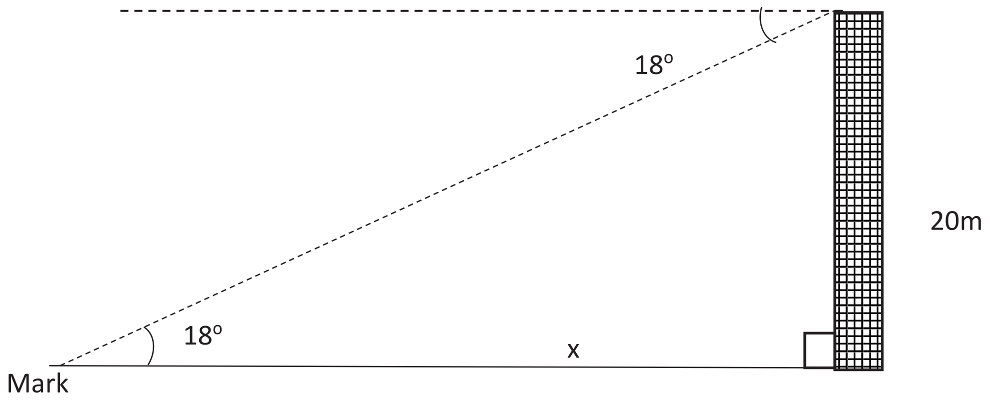


Figure 4

We let the distance of the mark from the building be x.

From the sketch, $tan18^o = \frac{20}{x}$

$$x \times tan18^o = x \times \frac{20}{x}$$

$$x = \frac{20}{tan18^o}$$

$$x = \frac{20}{tan18^o} = 61.6m$$

ACTIVITY

A bird originally resting on a horizontal ground flies 10m along a straight path to the top of a tree 7m tall. Find the angle of depression of the original position of the bird from the top of the tree.

LESSON 3: APPLYING THE KNOWLEDGE OF TRIGONOMETRICAL RATIOS TO REAL LIFE SITUATION

LEARNING OUTCOME

By the end of this lesson, you should be able to apply the knowledge of trigonometrical ratios to real life situations.

INTRODUCTION

EXAMPLE. *A vertical tower 35m tall stands by a lakeshore. The angle of depression of a ship from the top of the tower is initially 10º. After 10s, the angle of depression is 5º. Find the:*

(a) initial position of the ship from the foot of the tower.
(b) speed of the ship.

Sketch (Angle of depression is equal to the angle of elevation)

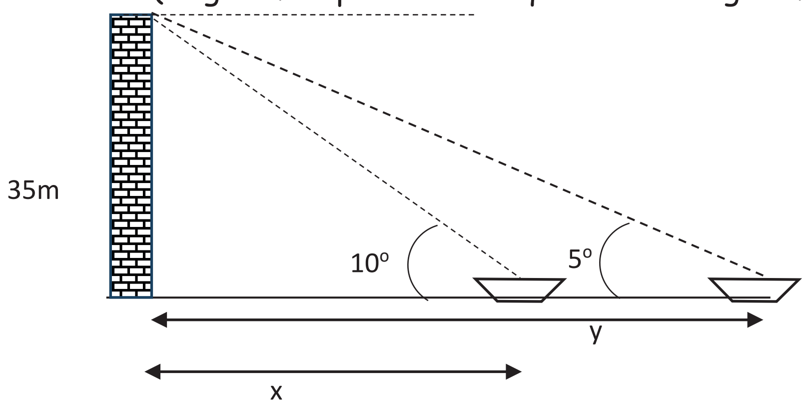


Figure 5

(a) We let the initial distance be x and the final distance be y.

From the sketch, $tan10^o = \frac{35}{x}$

$$x \times tan10^o = x \times \frac{35}{x}$$

$$x = \frac{35}{tan10^o}$$

$$x = \frac{35}{tan10^o} = 198.5m$$

(b)

From the sketch, $tan5^o = \frac{35}{y}$

$$y \times tan5^o = y \times \frac{35}{y}$$

$$y = \frac{35}{tan5^o}$$

$$x = \frac{35}{tan5^o} = 400.1m$$

Change in distance = 400.1- 198.5 = 201.6m

Speed $= \frac{distance}{time} = \frac{201.6}{10} = 20.16ms^{-1}$

ACTIVITY

1. Two buildings A and B are along a straight line with a telephone mast. The telephone mast is in between them at distances are 50m and 70m respectively. If the angle of elevation of the top of the mast from A is 16°, find the:
   (a) height of the mast.
   (b) angle of depression of B from the top of the mast.
2. A boy 1.5m tall stands 55m from a vertical tree and finds that the angle of elevation of the top of the tree is 20°. Find the height of the tree.

TOPIC: TRIGONOMETRY

By the end of this topic, you should be able to:

i) state units of measures.
ii) convert units from one form to another.
iii) calculate the surface area of three-dimensional figures (prism, cylinder, right pyramid, sphere, right cone,
iv) calculate the volume of some figures (e.g. cubes and pyramids.

MATERIALS REQUIRED

You will need a pen/pencil, mathematical set, simple calculator mathematical tables and exercise book to take important notes and try out the activities.

LESSON 1: STATING THE UNITS OF MEASURE AND CONVERTING FROM ONE FORM TO ANOTHER

LEARNING OUTCOME.

By the end of this lesson, you should be able to state the units of measure and convert from one to another.

INTRODUCTION

In the work we are going to do, we need **units of measure to give meaning to the final figures**. For example, if you are to buy a piece of land advertised as '3 at 4.5 millions'. This statement is not clear. But '3 km^2 for 4.5 millions' makes it clear. The units of measure of km^2 give meaning to the number 3.

A. **Units of area**. Calculations of area involves the multiplication (or product) of two distances.

ACTIVITY 1

Draw a rectangle of length 4cm and width 3cm.
Calculate the area of the rectangle you have drawn. (Remember to put the units)
How have you come up with that unit you put?
Repeat these three activities above with a circle of radius 3cm and a square of side 4cm.

You may have noticed why cm^2 becomes a unit of area. So, we will frequently use **cm^2**.

<u>Converting</u>

We can also use m^2. $1m^2 = 1m \times 1m = 100cm \times 100cm = 1.0 \times 10^4\ cm^2$

$1cm^2 = 1cm \times 1cm = \frac{1}{100} \times \frac{1}{100} = 1.0 \times 10^{-4} m^2$

EXAMPLE

Convert:

(a) $30m^2$ to cm^2.

(b) $480cm^2$ to m^2.

(a) $30m^2 = 30 \times 10^4 cm^2$

(b) $480cm^2 = 480 \times 10^{-4}\ m^2$

ACTIVITY 2

Convert the following:

(a) $2300cm^2$ to m^2.

(b) $46m^2$ to cm^2.

B. **Units of volume**. Calculations of volume involve the multiplication (or product) of three distances.

ACTIVITY 3

Draw a rectangular box of length 4cm, width 3cm and height 2cm. (or with dimensions 2cmX3cmX4cm).

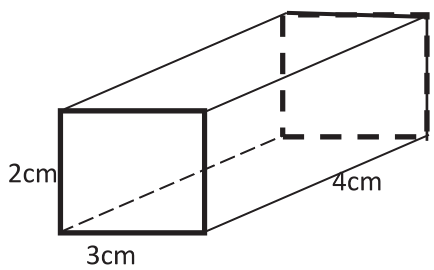


Figure 1

Notice that your box looks like figure 1. (The dotted lines represent the edges that are hidden behind the box as you may view from one side.

Calculate the volume of the rectangle you have drawn. (Remember to put the units)

How have you come up with the unit you have put?

You may have noticed why cm^3 becomes a unit of volume. So, we will frequently use **cm^3 or cubic cm (c.c.)**

You may have heard references in measurement such as 2000c.c.

In day to day life, litres are commonly used.

You may have gone to school with / carried a 20 litre jerrycan, bought a litre of milk etc.

ACTIVITY 4

When you find time, you can look around your home and identify the types of volume measures.

<u>Conversion</u>

$1cm^3$ = 1cm X 1cm X 1cm $\frac{1}{100}m \times \frac{1}{100}m \times \frac{1}{100}m = 1.0 \times 10^{-6}m^3$

$1m^3$ = 100cm X 100cm X100cm = 1.0×10^6 cm^3.

1 litre = $1000cm^3$.

$1m^3$ = 1000 litres

EXAMPLE

Convert:

(a) $24cm^3$ to m^3.

(b) $3.25m^3$ to cm^3.

(c) $283000cm^3$ to litres.

(a) $24cm^3 = 24 \times 1.0 \times 10^{-6} = 24 \times 10^{-6}\ m^3$.

(b) $3.25m^3 = 3.25 \times 1.0 \times 10^6 = 3.25 \times 10^{6}$.

(c) $283000cm^3 = \frac{283000}{1000} = 283 litres$.

ACTIVITY 5

1. Convert the following:

(a) $78cm^3$ to m^3.

(b) $42.11m^3$ to cm^3.

(c) $3560000cm^3$ to litres.

2. $1m^3$ of water costs 3200 Uganda shillings. Find the cost of
(a) $15m^3$ of water
(b) 20 litres of water.

LESSON 2: CALCULATING THE SURFACE AREA OF THREE-DIMENSIONAL FIGURES

LEARNING OUTCOME

By the end of this lesson, you should be able to calculate the surface area of three- dimensional figures.

INTRODUCTION

We find the surface area of three- dimensional figures by adding the areas of all its faces.

1. **Surface area of prisms.**

   A prism is a solid having identical and parallel ends or bases.
   Let us imagine we have a loaf of cake with triangular ends as shown in figure 2.

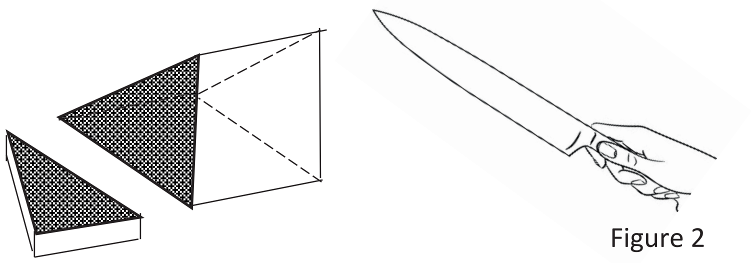


Figure 2

If we cut a slice section parallel to the ends, we get congruent shaded faces. Such a face is called a **cross-section**. Any plane cut made parallel to the ends produces a cross-section of the same shape. We call such a solid a **prism**. Prisms are identified by the shapes of their cross-section.
When the edges join the end faces at right angles, the prism is a **right prism.**
Do you know of any prism?

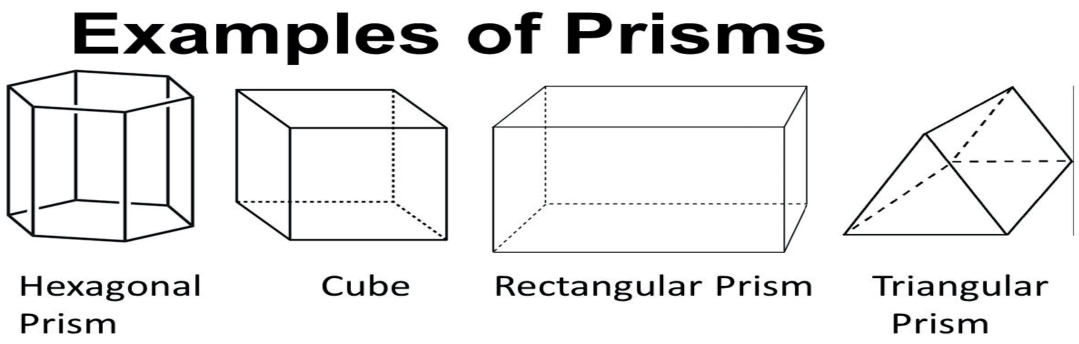


Figure 3

EXAMPLE

Figure 4 shows a right triangular prism.

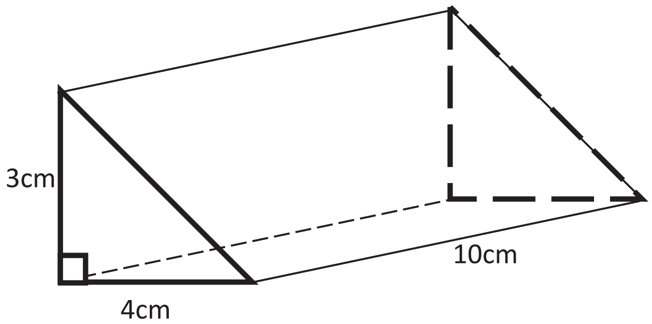


Figure 4

Find the surface area.

Area of the two ends $= 2 \times \left(\frac{1}{2} \times base \times height\right) = 2 \times \left(\frac{1}{2} \times 4 \times 3\right) = 12cm^2$

Area of base = length X width = 10 X 2 = 20cm^2.

Area of vertical face = length X width = 10 X 2 = 20cm^2.

Width of slanting face = $\sqrt{3^2 + 4^2} = 5cm$

Therefore, area of slanting face length X width = 10 X 5= 50cm^2.

Total surface area = 12 + 20 +20 + 50 = 102cm^2.

ACTIVITY 6

Find the surface area of each of the following prisms.

1.

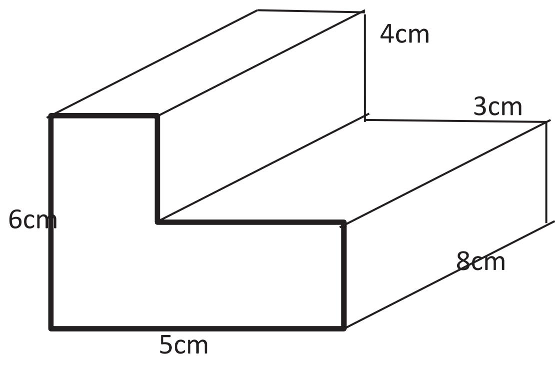


Figure 5

2.

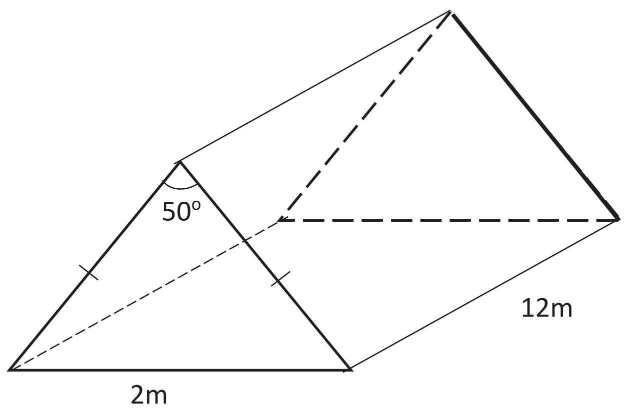


Figure 6

3.

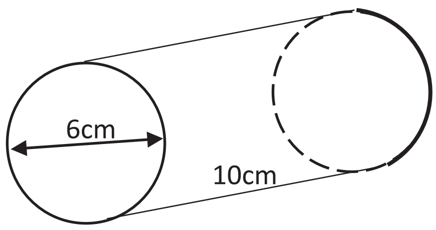


Figure 7

2. **Surface area of pyramids.**
   In Term 3 of Senior 2, you came across nets of solids. The pyramids were among them. They have a **base** and an **apex (vertex)** above the base. We

name them according to the shape of the base. A pyramid where the vertex is directly above the middle of the base is a **right pyramid.** E.g. in figure 8

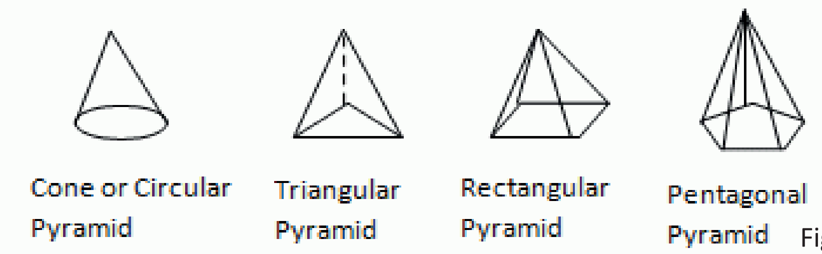


Figure 8

EXAMPLE

Figure 9 shows a rectangular pyramid.

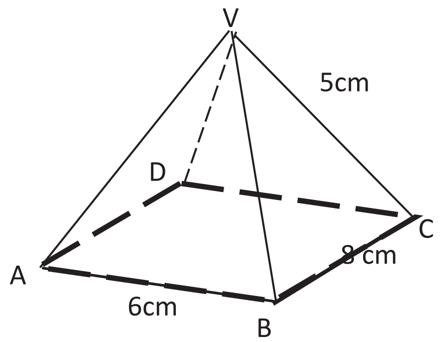


Figure 9

Find the surface area of the pyramid.

Height of triangular side AVB = $\sqrt{5^2 - 3^2} = 4cm$

Height of triangular side BVC = $\sqrt{5^2 - 4^2} = 3cm$

Area of opposite triangular faces AVB and DVC $= 2 \times \left(\frac{1}{2} \times 6 \times 4\right) = 24cm^2$

Area of opposite triangular faces BVC and DVA $= 2 \times \left(\frac{1}{2} \times 8 \times 3\right) = 24cm^2$

Area of the base = 6 X 8 = 48 cm^2

Total area = 24 + 24 + 48 = 96cm^2.

ACTIVITY 7

(a) Draw the sketch of a square pyramid with slant height of 13cm and a side of the base at
10cm.

(b) Imagining it is solid, find the surface area.

3. **Surface area of cones**.
   We add up the area of the circular base and that of the curved surface.

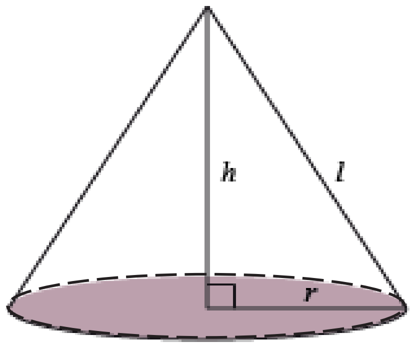


Figure 10

Let us consider a right circular cone of base radius r , height, h and slant height l as shown in figure 10.
Surface area = area of circular base + area of curved side.
$= \pi r^2 + \pi rl. = \pi r(r + l)$

EXAMPLE. *Find the surface area of a cone with a base radius of 5cm and slant height of 13cm.*

Surface area = $\pi r(r + l)$

$= \pi 5\,(5 + 13) = 282.74\text{cm}^2$

ACTIVITY 8
Find the surface area of a right solid cone of base radius 12cm and vertical height 15cm.

4. **Surface area of frustum.**
   When a top part of a cone is cut parallel to the base and removed, the remaining lower section is called a **frustum.** This is illustrated in Figure 11.

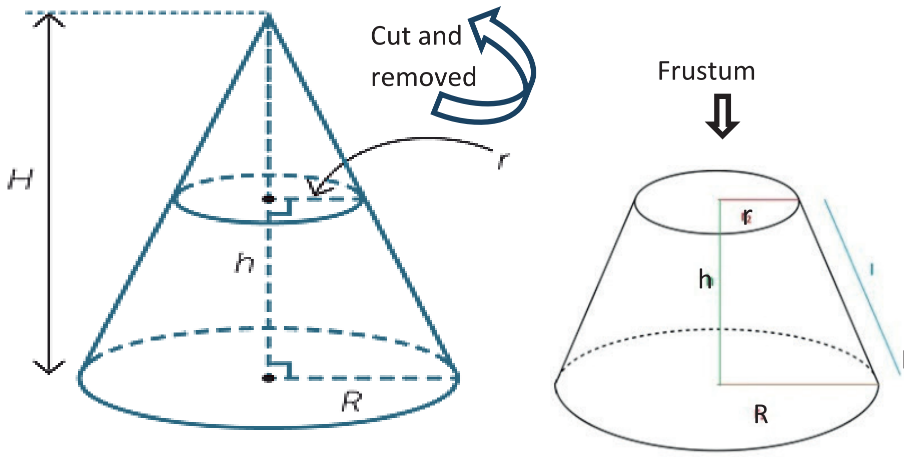


Figure 11

The surface area = (curved area of original cone) – (curved area of removed cone) + (area of the circular ends).

EXAMPLE

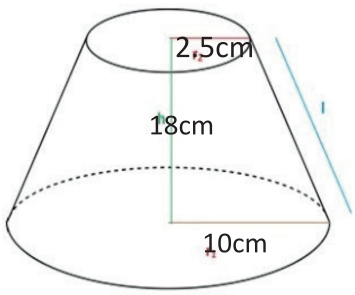


Figure 12

Figure 12 shows a solid frustum with radii of the ends as 2.5cm and 10cm respectively. If the height is 18cm, find the surface area.

We take a vertical section and complete the cone as shown in figure 13. Let the height of the removed cone be *x*.

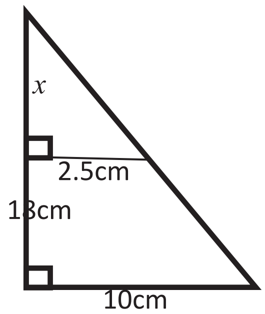


Figure 13

We now find x by using similar triangles.

$$\frac{x}{18+x} = \frac{2.5}{10}$$
$$4x = 18 + x$$
$$3x = 18$$
$$x = 6cm$$

We then find the slant heights by using Pythagoras theorem
For the original cone, $L = \sqrt{10^2 + 24^2} = 26cm$
For the removed cone, $l = \sqrt{6^2 + 2.5^2} = 6.5cm$
Surface area of:-

Curved part of original cone = $\pi 10\ (10 + 26) = 1130.97\text{cm}^2$.

Curved part of removed cone = $\pi 2.5\ (2.5 + 6.5) =$ 70.69cm^2

Circular ends= $\pi \times 10^2 + \pi \times 2.5^2 = 333.79\text{cm}^2$

Therefore, total surface area = $1130.97 - 70.69 + 333.79 = 1394.07\text{cm}^2$.

ACTIVITY 9.

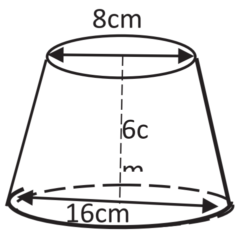


Figure 14

Figure 14 shows a solid frustum. Find the surface area in cm^2.

5. **Surface area of spheres.**
   Did you know that ball is an example of a sphere? Get a ball and identify the surface area.
   Name other spherical objects around you.
   Figure 15 shows a sphere and hemispheres.

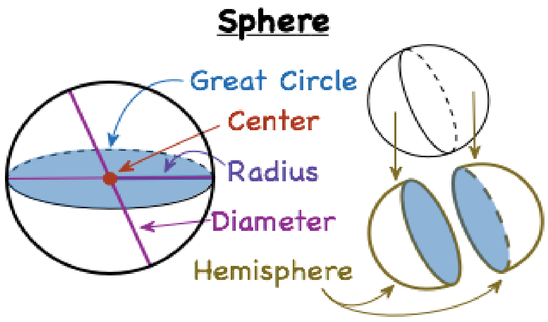


Figure 15

The surface **area of a sphere** of radius R is given by the formula $\mathbf{A = 4\pi R^2}$.

A sphere cut into two halves form two hemispheres.

The surface area of a hemisphere of radius R is $\mathbf{A = 3\pi R^2}$.

Example

Find the surface area of a sphere of radius 7cm.

$A = 4\pi R^2$

$= 4 \times \pi \times 7^2 = 615.75\text{cm}^2$.

ACTIVITY 10

1 Find the surface area of a sphere of radius 3m.

2. Find the surface area of a solid hemisphere of radius 12cm.

LESSON 3: CALCULATING THE VOLUME OF SOME FIGURES (E.G. CUBES AND PYRAMIDS)

LEARNING OUTCOME.

By the end of this lesson, you should be able to calculate the volume of some figures.

INTRODUCTION

You are now going to find the volume of some solids. This is the space inside or occupied by the solids.

1. **Volume of prisms.**
   We now know what a prism is.
   In general, the volume of any prism is **V = area of cross section X length**.

EXAMPLE

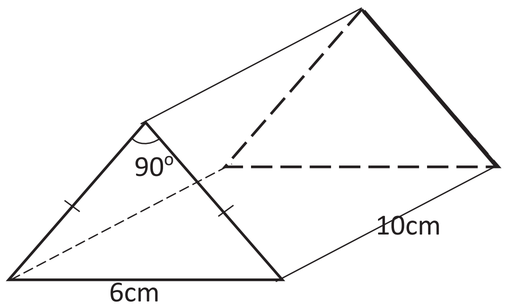


Figure 16

Figure 16 *shows a triangular prism. Find the volume of the prism.*

Let the height of the triangular cross-section be h.

$tan45^o = \frac{3}{h}$

$1 = \frac{3}{h} \therefore h = 3cm$

Area of cross-section = $\frac{1}{2} \times b \times h = \frac{1}{2} \times 6 \times 3 = 9cm^2$

Volume, **V = area of cross section X length**

= 9 X 10 = 90cm^2.

ACTIVITY 11

Find the volume of a right circular prism (cylinder) of radius 8cm and height 14cm.

2. **Volume of cones and pyramids.**

   Note that a cone is a circular pyramid.

   Volume of a pyramid, $\boldsymbol{V = \frac{1}{3} \times base\ area \times vertical\ height}$.

EXAMPLE

A rectangular pyramid has a base of length 5cm and width 3cm. If the vertical height is 6cm, find the volume.

$$\boldsymbol{V = \frac{1}{3} \times base\ area \times vertical\ height}$$

$$= \frac{1}{3} \times (5 \times 3) \times 6 = 30cm^3$$

ACTIVITY 12

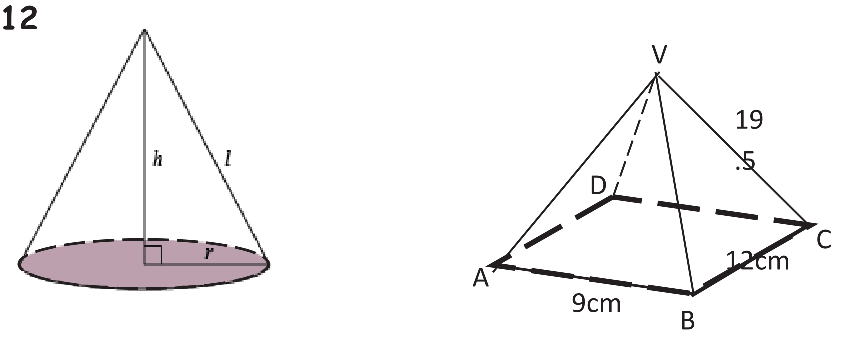


Figure 17

(I) (II)

Figure 17 shows two pyramids (I) and (II). Find the volume of each.

3. **Volume of frustum**

   You may recall the work that we did when finding the surface area of a frustum. The work involved completing the original cone of the frustum. We will have to do the same here.

   So that the volume of a frustum, **V = (volume of original cone) – (volume of removed cone)**.

EXAMPLE

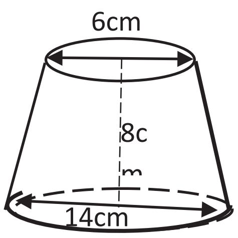


Figure 18

Figure 18 shows a right circular frustum. Find the volume.

We take a vertical section and complete the cone as shown in figure 19.
Let the height of the removed cone be *x*.

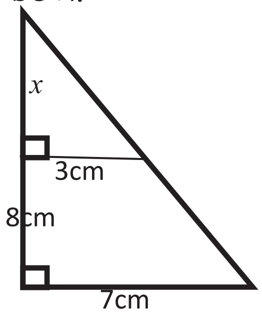


Figure 19

We now find *x* by using similar triangles.

$$\frac{x}{8+x} = \frac{3}{7}$$

$7x = 24 + 3x$

$4x = 24$

$x = 6cm$

Volume of original cone $V = \frac{1}{3} \times \textbf{\textit{base area}} \times \textbf{\textit{vertical height}}$

$= \frac{1}{3} \times \pi \times 7^2 \times 14 = 718.38cm^3$

Volume of removed cone $= \frac{1}{3} \times \pi \times 3^2 \times 6 = 56.55cm^3$

Volume of frustum = 718.38 – 56.55 = 661.83cm^3

ACTIVITY 13

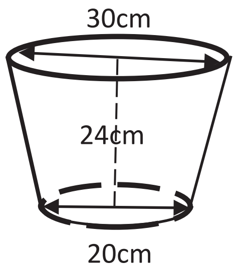


Figure 20

Figure 20 shows a small bucket in the shape of a hollow frustum. If it is to be completely filled with water, find the volume of water required in litres.

4. Volume of spheres.

The volume of a sphere of radius R is given by the formula $\boldsymbol{Volume} = \frac{4}{3}\pi R^3$.

ACTIVITY 14

Use the formula for volume of a sphere to find the expression for the volume of a hemisphere.

EXAMPLE

Find the volume of a sphere of radius 12cm.

$Volume = \frac{4}{3}\pi R^3$.

$= \frac{4}{3} \times \pi \times 12^3 = 7238.23cm^3$

ACTIVITY 15

1. Find the volume of a sphere of radius 2m.
2. Calculate the radius of a solid hemisphere whose volume is $19{,}404cm^3$.
3.

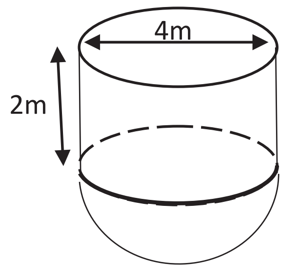


Figure 21

Figiure 21 shows an open tank made of a hollow cylindrical shell joined to a hollow hemispherical shell of diameter 4m. Find the volume of the tank in litres.

END